Inventory

Inventory can be termed as stock on hand at a given point of time which may be held for a purpose of later use or sale. It has an economic value and it may include raw material, work in process, semi finished or sub assembly and final product. In inventory control our aim is to manage inventory in such a manner that day to day working run smoothly without any delay but at min of cost.

Inventory Cost

- Purchase Cost
- Ordering or setup cost
- Holding or Carrying cost
- Shortage or stockout cost

Purchase cost :- It is the cost of purchasing inventory and it depend upon the quantity or bulk purchased.

$$\boxed{P.C = \text{No. of units} \times \text{cost per unit}}$$

Ordering cost :- when the inventory is purchased from outside the cost associated with bringing ~~sut~~ ~~shut down~~ inventory within production system is termed as ordering cost. It include cost of ~~tender~~, paper work cost, communication cost, processing cost, inspection cost, transportation cost etc.

Setup Cost :- when inventory is produced internally the cost associated with bringing shutdown production system again into starting position is termed as setup cost. It include maintenance cost of m/c, schedule chart preparation cost, cost of bringing~~ing~~ Raw material, arrangment of worker, tool, equipment etc.

$$O.C = No.\ of\ order \times cost/order$$

$$S.C = No.\ of\ \text{~~order~~ setup} \times cost/setup$$

Holding or carrying cost :- It is the cost associated with storing, keeping and maintaining inventory within production system. It include storage cost, handling cost, damage and depreciation cost, insurance cost, interest etc. This cost depends upon the quantity and period for which inventory is stored.

$$H.C = \text{Avg. Inventory for a period} \times \text{holding cost/unit/time}$$

Shortage or stockout : Shortage simply means absense of inventory and loss associated with not serving the customer ~~is termed as~~ shortage or stockout cost. It include pot. profit % loss, goodwill loss, fast transportatio cost, discount etc

$$S.C = \text{Avg. units short for a period} \times \text{Shortage cost/unit/time}$$

Inventory classification :-

i) **Transit or pipeline** →

Inventory cannot provide service while in transportation and such inventory is called transit or pipeline inventory

ii) **Buffer or Safety stock** →

1) d' = 15 units/day > d = 10 units/day > d'' = 6 units/day

2) LT' = 10 days > LT = 6 day > LT'' = 4 days.

conditions for using S.S.

$\boxed{ROL = 60\ units}$

S.S is not used

It is the reserve stock kept throughout the year and it is held for protecting against the fluctuation in the demand rate and lead time. It is not used under normal working condition and used only during adverse condition to prevent stock out.

Lead Time (LT) is the time gap b/w placing an order and inventory on hand so that it can be used or consumed.

iii) Seasonal Inventory :- The demand for these inventory item changes with seasonal variation.

iv) Anticipation Inventory :- These inventory are build up due to some anticipated demand in future like big selling forecast, govt. policy change, price hike, strike, shut down etc.

characteristics of inventory model :-

i) Dependent and Independent demand inventory item.

No. of cars demand /day ← Independent ← Sales & Marketing

↑ Forecasting

↓

$\boxed{\text{20 cars/day}}$

↓

100 tyres / day ← Dependent ← Production

↑ MRP

Dependent :- The demand for these items is directly related or linked to demand of any other item usually of higher level of which it becomes the part.

Independent :- The demand for these items is not directly related or linked to any other item. It is difficult to compute and is projected with the help of forecasting.

2) Inventory Review System :-

Review system

Perishable items

OC ↑ | SS ↑ H.C ↑

Q - system (Preferred more) (It gives warning before stock finishes)
or
Fixed order system
or
Reorder Level system
or
Two - bin system
(single Inventory & single order)

P - system
or
Fixed Period system
or
Period Review system
(multiple Inventory & single order)

S.S - system (combination of both)

eg - (S-S system)
or → optimum Replenishment policy

if fast Q -

ROL = 50 units ← Q

if slow P -

ROP = 45 days ← P

Ri-order point

d' = 15 d'' = 9

d = 10 units/day
LT = 5 day
ROL = 50 units
Q = 500 units / order

450	50

Fixed order System : In this system as inventory decreases to ROL of fresh order for fixed quantity is placed at that point. In this system size of order is fixed but the time of order is variable.

Fixed Period System :- In this system inventory is reviewed after a fixed period of time at the fresh order of variable quantity is placed at that point. In this system size of order is variable but the time of order is fixed.

3) **Deterministic and Probablistic** :-

Deterministic – In these model demand rate and LT remain fixed and const so we need not to carry S.S.

Probablistic – These model represent the real world condition where there is fluctuation in the demand rate and Lead time. In these model we need to carry S.S to prevent safety stock out during adverse condition.

Notation –

D → Annual or yearly demand of Inventory (unit/yr)

Q → Quantity to be order at each order point (unit/order)

N → No. of orders placed in a year.

$$\boxed{N = \frac{D}{Q}} \text{ order/year}$$

T → Time length of one inventory cycle or time gap b/w 2 sucessive order. (Year/order)

$$\boxed{T = \frac{1}{N}}$$ T.N = 1 (units should be same)

→ N = 4 order/yr.

T = 3 month/order

→ T = $\frac{1}{4}$ yr/order

$C \rightarrow$ cost of purchasing 1 unit of inventory (Rs/unit)

$C_o \rightarrow$ cost of placing 1 order (Rs/order)

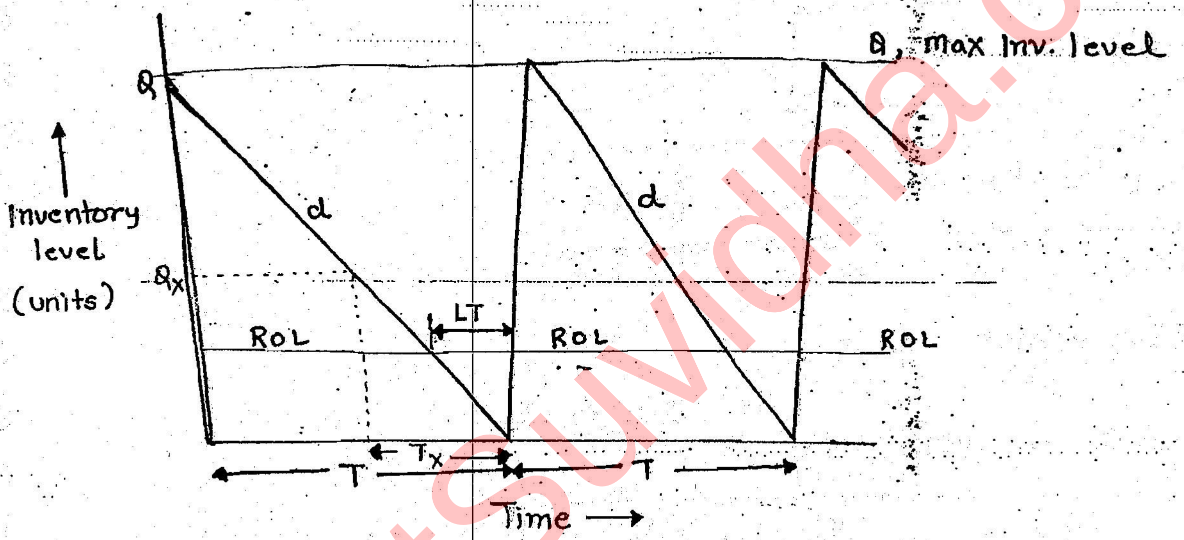
$C_h \rightarrow$ cost of holding 1 unit in inventory for 1 complete year (Rs/unit/year)

Deterministic -

1) Economic Order Quantity (EOQ)

Harris - wilson model

Infinite Rate of Replenishment



$Q_x = T_x d$

$Q = T \cdot d$

$\boxed{ROL = d \cdot LT}$

$$d = \frac{Q}{T} = \frac{Q_x}{T_x} = \frac{ROL}{LT}$$

Total Annual Cost -

TC = Purchase cost (PC) + Ordering cost (OC) + Holding cost (HC)

where, $PC = D \cdot C$

$OC = N \cdot C_o = \frac{D}{Q} C_o$

H.C for period T

ex, 20, 15, 10, 5, 0

C_h = Rs 2/unit/day

H.C = Rs 100

$Q_{avg} = \frac{50}{5} = 10$ units

$C_h = 2$

H.C = 10 x 5 x 2

= 100 ₹

when more interval

$Q_{avg} = \frac{20+0}{2} = 10$

$\therefore HC_{(T)} \rightarrow \frac{Q}{2} C_h T$

Annual HC = $\frac{Q}{2} C_h T \cdot N$ $\qquad T \cdot N = 1$

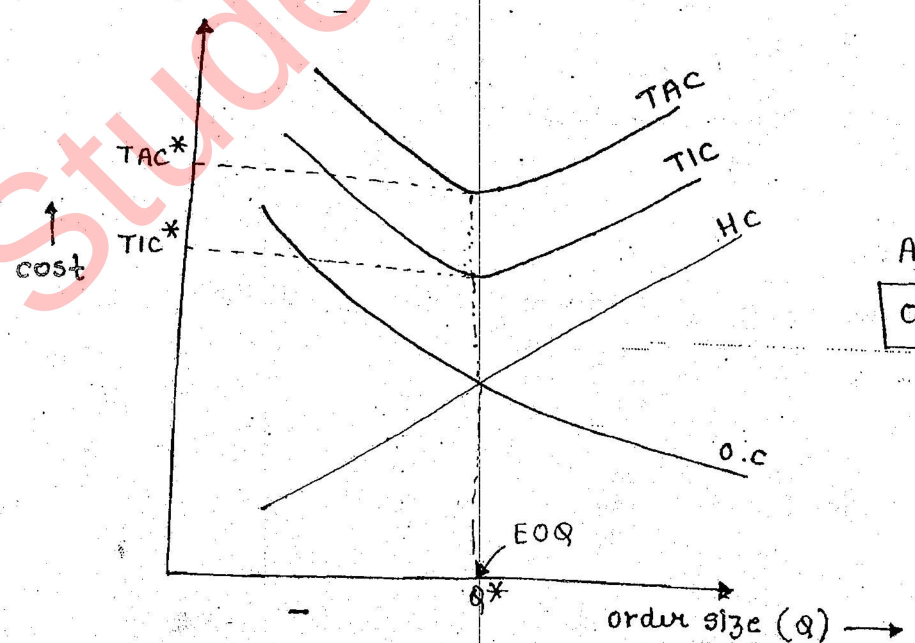
$$\boxed{\text{Annual HC} = \frac{Q}{2} C_h}$$

$$\therefore \boxed{TAC = \underbrace{DC}_{\text{const}} + \underbrace{\frac{D}{Q} C_o}_{Q\uparrow\, C_o\downarrow} + \underbrace{\frac{Q}{2} C_h}_{Q\uparrow\, C_h\uparrow}}$$

Total variable cost
or
Total inventory cost = O.C + H.C

$$\boxed{TIC = \frac{D}{Q} C_o + \frac{Q}{2} C_h}$$

$$\boxed{TAC = TIC + D.C}$$



At EOQ –

$$\boxed{OC = HC}$$

The ordering quantity Q^* at which holding cost becomes equal to ordering cost and the total Inventory cost is min. is known as Economic Order Quantity (EOQ).

At EOQ →

$$OC = HC$$

$$\frac{D}{Q^*} C_o = \frac{Q^*}{2} C_h$$

$$\boxed{Q^* = \sqrt{\frac{2 D C_o}{C_h}}} \qquad \boxed{TIC^* = \frac{D}{Q^*} C_o + \frac{Q^*}{2} C_h}$$

But as at EOQ –

$$\frac{D}{Q^*} C_o = \frac{Q^*}{2} C_h$$

$$TIC^* = 2 \cdot \frac{Q^*}{2} \cdot C_h$$

$$\boxed{TIC^* = \sqrt{2 D C_o C_h}} \qquad \boxed{TIC^*_{(Q)} = \frac{D}{Q} C_o + \frac{Q}{2} C_h}$$

only for eoq –   for EOQ or Non-EOQ

For TIC to be minimum –

$$\frac{dTIC}{dQ} = 0$$

$$\frac{C_h}{2} - \frac{D}{Q^{*2}} C_h = 0 \xrightarrow{2^{nd}\ diff} \frac{2 D C_o}{Q^3} \rightarrow +ve$$

$$Q^* = \sqrt{\frac{2 D C_o}{C_h}}$$

Note → when H.C is given in terms of interest or percentage it always correspond to unit price of inventory and the interest rate should be always yearly.

$$C_h = i\% \text{ of } C$$

($i\%$ → yearly)

eg – C = Rs 50/unit

$i\%$ = 1.5 % / month

$i\%$ = 18 % / year

$C_h = 0.18 \times 50 \Rightarrow$ Rs 9/unit/yr

Q - For a inventory, TIC at order size of 600 units and 1200 units are equal. Then det. 'EOQ'

→ $TIC(Q) = \frac{D}{Q} C_o + \frac{Q}{2} C_h$

$TIC(600) = TIC(1200)$

$$\frac{D}{600} C_o + 300 C_h = \frac{D}{1200} C_o + 600 C_h$$

$$D C_o \left[\frac{1}{600} - \frac{1}{1200}\right] = C_h (600 - 300)$$

$$\frac{D C_o \times \cancel{600}}{\cancel{600} \times 1200} = 300 C_h$$

$$2 \times \frac{D C_o}{C_h} = 300 \times 1200 \times 2$$

$$Q^{*2} = 600 \times 1200$$

$$Q^* = 848.52 \text{ unit/order}$$

if $TIC(Q_1) = TIC(Q_2)$ then $Q^* = \sqrt{Q_1 Q_2}$

Q → Determine EOQ value when annual demand is worth Rs 50,000. C_o is 4% of order value and C_h is 10% of avg Inventory value.

(value ↳ Rs)

→ $DC = 50,000$ Rs

$C_o = 4\% \, (Q^* c)$

$C_h = 10\% \, c$

$$Q^* = \sqrt{\frac{2 D C_o}{C_h}}$$

$$Q^* = \sqrt{\frac{2 \times 50000}{c} \times \frac{0.04 \, Q^* \cancel{c}}{0.1 \cancel{c}}}$$

$$Q^{*\cancel{2}} = \frac{40,000 \, \cancel{Q^*}}{c}$$

$$Q^* c = 40,000 \text{ Rs}$$

Q → The demand of soap for a shop keeper is 40 kg/day and he purchased it from retailer at the rate of Rs 50/kg. C_o = Rs 200/order, C_h = Rs. 0.1/kg/day, LT = 13 days and their current order policy is to order 200 kg every 5 day. Then det

i) EOQ

ii) Amount of saving with EOQ compared to current order policy in ~~1~~30 days.

iii) ROL corresponding to EOQ –

→ $Q = 200$ kg/order

$d = 40 \times \cancel{365}$ kg/year

@ D & C_h should be same unit

$$EOQ = \sqrt{\frac{2DC_o}{C_h}}$$

$$Q^* = \sqrt{\frac{2 \times 40 \times \cancel{365} \times 200}{0.1}} \Rightarrow \sqrt{160000} = \underline{\underline{400}}$$

(ii) $D = 30 \times 40 = 1200$ kg/month

$C_h = 0.1 \times 30 = 3$ Rs/kg/month

$Q = 200$ kg/order

$$TIC = \frac{D}{Q}C_o + \frac{Q}{2}C_h$$

$$= \frac{1200}{200} \times 200 + \frac{200}{2} \times 3$$

= 1500 Rs

EOQ ⇒ $Q^* = 400$ kg/order

$$TIC = \frac{1200}{400} \times 200 + \frac{400}{2} \times 3$$

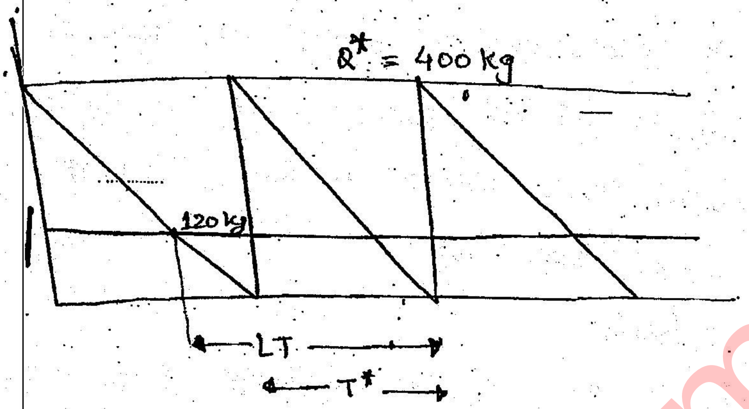
$= 1200$ RS

Saving = 300

3) ROL = LT × d

= 13 × 40

= 520 kg



$T^* = \frac{Q^*}{d}$

$T^* = 10$ days

$\boxed{LT > T^*}$

∴ Effective LT = LT − T*

= 13 − 10 = 3 days

ROL = 3 × 40 = 120 kg

Q→ For a production system −

D = 18000 unit/yr

c = Rs 8/unit

C_o = Rs 240/order

C_h = 12% of c

LT = 10 days

300 working day/yr

determine −

i) Q*

ii) N*

iii) T*

iv) TIC*

v) ROL

vi) No. of Days of stock at ROP

vii) Amount of saving with EOQ against earlier practice of 4 order in a year

viii) Increase in total cost associated with −

a) ordering 40% more than EOQ

b) ordering 40% less than EOQ.

1) $Q^* = \sqrt{\frac{2DC_o}{C_h}}$

$C_h = \frac{12}{100} \times 8$

$= \sqrt{\frac{2 \times 18000 \times 240}{0.96}}$

3000

ii) $N^* = \frac{D}{Q^*}$

$= \frac{18000}{3000} = 6$ order/yr

iii) $T^* = \frac{1}{N^*} = \frac{1}{6}$ yr/order. $\Rightarrow$ 50 days/order.

iv) $TIC^* = \sqrt{2DC_oC_h}$

$= \sqrt{2 \times 18000 \times 240 \times 0.96} = Rs\ 2880$

v) $ROL = LT \times d$

$d = \frac{18000}{300} = 60$ units/day.

$ROL = 10 \times 60 = 600$ units

$T = T^* \times N^*$

$T = 50 \times 6 = 300$

$d = \frac{Q}{T}$

vi) $LT = 10$ days

vii) $N = 4$ order/yr

$Q = \frac{18000}{4} = 4500$

$TIC = N.C_o + \frac{Q}{2} C_h$

$= 4 \times 240 + \frac{4500}{2} \times 0.96$

$= Rs\ 3120$

$\therefore$ Saving $= 3120 - 2880 = Rs\ 240$

viii) a) 40% more than EOQ –

$Q = 1.4 Q^* = 4200$ units/order

$TIC = \frac{18000}{4200} \times 240 + \frac{4200}{2} \times 0.96$

$= Rs\ 3044.57$

Increase → Rs 164.57

b) 40% less than EOQ —

$Q = .6\, Q^* = 1800$ units / order

TIC = Rs 3264

Increase = Rs 384

Conclusion → when Q is dec. 40% then the cost increases in both cases but at faster rate.
Hence curve is not symmetrical.

$$\boxed{\frac{TIC(Q)}{TIC(Q^*)} = \frac{1}{2}\left[K + \frac{1}{K}\right]}$$

↓

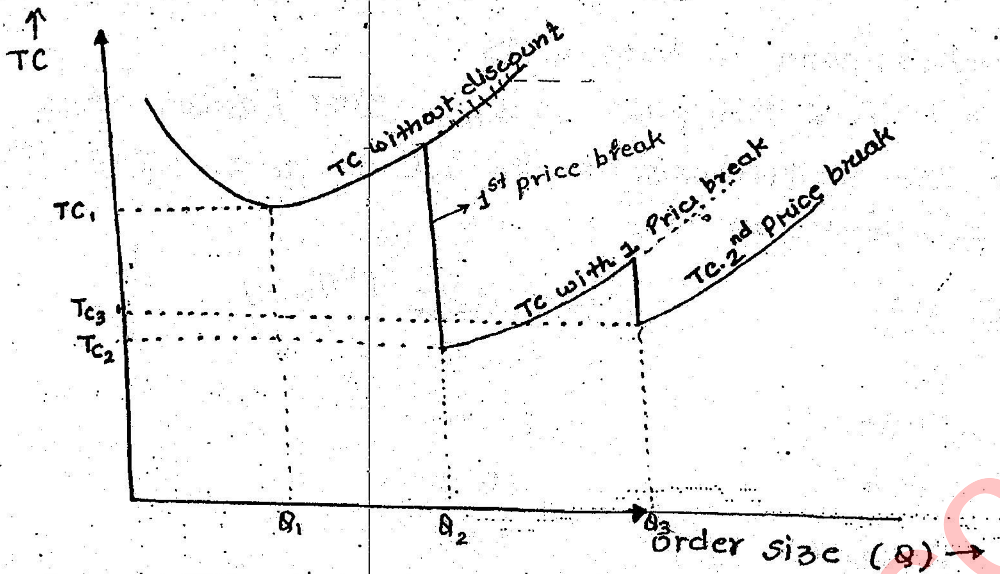
Model Sensitivity
or
Model Robustness

$$\boxed{Q = KQ^*}$$

$$\frac{TIC(Q)}{2880} = \frac{1}{2}\left[0.6 + \frac{1}{0.6}\right]$$

TIC (Q) = Rs 3264

2) EOQ with Price break or Quantity discount



$$TC = \boxed{DC} + \frac{D}{Q} C_0 + \frac{Q}{2} C_h$$

$Q \rightarrow 999$ to 1000

$999 \rightarrow C = 10\%$

$1000 \rightarrow C = 9\%$ discount

$D.C = (20,000 \cancel{*}) \downarrow \longrightarrow 1^{st}$ Price break.

In some condition discount is offered on unit price of inventory for large quantity purchased and this discount takes the form of price break. As discount is always offered on unit price of inventory so in order to get the best order size we need to consider purchasing cost along with ordering and holding cost. In this problem first we compute feasible EOQ and then total cost is computed at feasable EOQ and next higher order size having price break wherever the total cost comes out to be minimum give the best order size.

Q - Annual demand = 8000 units

HC = 10% of unit price ; C_o = 1800 / order and Item can be purchased in the lot as given below. Then det the best order size.

Lot size	Unit Price (Rs/unit)
1 - 999	220
1000 - 1499	200
1500 - 1999	190
2000 & above	185

→ $$EOQ = \sqrt{\frac{2DC_o}{C_h}} = \sqrt{2 \times 8000 \times 1800}$$

convention - we know that -

$$EOQ^* \text{ ie } Q^* = \sqrt{\frac{2DC_o}{C_h}}$$

~~where C_h = 10% of C~~

Starting from the lowest unit price and searching feasible EOQ

for 185 = C -

$C_h = .1 \times 185 = 18.5$

$$Q^* = \sqrt{\frac{2 \times 8000 \times 1800}{18.5}} = 1247.7 \text{ units/order} -$$

Since it is not feasible as for C = Rs 185, Q ≥ 2000.

Proceeding to next higher unit price of Rs 190 /unit.

$$Q^* = \sqrt{\frac{2 \times 8000 \times 1800}{19}} = 1231.17 \text{ units/order} -$$

Again not feasible -

for C = 200 -

$$Q^* = \sqrt{\frac{2 \times 8000 \times 1800}{20}} = 1200 \text{ units/order}$$

It is feasible as for C = Rs 200/unit, Q must be between 1000 to 1499.

Now we compute total cost at feasible EOQ i.e. $Q^* = 1200$ and the next higher price break point of $Q = 1500$ and $Q = 2000$.

At 1500 – $TC = DC + \frac{D}{Q}C_o + \frac{Q}{2}C_h$

$$= 28{,}000 \times 190 + \frac{28000}{1500} \times 1800 + \frac{1500}{2} \times \cancel{(20)}\,(19)$$

$$= 1624000$$

At 1200 – $TC = 8000 \times 200 + \frac{8000}{1200} \times 1800 + \frac{1200}{2}(20)$

$$= 1543840$$

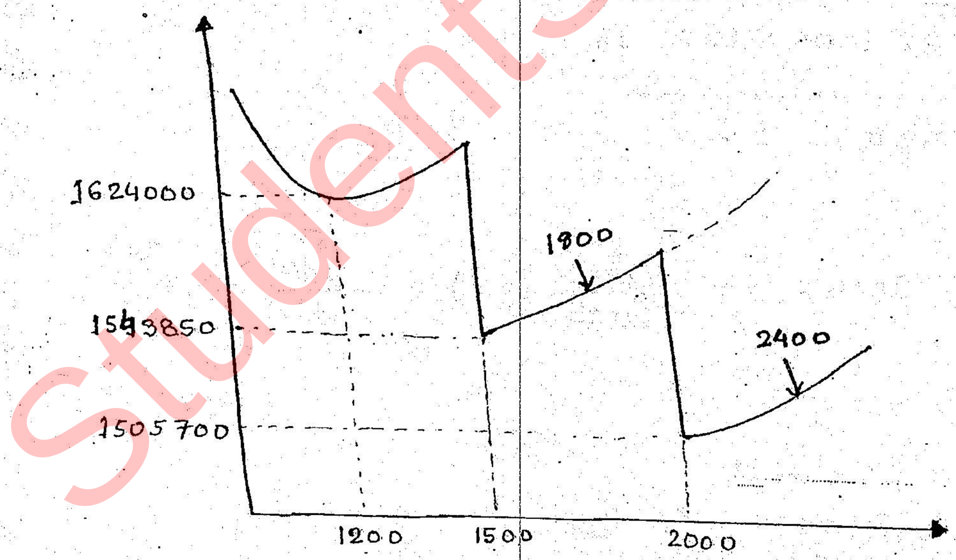
At 2000 → $TC = 8000 \times 185 + \frac{8000}{2000} \times 1800 + \frac{2000}{2}(18.5)$

$$= 1505700$$ → best cost.

At 1800 → $TC = 8000 \times 190 + \frac{8000}{\cancel{1800}} \times \cancel{1800} + \frac{1800}{2}(19)$

$$= 1545100$$

At 2400 → TC = Rs 1508200



Q → $D = 2000$ units/yr

$C = $ Rs 1/unit

$C_0 = $ Rs 10/order

$C_h = $ Rs 0.16/unit/yr

i) find Q^*, TIC^*

ii) If $Q = 1000$, 5% discound on c

if $Q = 2000$, 7% discount on c

Det. best order size ?

→ i) $Q^* = \sqrt{\frac{2DC_0}{C_h}} = \sqrt{\frac{2 \times 2000 \times 10}{0.16}} = 500$

$TIC^* = 2000 \times 1 + \frac{2000}{500} 10 + \frac{500}{2} \times 0.16$

$\Rightarrow 2000 + 40 + 40$

$= 2080$

$TIC^* = \sqrt{2DC_0C_h}$

$= \sqrt{2 \times 2000 \times 10 \times .16}$

$= \sqrt{400 \times 16} \rightarrow$ Rs 80

ii)

a) $TIC(1000) = 2000 \times 1 + \frac{2000}{1000}(10) + \frac{1000}{2}(.05)$

$= 12045$

$C = $ Rs 0.95/unit

T.C = 2000 ← best

b) TIC (2000) → C = Rs 0.93/unit , TC = 2030

c) TIC (500) → TC → 2080