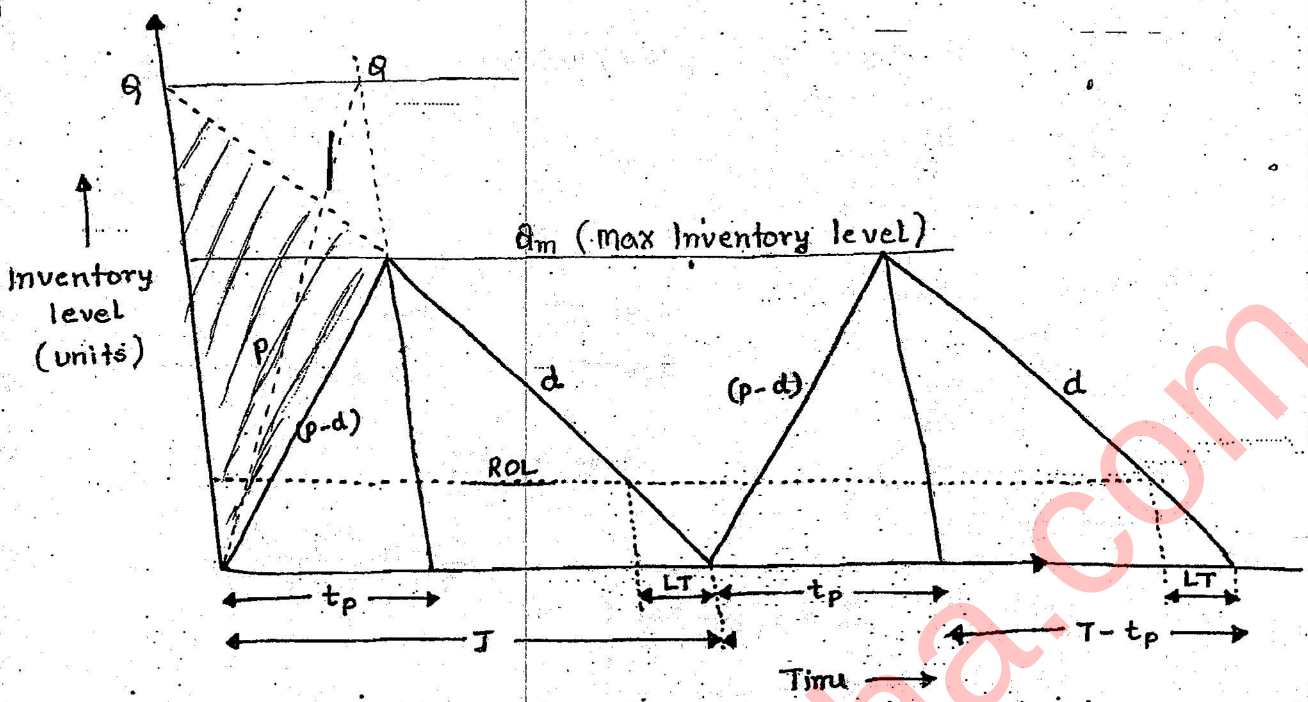
3) Production or Built-up Model :-



$p = 100$ units / day

$d = 20$ units / day

$(p-d) = 80$ units / day

LT → set up time

t_p → production cycle time

This model is similar to first model EOQ, the only difference is inventory built up is gradual rather than instantaneous.

p → production or built up rate

d → demand or consumption rate.

t_p → production or manufacturing cycle time.

Q → units per set up.

C_o → Rs / setup

$$\boxed{Q = t_p \cdot p} \qquad t_p = \frac{Q}{p}$$

$$Q_m = t_p (p-d)$$

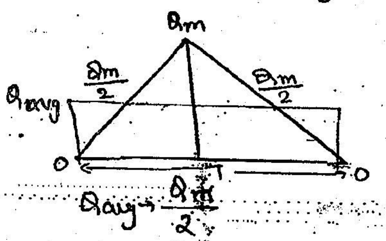
$$\boxed{Q_m = \frac{Q(p-d)}{p}} \qquad Q_m < Q$$

$T.I.C = \text{setup cost (SC)} + \text{Holding cost (HC)}$

$\rightarrow S.C = \text{No. of setup} \times \text{cost/setup}$

$$\boxed{S.C = \frac{D}{Q} \times C_0}$$

$\rightarrow HC = Q_{avg} \times C_h$



Area $\square$ = Area Δ

$Q_{avg} \times \cancel{T} = \frac{1}{2} \times Q_m \times \cancel{T}$

$Q_{avg} = \frac{Q_m}{2}$

$= \frac{1}{2} Q \left(\frac{p-d}{p}\right)$

$$\boxed{HC = \frac{Q}{2} C_h \left(\frac{p-d}{p}\right)}$$

$$\therefore \boxed{TIC = \frac{D}{Q} C_0 + \frac{Q}{2} C_h \left(\frac{p-d}{p}\right)}$$

for TIC to be minimum

$$\frac{d(TIC)}{dQ} = 0$$

$$\frac{C_h}{2}\left(\frac{p-d}{p}\right) = \frac{D}{Q^{*2}} C_0$$

$$\boxed{Q^* = \sqrt{\frac{2DC_0}{C_h}\left(\frac{p}{p-d}\right)}}$$

$\sqrt{\frac{p}{p-d}} \rightarrow$ production factor > 1

1st EOQ model is special case of $\uparrow$ model

$p \rightarrow \infty$

$$\sqrt{\frac{p\ 1}{1 - \frac{d}{p}}} = 1 \qquad \frac{d}{p} \rightarrow 0,\ p \rightarrow \infty$$

At EOQ $\rightarrow$

SC = HC

$$\boxed{TIC^* = \sqrt{2DC_0C_h\left(\frac{p-d}{p}\right)}}$$

$\rightarrow < 1$

Q → A company req. 12000 units of a product X in a year. ~~component~~ product X is made in 30 batches of 400 units each on a m/c that makes 8 unit per hour. Company operates for 4000 hr / year and req 500 Rs to set up the m/c. C_h = 15 Rs/unit/year. Find out wether the existing production plan is optimum. If not suggest the new plan and amount of saving possible. Also determine production cycle time, max inventory level and production run corresponding to optimum condition.

→ $D = 12000$ units

$Q = 400; \quad N = 30$

$p = 8$ unit/hr

C_o = Rs 500/set up

C_h = Rs 15/unit/year

4000 hr/year ÷

$$d = \frac{12000}{4000} = 3 \text{ units/hr}$$

i) $$Q^* = \sqrt{\frac{2DC_o}{C_h}\left(\frac{p}{p-d}\right)} = 1131.37 \text{ units/setup}$$

↳ cannot be in decimal

TIC^* = Rs 10606.66

$$N^* = \frac{D}{Q^*} = \frac{12000}{1131.37} = 10.6 \text{ setup/yr} \quad \times$$

so finding diff value of N for which N & Q will be whole No.

$N = 11, \quad Q = 1090.9$

$N = 12, \quad Q = 1000$

$N = 10, \quad Q = 1200$

a) at $N = 12$, $Q = 1000$

$$TIC = N \cdot C_o + \frac{Q}{2} C_h \left(\frac{p-d}{p}\right)$$

$$= 12 \times 500 + \frac{1000}{2} \times 15 \times \frac{5}{8} \Rightarrow \text{Rs } 10687.5$$

b) at N = 10 , Q = 1200

TIC = Rs 10625 ← (min)

→ Best order cost = Rs 10625

current policy – N = 30 , Q = 400

TIC = Rs 16875

Saving = 16875 – 10625

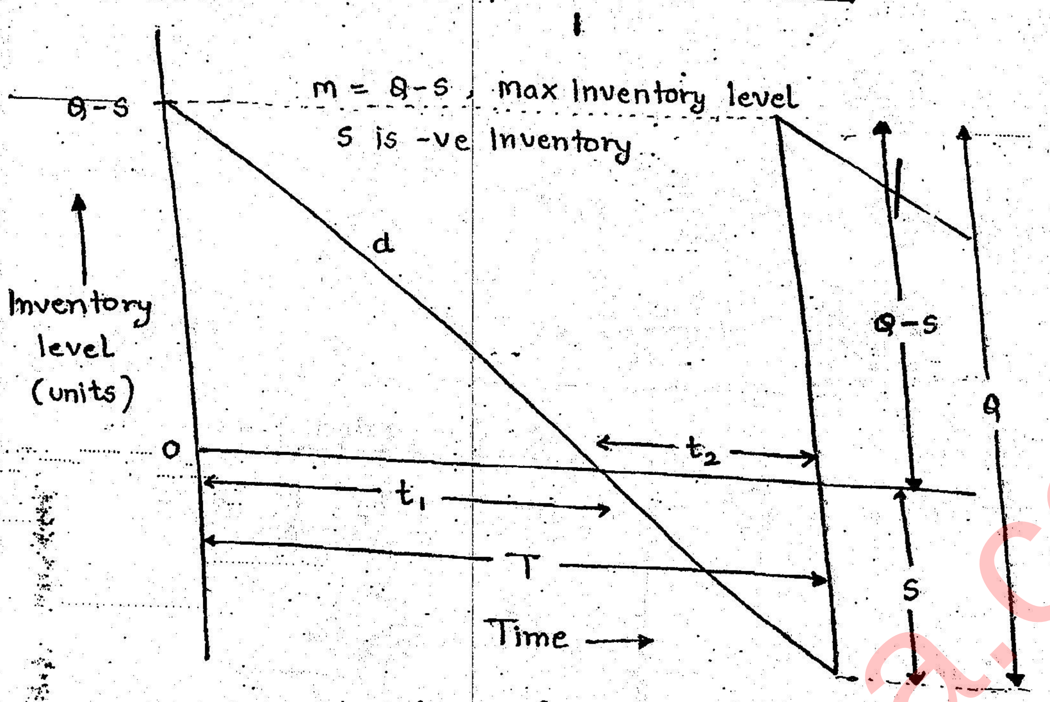
= Rs 6250

(b) – $t_p = \frac{Q}{P} = \frac{1200}{8} = 150$ ~~setup~~ hr/setup.

$Q_m = Q\left(\frac{P-d}{P}\right) = 750$ units/setup.

$T = \frac{Q}{d} = \frac{1200}{3} = 400$ hrs/setup

Shortage or Stockout or Backorder



This model is similar to first model EOQ. The only difference is that shortages are allowed. Planned shortage or backorder is ~~the condition when a~~ customer places an order and finds that inventory is out of stock, then he wait for next shipment to get his order fulfilled.

$S \rightarrow$ no. of unit short, or backordered

$C_b \rightarrow$ backorder or shortage cost / unit / year

TIC = OC + HC + Shortage cost (S.C)

$$\boxed{OC = \frac{D}{Q} C_o}$$

$$\boxed{HC = \frac{(Q-S)^2}{2Q} \times C_h}$$

$$\boxed{SC = \frac{S^2}{2Q} \times C_b}$$

HC for period T –

$$HC = \left(\frac{Q-S}{2}\right) t_1 \times C_h \quad \text{— (1)}$$

as $(Q-S) = t_1 d$

$Q = T.d$

on dividing -

$$\frac{t_1}{T} = \frac{Q-S}{Q}$$

$$t_1 = \left(\frac{Q-S}{Q}\right) \cdot T$$

putting in (1) –

H.C for period T –

$$= \frac{Q-S}{2Q} \times \frac{Q-S}{Q} \cdot T \cdot C_h$$

$$= \cancel{\left(\frac{Q-S}{Q}\right)^2 C_h T} \quad \frac{(Q-S)^2}{2Q} C_h T$$

$$\therefore \text{Annual H.C} = \cancel{\left(\frac{Q-S}{Q}\right)} \quad \frac{(Q-S)^2}{2Q} C_h T \cdot N \quad ; \; TN \rightarrow 1$$

S.C for period T –

$$SC = \frac{S}{2} t_2 C_b \quad \text{— (1)}$$

$$S = t_2 d$$

$$Q = T \cdot d$$

on dividing –

$$\frac{t_2}{T} = \frac{S}{Q}$$

$$t_2 = \frac{S}{Q} T$$

$$S = \frac{S^2}{2Q} \cdot C_b T$$

SC for period T –

$$S.C = \frac{S}{2} \cdot \frac{S}{Q} C_b T$$

$$SC = \frac{S^2}{2Q} C_b T$$

$$\text{Annual S.C} = \frac{S^2}{2Q} C_b (T.N) \rightarrow 1$$

$$\therefore \boxed{TIC = \frac{D}{Q} C_o + \frac{(Q-S)^2}{2Q} C_h + \frac{S^2}{2Q} C_b}$$

$$\boxed{Q^* = \sqrt{\frac{2DC_o}{C_h}} \sqrt{\frac{C_b + C_h}{C_b}}} \rightarrow \text{cost factor} > 1$$

$$C_b \rightarrow \infty$$

$$\sqrt{1 + \frac{C_h}{C_b}} \quad \frac{C_h}{C_b} \rightarrow 0, \; C_b \rightarrow \infty$$

$C_b \neq 0$ if so there is no use of keeping inventory.
there is no loss when inv. is zero.

$$C_b >>> C_h$$

At EOQ –

$$\boxed{OC = HC + SC}$$

$$\boxed{TIC^* = \sqrt{2DC_oC_h} \sqrt{\frac{C_b}{(C_b + C_h)}}} \rightarrow < 1$$

optimum no. of unit short or Backorder (S^*) :-

$$(Q^* - S^*) \times C_h = S^* \times C_b$$

TIC → partial diff. wrt Q & T

$$\frac{Q^* - S^*}{S^*} = \frac{C_b}{C_h}$$

adding 1 both side

$$\frac{Q^*}{S^*} = \frac{C_b + C_h}{C_h}$$

$$\boxed{S^* = Q^* \left(\frac{C_h}{C_b + C_h}\right)}$$

max inventory = $M^* = Q^* - S^*$

$$\boxed{M^* = Q^* \left(\frac{C_b}{C_b + C_h}\right)}$$

Q - A dealer supplies following information D = 10 000 units ; C_o = Rs 10/order ; C_h = 20% of C ; C = 20 Rs/unit.
Dealer is considering the possibility of back ordering and he has estimated that the annual cost of backordering is 25% of unit price. Determine -

a) best order size

b) No. of unit backorder or short.

c) would you recommend for backordering if so the annual cost saving by adopting the policy of backordering.

→

$$C_h = \frac{20}{100} \times 20 = 4$$

$$C_b = \frac{25}{100} \times 20 = 5$$

$$Q^* = \sqrt{\frac{2DC_o}{C_h}} \sqrt{\frac{C_b + C_h}{C_b}}$$

$$= \sqrt{\frac{2 \times 10000 \times \cancel{10} \times 9}{{}_2\cancel{4} \times 5}} \rightarrow 300$$

b) $S^* = Q^* \left(\frac{C_h}{C_b + C_h}\right)$

$= \frac{300 \times 4}{9} = \frac{400}{3} = 133.33$

c) With backorder –

$TIC^* = \sqrt{2 D C_o C_h} \sqrt{\frac{C_b}{C_b + C_h}}$

$=$ Rs 666.67

without backorder –

$TIC^* = \sqrt{2 D C_o C_h}$

$=$ Rs 894.42

→ Combination of Production and Shortage model –

$$\boxed{Q^* = \sqrt{\frac{2 D C_o}{C_h}} \sqrt{\frac{P}{(P-d)}} \sqrt{\frac{C_b + C_h}{C_b}}}$$

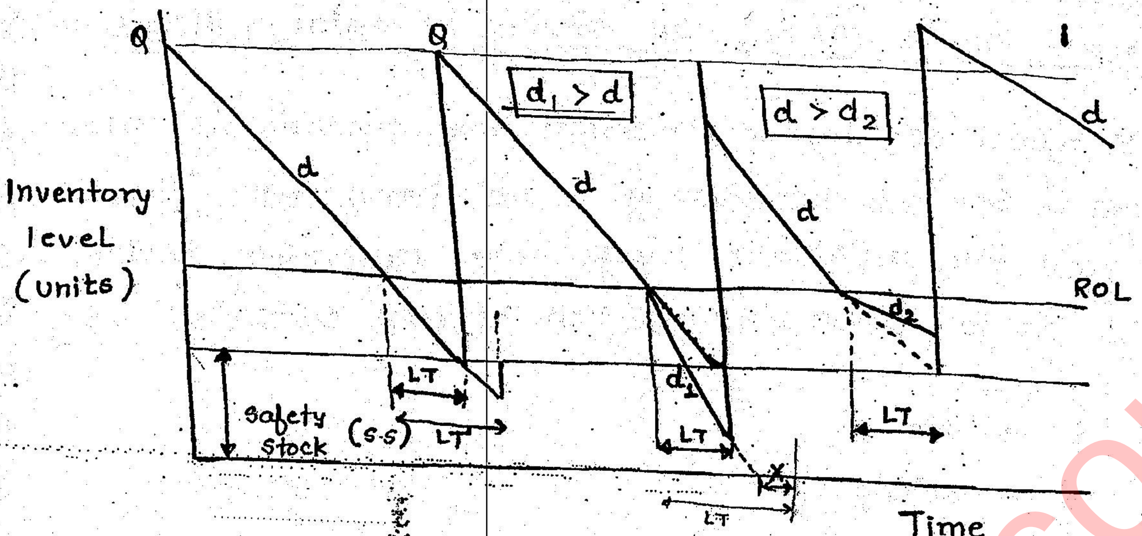
$$\boxed{TIC^* = \sqrt{2 D C_o C_h} \sqrt{\left(\frac{P-d}{P}\right)} \sqrt{\frac{C_b}{C_b + C_h}}}$$

a) case 1 : $P \rightarrow \infty$, Shortage

b) case 2 : $C_b \rightarrow \infty$, production

c) case 3 : $P \rightarrow \infty$ and $C_b \rightarrow \infty$, EOQ

5. Probablistic Model.



Factors encouraging higher s.s →

1) when the demand rate and LT variation are more at fluctuating
2) when the inventory holding cost is less and is not of much concern
3) when the loss due to absence of inventory i.e. shortage cost is very high.
4) when the no of orders in the years are more
5) To have better customer satisfaction

ROL = Average Demand during LT + Safety Stock (SS)

$$\boxed{ROL = LT \times d + SS}$$

$$Q_{max} = Q + SS$$

$$Q_{min} = S.S$$

$$Q_{avg} = \frac{Q}{2} + S.S$$

II Probabilistic

1) Demand - Profit model or static Inventory model -

In this model demand is uncertain and decision is based upon single order i.e. reordering is not permitted. This model is used for perishable items like vegetables, fruits, flowers etc, or for those items which become outdated very fast.

$D \rightarrow$ Demand

$S \rightarrow$ supply

$p \rightarrow$ profit / unit

$l \rightarrow$ loss / unit

case 1 $\rightarrow$ if $D > S \rightarrow \boxed{(D-S)\,p}$ potential profit loss

case 2 $\rightarrow$ if $S > D \rightarrow \boxed{(S-D)\,l}$

where,

$$p = S_p - C + C_b$$
$$l = C - C_s + C_h$$

where $p \rightarrow$ potential profit loss /unit for not meeting the demand

$S_p \rightarrow$ S.P per unit

$C \rightarrow$ Purchasing cost per unit

$C_b \rightarrow$ backorder or shortage loss per unit

$l \rightarrow$ unsold item loss per unit.

$C_s \rightarrow$ ~~Savl~~ Salvage or scrap value $\rightarrow$ product have some fixed value left even after its validity

$C_h \rightarrow$ holding cost per unit.

In this model in order to maximise our profit we select the ordering quantity (S) in such a manner that -

$$\boxed{P(S-1) < \frac{p}{p+l} \leq P(S)}$$

where $P(s-1) \rightarrow$ cummulative probability for the demand of $(s-1)$ unit

$P(s) \rightarrow$ cummulative prob. for the demand of (s) unit.

Demand	Probability	Comm. Prob
1	0.07	0.07
2	0.10	0.17
3	0.09	0.26
4	0.13	0.39
5	0.10	0.49
6	0.08	0.57
		⋮
		1

let $\frac{p}{p+h} = 0.46$

$P(s) = 0.49$

$P(s-1) = 0.39$

$\therefore s = 5$

by ordering 5 unit we will have max profit.

* Special case –

if $c = c_s = s_p$

$p = c_b$, $h = c_h$

$$\boxed{P(s-1) < \frac{c_b}{c_b + c_h} \leq P(s)}$$

Q – The shopkeeper purchases seasonal product at the beginning of the season and cannot reorder. Item cost 200 and sold at Rs 350 each. For any item that cannot meet on demand he had estimated a good will loss of Rs 50. Any item unsold will have a salvage value of Rs 100 and the holding cost during the period is 10% of unit price. Find optimised no. of product to maximise the profit.

→

Demand	Probability	Cum. Prob.
1	0.07	0.07
2	0.10	0.17
3	0.08	0.25
4	0.09	0.34
5	0.13	0.47
6	0.12	0.59
7	0.11	0.70
8	0.06	0.76
9	0.09	0.85
10	0.11	0.96
11	0.04	1

0.70 → $p(s)$

$P = 350 - 200 + 50$

$P = 200$

$d = 200 - 100 + 20 = 120$

$\frac{200}{320} = 0.625$

Q - Find the shortest cost range when the holding cost is Rs 2 and the demand and prob. distribution is as given below with optimum stock level of 7 unit.

Demand	Prob	Cum. Prob
1	0.06	0.06
2	0.09	0.15
3	0.14	0.29
4	0.10	0.39
5	0.11	0.50
6	0.16	0.66
S 7	0.05	0.71
8	0.14	0.85
9	0.09	0.94
10	0.06	1.0

$C_h = 2$

$p(s) = 0.71$

$p(s-1) = 0.66$

$$\underbrace{0.66 < \frac{C_b}{C_b + C_h} \leq 0.71}$$

Rs $3.88 < C_b \leq$ Rs 4.89

2 - Service Level Model :-

— This model is preferred where the different cost factor involve with inventory or not known exactly. It is based upon probability theory and the amount of S.S is kept a/c to the level of service management wants to achieve.

$$\boxed{\text{Service Level} = \left(\frac{\text{Number of units supplied without delay}}{\text{Total number of units demanded}}\right)_{LT}}$$

S → 0 to 1 or 0 to 100%

95% service level is the standard value and it means that 95% of the customers order on an average is fulfilled during LT and only 5% of the customers order on an average are rejected during LT.

When the demand during Lead Time may be aproximated by an normal distribution with certain average ($\bar{x}$ or μ) and standard deviation (σ), then ROL is given by

$$\boxed{ROL = \bar{X} + Z\sigma}$$

where $SS = Z\sigma$

where,

$\bar{X}$ → average demand during LT.

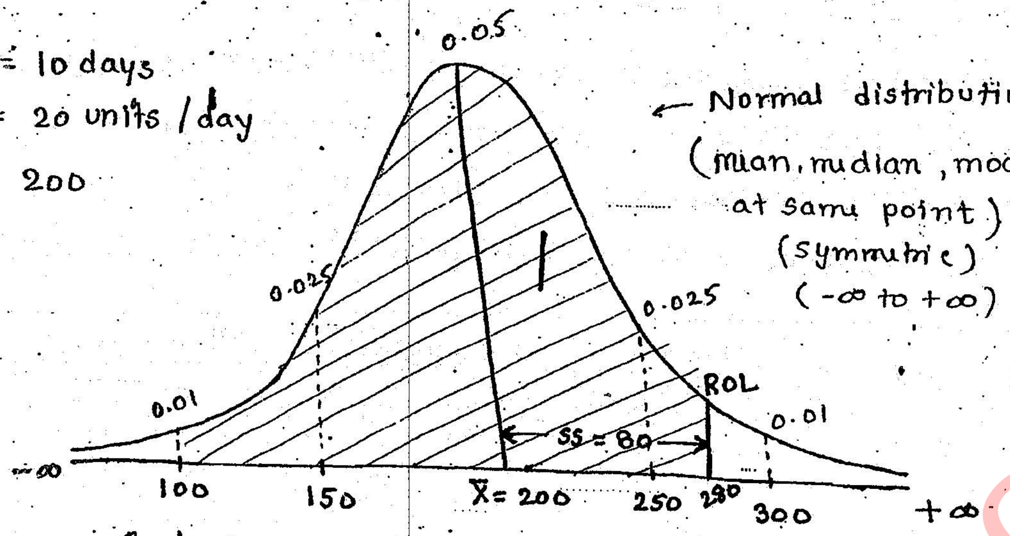
$\bar{X} = LT \times d$

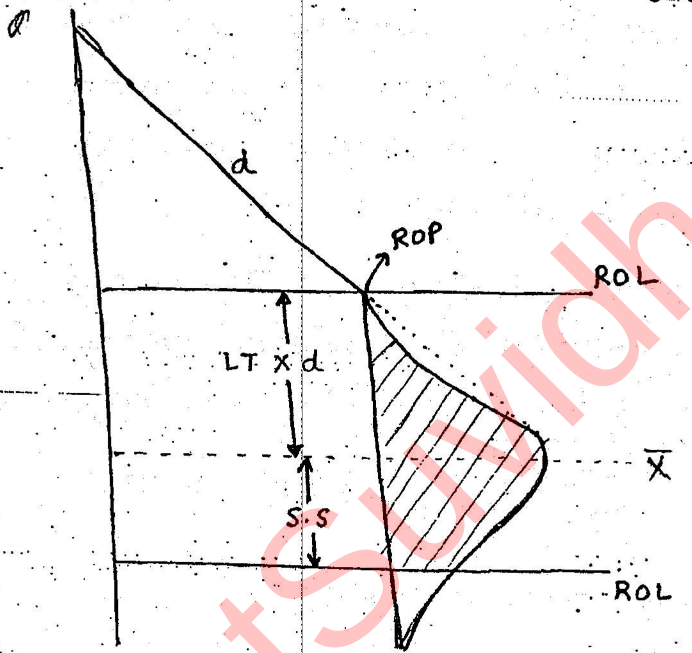
σ → S.D for the demand variation during LT

Z → Standard normal variate whose value depend upon the service level required.

Z		Service level (%)
0.84	——	80%
1.24	——	90%
* → 1.645	——	95%
2.33	——	99%

LT = 10 days
d = 20 units / day
$\bar{X} = 200$





S.D → std. deviation → σ → deviation from the mean value.

x_1, x_2, x_3

$$\bar{x} = \frac{x_1 + x_2 + x_3}{3} \quad ; \quad \sigma = \sqrt{\frac{(x_1-\bar{x})^2 + (x_2-\bar{x})^2 + (x_3-\bar{x})^2}{3}}$$

(more S.S) ∵ more fluctuation
more variation

$S.S = \underset{\text{const.}}{z}\,\sigma$ → main parameter for S.S

more 100

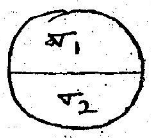
S.S 20 → $\mu = 60$	70, 50 → $\mu = 60$
80, 40 → $\mu = 60$	60, 60 → $\mu = 60$

Note :- L.T is 1 complete cycle and σ should be always corresponding to LT while computing S.S.

Let 1 complete cycle consist of 2 parts –

1st half $= \sigma_1$

2nd half $= \sigma_2$



$\sigma^2 = \sigma_1^2 + \sigma_2^2$

$\sigma = \sqrt{\sigma_1^2 + \sigma_2^2} \rightarrow$, $SS = z \cdot \sigma$

Q → Average weakly demand is of 800 units and weakly S.D is of 100 unit. Holding cost is Rs 0.2 /unit/ ~~month~~ weak with the lead time of 4 weak. Unit price of inventory is Rs 40. Then for 95% service level determine –

i) Safety stock

ii) ROL

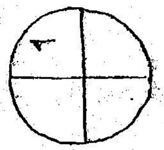
iii) Annual cost of maintaining S.S.

→ $z = 1.64$

~~$\bar{X}$ = 800/unit/weak~~ $d = 800$ unit/weak.

$\sigma = 100$ unit / weak, $\sigma = 100 \times 4 = 400$ unit

$LT = 4$ weak



i) $SS = z \cdot \sigma$

$= 1.645 \times 100 = 164.5$ unit

ii) $ROL = \bar{X} \cdot$ ~~LT~~ $+ SS$

$\bar{X} = LT \times d$

$=$ ~~800 + 164.5~~

$= (4 \times 800) + 164.5$

$= 3200 + 164.5 = 33$

i). As LT is of 4 weak and σ is given weekly so converting, σ corresponding to LT:

$$\sigma'^2 = \sigma^2 + \sigma_3^2 + \sigma^2 + \sigma^2$$

$$\sigma'^2 = 4\sigma^2$$

$$\sigma' = 2\sigma = \underline{200 \text{ unit}}$$

$$S.S. = Z\sigma'$$

$$= 1.645 \times 200$$

$$= \underline{\underline{329 \text{ units}}}$$

ii) $ROL = (LT \times d) + S.S$

$$= (4 \times 800) + 329$$

$$= \underline{\underline{3529 \text{ unit}}}$$

iii) $SS \times C_h$

$C_h = 0.2/\text{unit}/\text{week} \rightarrow \times 52$ weak/year (Annual)

$C_h = 10.4/\text{unit}/\text{yr}$

$= 329 \times 10.4 \Rightarrow \underline{\underline{3421.6 \text{ Rs}}}$

Q → Actual demand → 220, 190, 210, 270, 180, 230

find S.S –

→ S.S = Max Dem. during LT – Avg. Demand

SS = 270 – 216.6

= $\underline{\underline{54}}$

Q- For the production system Annual demand is 20,000 units and cost of 1 unit is ₹40, C_o = ₹120/order, C_h = ₹1.2/unit/yr and past lead times are 15, 20, 18, 12, 22, 27 days. If there are 250 working days in a year then calculate –

i) EOQ

ii) S.S

iii) ROL

iv) Average stock in inventory.

→ $D = 20{,}000$ $\qquad d = \frac{20000}{250} = 80$ units/day

$C = 40$

$C_o = 120/\text{order}$

$C_h = 1.2/\text{unit/yr}$

i) $EOQ = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{\frac{2 \times 20000 \times 120}{1.2}} = 2000$

ii) $SS = \cancel{Z \times \sigma}$

= Max Dem. during LT – Avg Dem. during LT

= Max LT × d – Avg LT × d

= (27 × 80) – (19 × 80)

= 640 units

iii) ROL = (Avg LT × d) + SS

= (19×80) + 640 = 2160 units

iv) $Q_{avg} = \frac{Q^*}{2} + S.S$

= 1000 + 640

= 1640 units

Inventory Classification and Control

1) ABC control → (Pareto Law or 80-20 Law)

80 → budget %, 20 → Item %

	Usage %	Item %
A	50-60%	10-20%
B	30-40%	30-40%
C	10-20%	50-60%

Items	Item %	Demand	Price (c)	Usage Value (D·C)	Usage %	Usage % desc order	
1	10%	100	200	20000	$\frac{20000}{\Sigma x} \times 100$	32%	54% A
2	10%	60	70	4200	$\frac{4200}{\Sigma x} \times 100$	22%	
3	10%	200	400	80000	$\frac{80000}{\Sigma x} \times 100$	18%	33% B
4	10%					9%	
5	10%					6%	
⋮	10%					5%	13% C
10				Σx		4.5%	

In ABC control inventory items are classified into A, B and C category depending upon their usage value. For A category items inventory is kept almost nill and frequent review is done on the other hand for C category item large amount of inventory is kept and it is reviewed after a long period.

2) VED (Vital Essential and Desirable) :-

Inventory are classified on the basis of importance of inventory items for the production system.

3) HML (High, medium and Low) :-

Inventories are classified on the basis of unit price of inventory items.

4) SDE (scarce Difficult Easy) :-

Inventories are classified on the basis of availability of inventory items for the production system.

---

7) O.C = HC

$N \times 500 = 2000$

$N = 4$ order/yr

8)

D	Single order	cost
50	105 - 50 = 55	200
	55×1	+55
0	55×1	+ 55
15	55 - 15 = 40	+ 40
	40×1	+ 20
20	40 - 20 = 20	
	20×1	270
20	20 - 20 = 0	

when double ordering -

D	Double order	cost
50	65 - 50 = 15	100
	15×1	+ 15
0	15×1	+ 15
15	15 - 15 = 0	+ 100
20	40 - 20 = 20	+ 20
	20×1	250
20	20 - 20 = 0	

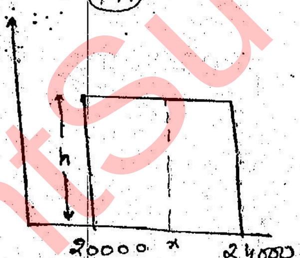
13 → $p = 1.2$/unit

$\lambda = 0.8$/unit

$\frac{p}{p+\lambda} = 0.6$

60% = service level

$P(S-1) < \frac{p}{p+\lambda} \leq P(S)$



Area = 1

$4000 \times h = 1$

$h = \frac{1}{4000}$

Area = 0.6

$(x - 20,000)\frac{1}{4000} = 0.6$

$x = 22400$ units

15 →

LT		freq	%	comm%
.5	0-1	4	5%	5%
1.5	1-2	8	10%	15%
2.5	2-3	20	25%	40%
3.5	3-4	24	30%	70%
4.5	4-5	16	20%	90%
5.5	5-6	4	5%	95%
6.5	6-7	4	5%	100%

S.S = (max LT − Avg LT) × d

max LT ⇒ 5 week

Avg LT = $\frac{0.5 \times 4 + 1.5 \times 8 + 2.5 \times 20 + \ldots}{80}$

Avg LT = 3.35

18 → 10 − 70%

SS ← 14 − 85%

16 − 90%

ROL = (LT × d) + SS

= 38 × 4 + 14 = 166