"Arrays" :-

"1-D Array" :-

int a[10] = {10, 20, 30 - - 100} /* a is an Array of 10 elements, all are int */

1000	1002	1004	1006	1008	1010	1012	1014	1016	1018
10	20	30	40	50	60	70	80	90	100
0	1	2	3	4	5	6	7	8	9

→ 1018-1019

Ex ① :-

Loc (a[8]) = 1000 + (8 − 0) × 2 ← int

= 1000 + 16 ⇒ 1016

* Array size = J − i + 1

1019 − 1000 + 1

⇒ 20 int

2 ⇒ 10 element

Loc (a[4]) = 1000 + (4 − 0) × 2

= 1000 + 8 ⇒ 1008

Random Access possible (due to this formula). O(1) time

Ex: ② :-

A [150 - - - - 700]

Total elements = (J − i + 1) 700 − 150 + 1

= 550 + 1 = 551

Base Addr = 1000

Size of element (C) = 10

Loc (A [575]) = 1000 + (575 − 150) × 10

= 1000 + 4250 = 5250

Ex: ③

A [−500 - - - - - - −25]

B.A. = 0 C = 5

total elements

= ~~500 + 25 + 1~~

= −25 + 500 + 1

= 475 + 1 = 476

Loc (A[−150]) = 0 + (−150 + 500) × 5

= (350) × 5

= 1750

Note :-

A [lb .... ub] # of elements :- $ub - lb + 1$

B.A. = BA element size = 'C'
Base Add.

$$\boxed{Loc(A[i]) = B.A. + [i - lb] \times C}$$

* **Note :-**
By default Array index will always start from '0' but not from '1' because no need to calculate offset value everytime.

"2D - Arrays" :-

int a [1 ... 4, 1 ... 5]

4 rows [4-1+1], 5 coloums [5-1+1] ⇒ 20 elements

Coloum major

	1	2	3	4	5
1	a_{11}	a_{12}	a_{13}	a_{14}	a_{15}
2	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}
3	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}
4	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}

Row major

4x5

Row major order

a

1000	02	04	06	08	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
a_{11}	a_{12}	a_{13}	a_{14}	a_{15}	a_{21}	a_{22}	a_{23}	a_{24}	a_{25}	a_{31}	a_{32}	a_{33}	a_{34}	a_{35}	a_{41}	a_{42}	a_{43}	a_{44}	a_{45}
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19

$$Loc(a[4][3]) = 1000 + [(4-1) \times 5 + (3-1)] \times 2$$

Row, Column, element

$$= 1000 + 34 = \underline{1034}$$

$$Loc(a[2][5]) = 1000 + [(2-1) \times 5 + (5-1)] \times 2$$

$$= 1000 + 18 = \underline{1018}$$

Ex: ②:-

$A[25 \ldots 250, 79 \ldots 527]$ Rows = 226, Coloums $448+1 = 449$

$B \cdot A = 1000$, $C = 10B$ Row major order

$$Loc(A[201][423]) = 1000 + [(201-25) \times 449 + (\cancel{527}\ 423-79)] \times 10$$

$$= 1000 + 793680$$

$$= \underline{794680}$$

Ex: ③:-

$A[-200 \ldots 200, -75 \ldots +75]$ Rows $200+200+1 = \underline{401}$, Colums $75+75+1 = \underline{151}$

$BA = 0$ $C = 5B$ Row major.

$$Loc[A[175][3]) = \cancel{1000}\ 0 + [(175+200) \times 151 + (3+75)] \times 5$$

$$= (283515)$$

★ **Note:-**

$A[lb_1 \ldots ub_1, lb_2 \ldots ub_2]$ Coloums $= ub_2 - lb_2 + 1$

Base Addr. = BA Size(element) = C Row major

$$\boxed{Loc[A[i][J]) = B \cdot A + [(i - lb_1) \times (ub_2 - lb_2 + 1) + (J - lb_2)] \times C}$$

nc no of colu

:- Coloum major order :-

Ex:- $A[25 \ldots 250, 79 \ldots 527]$

Rows: $250-25+1 = 226$ Coloums: $527-79+1 = 449$

Base Addr. = 1000 C = 10B

Coloum major order

go to the col 1st, Rows

$$Loc(A[201][423]) = \underset{1000}{B.A.} + ((423-79) \times 226 + (201-25)] \times 10$$

then Row

$$= 1000 + 779200 = \underline{780200}$$

Ex: ② $A[-200 \ldots +200, -75 \ldots +75]$

Rows: $200+200+1 = 401$ cols: $75+75+1 = 151$

B.A. = 0, C = 5B · Coloum major

$$Loc(A[175][3]) = 0 + ((3+75) \times 401 + (175+200)] \times 5$$

get col 1st, rows, then row

$$= \underline{158265}$$

★ Note :- $A[lb_1 \ldots Ub_1, lb_2 \ldots Ub_2]$

BA, C, Col major order

$$\boxed{Loc[A[i][J]] = B.A. + ((J-lb_2) \times \underbrace{(Ub_1-lb_1+1)}_{nr} + (i-lb_1)] \times C}$$

★ By default C Language or other programming Language follows "Row major order"

1. "3-D Array":-

int a [5 ... 15, 7 ... 21, 4 ... 11]

$\Rightarrow 11$, $\Rightarrow 15$, $\Rightarrow 8$

3-D:- Collection of 2D's

int a[11] [15] [8]

11 2D's, each of size (15×8)

5 (15×8) → Rows ⇒ 15, Cols - 8

6 (15×8)

7 (15×8)

⋮

15 (15×8)

Rows: 7, 8, 9, ..., 21

Cols: 4, 5, 6, ..., 11

int a [5 ... 15, 7 ... 21, 4 ... 11]

$\Rightarrow 11$, $\Rightarrow 15$, $\Rightarrow 8$

B.A. = 1000, C = 10B Row major

Loc (A[12] [20] [7]) = BA +

$$= ((12-5) \times 15 \times 8 + (20-7) \times 8$$

$$+ (7-4)] \times 10$$

$$= \underline{10\ 470}$$

2780 + 1000 = 3780

Colo major:-

$$Loc(A[12][20][7]) = 1000 + ((12-5) \times \underline{15 \times 8}$$

$$+ ((7-4) \times 15) + (20-7)] \times 10$$

$$= 9980$$

Note

i) $A[\, lb_1 \ldots \overset{\text{Row} \to nr}{ub_1},\ lb_2 \ldots \overset{\text{col} \to nc}{ub_2},\ lb_3 \ldots ub_3 \,]$

(required 2D) # of 2D's

BA, C, Row major

$$\boxed{Loc(A[i][j][k]) = BA + \big((\underbrace{i - lb_1) \times nr \times nc}_{\text{\# of 2D's}} + \underbrace{(j - lb_2) \times nc}_{\text{required row}} + \underbrace{(k - lb_3)}_{\text{col}}\big] \times \underbrace{c}_{\text{element size}}}$$

ii) $A[\, lb_1 \ldots ub_1,\ lb_2 \ldots ub_2,\ lb_3 \ldots ub_3 \,]$

BA, C, ~~Row~~ col major

$$\boxed{Loc(A[i][j][k]) = BA + \big(\underbrace{(i - lb_1) \times nr \times nc}_{\text{required 2D}} + \underbrace{(k - lb_3) \times nr}_{\text{required col}} + \underbrace{(j - lb_2)}_{\text{Row}}\big] \times C}$$

"Lower Triangular matrix":-

int a[1.... 4, 1... 4]

$$\begin{array}{c|cccc|}
 & 1 & 2 & 3 & 4 \\
1 & a_{11} & a_{12} & a_{13} & a_{14} \\
2 & a_{21} & a_{22} & a_{23} & a_{24} \\
3 & a_{31} & a_{32} & a_{33} & a_{34} \\
4 & a_{41} & a_{42} & a_{43} & a_{44}
\end{array}$$

→ zeros

4x4

Non-zero

$$\underset{\text{(Non zero)}}{\text{elements}} = \frac{n(n+1)}{2} \Rightarrow \frac{4(4+1)}{2} = 10$$

:- All Triangular matrices are square matrices.

:- Storing zero :- unnecessary space wastage.

* :- a[i][J] [If i ≥ J] ⟶ element belongs to lower triangle.

* :- a[i][J] [If J > i] ⟶ upper triangle return (0);

:- Starting position of Row & coloum must be same.

Ex: A[25--150, 25--150]

:- Storing in 'Row major order' :-

1000	02	04	06	08	10	12	14	16	18
a_{11}	a_{21}	a_{22}	a_{31}	a_{32}	a_{33}	a_{41}	a_{42}	a_{43}	a_{43}
0	1	2	3	4	5	6	7	8	9
1r	2-r		3-r			4-r			

Ex: ①

cols ← $\frac{n(n+1)}{2}$

$$Loc(A[4][3]) = \underset{1000}{B.A.} + \cancel{[(4-1) \times \frac{3 \times 4}{2} +]}$$

$$= 1000 + \left[\frac{(4-1)(4-1+1)}{2} + (3-1)\right] \times 2$$

$$= \boxed{1016}$$

Ex: ②

$A[25 \text{---} 150, 25 \text{---} 150]$

⇓ 126 ⇒ 126

Base Addr. = 1000 C = 10, Lower Triangle Matrix, Row major

$$Loc(A[125][105]) = 1000 + \left(\frac{(125-25)(125-25+1)}{2} + (105-25)\right] \times 10$$

$$= 1000 + \left(\frac{\overset{50}{\cancel{100}} \times 101}{\cancel{2}} + 80\right] \times 10$$

$$52300 \Leftarrow = 1000 + (5050 + 80) \times 10$$

Note

$A\,[lb_1 \text{---} ub_1,\ lb_2 \text{----} ub_2]$

B.A, C, Lower triangular matrix, Row major

$$Loc(A[i][J]) = B.A + \left(\frac{(i-lb_1)(i-lb_1+1)}{2} + (J-lb_2)\right) \times C$$

If $i \geq J$ else return (0);

– "Coloum major":

a	1000	02	04	06	08	10	12	14	16	18
	a_{11}	a_{21}	a_{31}	a_{41}	a_{22}	a_{32}	a_{42}	a_{33}	a_{43}	a_{44}
	0	1	2	3	4	5	6	7	8	9
		1-C				2-C		3-C		4-C

$Loc(A[4][3]) = \cancel{1000 + [(3-1)}$

$\cancel{1000 + [(4-0)(+1)}$ → all col crossed (now on right side)

$$Loc(A[4][3]) = 1000 + \left[\left(\frac{4 \times 5}{2} - \underbrace{\frac{2 \times 3}{2}}_{\text{unnecessarly crossed coloums}}\right) + (4-3)\right] \times 2$$

$(4-3)$ → Row

$$= \underline{1016}$$

Ex. - 2

$A\,[25 \text{---} \overset{176}{200},\ 25 \text{---} \overset{176}{200}]$

$B.A = 1000$, $C = 10B$, Lower Triangle matrix & Coloum major

$\frac{176-150}{26}$

$$Loc(A[150][90]) = 1000 + \left[\left[\frac{176 \times 177}{2} - \frac{111 \times 112}{2}\right] + (150-90)\right] \times 10$$

$$= \underline{95200}$$

or

$$= 1000 + \left[\frac{176 \times 177}{2} - \frac{(200-90+1)(200-90+2)}{2} + \right.$$

$(150-90)\ ] \times 10$

* **Note :-**

$A[lb_1 \ldots ub_1,\ lb_2 \ldots ub_2]$

B.A, C, Lower Triangle matrix, coloum major

$$Loc(A[i][j]) = B.A + \left[\frac{(ub_2 - lb_2)(ub_2 - lb_2 + 1)}{2} - \frac{(ub_2 - j + 1)(ub_2 - j + 2)}{2} + (i - j)\right] \times C$$

Lower triangle so $i \geq j$

$$= B.A + \left[\frac{(nc)(nc+1)}{2} - \frac{(ub_2 - j + 1)(ub_2 - j + 2)}{2} + (i - j)\right] \times C$$

:-

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

4x4

$Z = 3n - 2$ elements

Row major

$\underset{1^{st}}{n} + 1 + 1 \ldots + \underset{last}{n}$

49 rows

50th Row ⇒ $N + 48$

Coloum major

$\underset{1^{st}}{2} + 3 + 3 + 3 \ldots + \underset{last}{2}$

50th Coloum ⇒ $48 \times 3 + 2$