TRANSPORTATION

	D_1	D_2	...	D_n	Supply
F_1	c_{11}, x_{11}	c_{12}, x_{12}	...	c_{1n}, x_{1n}	a_1
F_2	c_{21}, x_{21}	c_{22}, x_{22}	...	c_{2n}, x_{2n}	a_2
$\vdots$	$\vdots$	$\vdots$		$\vdots$	$\vdots$
F_m	c_{m1}, x_{m1}	c_{m2}, x_{m2}	...	c_{mn}, x_{mn}	a_m
Demand	b_1	b_2	...	b_n	

No. of variable $= m \times n$ $\rightarrow 5 \times 4$

No. of equation $= m + n - 1$

$^{n}C_{m}$ ($n \rightarrow$ no. of variable, $m \rightarrow$ no. of eq^n) $\rightarrow$ $^{20}C_8 = 125970$

$$\text{min } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$$

$$\boxed{\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j}$$

The aim of transportation problem is to meet the demand and supply requirement in the most optimum and effective manner to minimize total transportation cost.

Feasible Solution :- A set of non -ve individual allocation which satisfy all the given constraint is called as feasible solution.

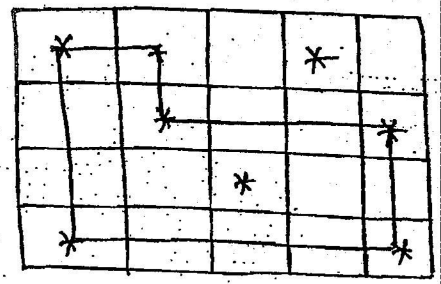
Basic feasible solution :- In a $m \times n$ transportation problem if the total no. of allocation is exactly equal to $m+n-1$ then the sol^n is called basic feasible solution.

Non-degenerate basic feasible solution :-

For $n \times m$ transportation problem sol^n is called Non degenerate when the following 2 conditions are satisfied -

i) total no. of allocation exactly equals to $m+n-1$

ii) these $m+n-1$ allocations must be at independent positions.



Dependent → can form loop

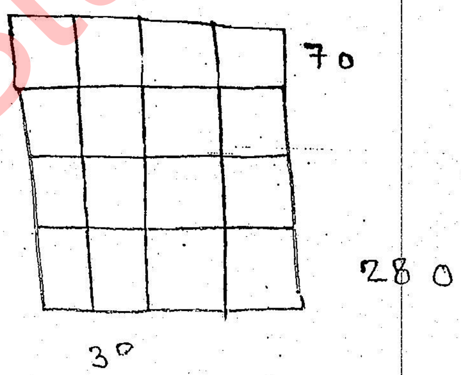
By independent position we mean that it is always impossible to form a closed loop by joining these allocation by the series of horizontal and vertical line from one allocated cell to another.

Note :- optimility test can only be performed when initial sol^n is non-degenerate.

Balanced and Unbalanced Transportation Problem :-

Unbalance → balance

→ add dummy



If the total supply from all the factory is equal to total demand from all the destination problem is called balanced otherwise unbalanced. If the problem is unbalanced, balance it by dummy source ~~and~~ or destination.

Q →

Find the optimum allocation to minimise total transportation cost.

	1	2	3	4	
P_1	20	30	50	17	7
P_2	70	35	40	60	10
P_3	40	12	60	25	18
	5	8	7	15	35

i) Initial solution :-

1) North west corner Rule :-

20 (5)	30 (2)	50	17	7/2/0
70	35 (6)	40 (4)	60	10/4/0
40	12	60 (3)	25 (15)	18/15
5	8/6/0	7/3/0	15	

$Z = 20 \times 5 + 30 \times 2 + 35 \times 6 + 40 \times 4 + 60 \times 3 + 25 \times 12$

$= \text{Rs } 1085$

ii) Row Minima

20	30	50	17 (7)	7/0
70	35 (8)	40 (2)	60	10/2/0
40 (5)	12	60 (5)	25 (8)	18/10/5/0
5/0	8/0	7/5/0	15/8/0	

$Z = 17 \times 7 + 35 \times 8 + 40 \times 2 + 40 \times 5 + 60 \times 5 + 25 \times 8$

$= 1179$ Rs

iii) Column - Minima method :

20 (5)	30	50	17 (2)	7/2/0
70	35	40 (7)	60 (3)	10/3/0
40	12 (8)	60	25 (10)	18/10/0
5/0	8/0	7/0	15/13/3/0	

$Z = 20 \times 5 + 12 \times 8 + 40 \times 7 + 17 \times 2 + 60 \times 3 + 25 \times 10$

$=$ Rs 940

iv) Least Cost Method or Method of Matrix Minima :

20	30	50	17 (7)	7/0
70 (3)	35	40 (7)	60	10/3/0
40 (2)	12 (8)	60	25 (8)	18/10/2/0
5/3/0	8/0	7/0	15/8/0	

$Z = 70 \times 3 + 40 \times 2 + 17 \times 7 + 40 \times 7 + 25 \times 8 + 12 \times 8$

$=$ Rs 985

v) Vogel's Approximation Method (VAM)

(OR)

Unit cost Penalty

20 5	30	50	17 2	7/2/0	3	13	33	33
70	35	40 7	60 3	10/3/0	5	5	20	20
40	12 8	60	25 10	18/10/0	13	13	35 ←	
5/0	8	7/0	15/5/3/0					
20 ↑	18	10	8					
	18 ↑	10	8					
		10	8					
		10	43 ↑					

Z = TC = Rs 940

In this method we write the difference between smallest and second smallest element in each row and column and write them below the respective row and column. Then we select the highest individual difference and the max possible allocation is done in the minimum cost cell of selected row or column. The row or column whose requirement become zero is striked off so that it cannot be considered again. Continue in the similar manner until all the allocations are done.

ii) Optimality :-

As the total no. of allocation is exactly is equal to m+n-1 = 6 and is at independent location. So optimality test can be performed

i) Stepping Stone :-

20 (5)	30	50	17 (2)
70 (+)	35 (3)	40 (7)	-3 (−) 60 0 = (3)
40	-3 12 (−) (8)	60	+3 25 (10) (+)

$+60 - 25 + 60 - 40 = +55$

$+40 - 20 + 17 - 25 = +12$

$+30 - 17 + 25 - 12 = +26$

$+35 - 60 + 25 - 12 = -12$

In this method we allocate 1 unit in an unallocated empty cell and compute the effect on the cost of matrix. It is a hit and trial method in which chances of making error are more.

ii) Modified Distribution (MODI - method)

or

UV Method

The steps involved are :-

1) Develop cost matrix for allocated cells only

$v_j \longrightarrow$

	v_1	v_2	v_3	v_4
u_1	20			17
u_2			40	60
u_3		12		25

$u_i \downarrow$

$v_1 = 0$

$u_1 + v_1 = 20$

$u_1 + v_4 = 17$

$u_2 + v_3 = 40$

$u_2 + v_4 = 60$

$u_3 + v_2 = 12$

$u_3 + v_4 = 25$

2) computing u_i and v_j values by taking $v_1 = 0$

	0	-16	-23	-3
20	20			17
63			40	60
28		12		25

3) Develop u_i and v_j ~~for~~ unallocated cells by entering the sumation of u_i and v_j value for unallocated cell

	4	-3	
63	47		
28		5	

4) Substract the cell values of u_i and v_j values for unallocated cell from the original cost matrix to get cell evaluation matrix

	26	53	
7	-12		
12		55	

5) If ~~any of~~ the cell value in the cell evaluation matrix is negative then the current solution is not optimum

6) In the cell evaluation matrix identify the cell with a most negative value, mark it and it is termed as identified cell.

7) Trace a path in the matrix such that it start from the identified cell and corner of the path should already have a allocation. Make identified cell as +ve and each other cell at the corner of the path alternatively -ve, +ve, -ve and so on.

8) Make a new allocation in the identified cell by entering the smallest allocation on the path that has been assigned the -ve sign. The basic cell whose allocation become 0 leave the solution

	26	53	
7	+3 −12		−3
12	−3	55	+3

- $u_i + v_j$ matrix ~~for unallocated~~ cell →

	0	−16	−11	−3
20	~~20~~	4	9	~~17~~
51	51	~~35~~	~~40~~	48
28	28	~~12~~	17	~~25~~

- cell evaluation matrix →

	26	41	
19			12
12		43	

Special case :-

1) Degeneracy :- when the no. of allocation become less than m+n-1 then optimality test cannot be performed and such a solution is called degenerate and the condition is known as degeneracy.

ii) Maximisation Problem :-

maxim → Min

40	80	50	90
100	30	70	20
70	60	10	60

60	20 (24)	50	10 (26)
0 (25)	70	30 (40)	80
30	40 (16)	90	40

Let

then $z = 24 \times 80 + 26 \times 90 + 25 \times 100 + 40 \times 70 + 16 \times 60$

Maximisation problem are solved by converting it into minimisation. This is done by substracting from the highest element all the element of matrix

Q - Unit transportation cost in Rs are given in the cost matrix below. Determine the initial feasible solution using vogels Approx and find the optimum distribution possible for the company.

	Dummy		D	E	F	G				
	0	A	~~44~~	~~50~~	~~40~~	39 180	180/0	39	1	1	1
	0 60	B	42 90	~~51~~	54 20	~~53~~	170/80 110/20/0	9 42	9	11	1
	0	C	41	40 100	42 100	45	200/100	40	1	1	3
	60/0		90/0	100/0	120	180/0	550				
							490				

0	1	1	2	6
0	3	10	2	6
0	3		2	6
	1	10	2	6
			2	6
			2	6

$z = Rs\ 20080$

for optimality

no of allocation $\neq$ $(m+n-1)$

$$6 = 5+3-1 = 7$$

as the total number of allocation is 6 which is less than $(m+n-1) = 7$, so the current solution is degenerate.

Now allocating infinitely small but positive value ϵ at vacant minimum cost cell such that all allocation remain in an independent position. In the final solution we put $\epsilon = 0$

	0	10	12	-3	-42
42			~~39~~	39	0
42	42		54		0
30		40	42		

	D	E	F	G	H
A	44	50	40	39 180	* 0 ϵ
B	42 90	51 20	54 0 − 20	53	0 60
C	41	80 40 100	120 42 100	45	* 0

→ form loop ✗

Z = Rs 20080

Z = Rs 20060

$$\begin{array}{r} +51 \\ -54 \\ +42 \\ -40 \\ \hline -1 \end{array}$$

* Gate

ASSIGNMENT

conditions -

1) Square matrix ($m = n$)

2) x_{ij} ~~or~~ $= 0$ or 1

all $a_i = 1$

all $b_j = 1$

	X		
	X		
X	$\square$	X	X
	X		

$\square$ → Allocation ($x_{ij} = 1$)

X → Non Allocation ($x_{ij} = 0$)

No. of variable $= n^2$

No. of eq^n $= 2n - 1$

$$\text{Min } z = \sum_{i=1}^{n} \sum_{j=1}^{m} c_{ij} x_{ij}$$

Assignment problem is a special case of transportation where $m = n$ and in every row and every column only one allocation is possible.

Q- 4 technitian are req. to perform 4 diff. job whose time are given in table below in hour. Assign the job to technitian to minimise total work time.

20	36	31	37
24	34	45	22
22	45	38	18
37	40	35	28

steps involved are :- a) Develop opportunity ~~appox~~ cost matrix.

i) Substract the smallest element in each row from every element of the corresponding row.

0	16	11	7
2	12	23	0
4	27	20	0
9	12	7	0

ii) Substract the smallest element in each column from every element of the corresponding column.

0	4	4	7
2	0	16	0
4	15	13	0
9	0	0	0

opp. cost matrix.

b) make allocation in the opportunity cost matrix

$\boxed{0}$	4	4	7
2	$\boxed{0}$	16	$\cancel{0}$
4	15	13	$\boxed{0}$
9	$\cancel{0}$	$\boxed{0}$	$\cancel{0}$

$Z = 20 + 34 + 18 + 35$

$= 107$ hr

* If the total no. of allocation is exactly equal to size of matrix then the current sol^n is optimum otherwise perform optimality.

Q → Solve the following assignment problem for minimization.

20	30	40	50
40	50	60	70
70	80	90	80
30	50	80	40

Applying row & column

$\cancel{0}$	$\boxed{0}$	$\cancel{0}$	20
$\cancel{0}$	$\cancel{0}$	$\boxed{0}$	20
$\cancel{0}$	$\cancel{0}$	$\cancel{0}$	$\boxed{0}$
$\boxed{0}$	10	30	$\cancel{0}$

$Z = 30 + 60 + 80 + 30$

$= 200$ Rs.

Q → Solve the following assignment problem for minimisation.

9	22	58	11	19	27
43	78	72	50	63	48
41	28	91	37	45	33
74	42	27	49	39	32
36	11	57	22	25	18
3	56	53	31	17	28

✓(4)	✓(2a)				✓(2b)	
~~0~~	13	49	$\boxed{0}$	~~0~~	13	
~~0~~	35	29	5	10	$\boxed{0}$	✓(3a)
13	~~0~~	63	7	7	~~0~~	✓(1)
47	15	$\boxed{0}$	20	2	~~0~~	
25	$\boxed{0}$	46	9	4	2	✓(3b) — x
$\boxed{0}$	53	50	26	4	20	✓

As the number of allocation is 5 which is less than the size of matrix, so the current sol^n is not optimum.

Now we proceed to find the minimum no. of line require to cover all zero at least once and the steps involved are –

i) Mark all row for which assignment has not been made. (3rd row)

ii) Mark all column which have unassigned zero in the marked row (2nd and 6th column).

iii) Mark all row which have assignment in the marked column (2nd and 5th row)

iv) Continue step 2 and 3 untill chain of marking is completed.

v) Draw the minimum no. of lines through unmarked row and marked column to cover all zero atleast once.

vi) Select the smallest element that do not have line through them substracted from all the element and add it to the element at the intersection of two line and leave the remaining element of matrix unchanged. Make allocation in the new opportunity cost matrix.

4	17	49	$\boxed{0}$	$\cancel{0}$	17
$\boxed{0}$	35	25	1	6	$\cancel{0}$
13	$\cancel{0}$	59	3	3	$\boxed{0}$
51	19	$\boxed{0}$	20	2	4
25	$\boxed{0}$	42	5	$\cancel{0}$	2
$\cancel{0}$	53	46	22	$\boxed{0}$	20

no. of allocation = n or m = 6

optimum =

$Z = 142$