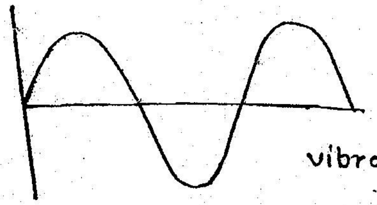
* Mechanical Vibration

To and fro periodic motions about their eq^m position.

Harmonic motions
↓
oscillation.

- Mean Position
- Eq^m Position
- Zero Position

sine wave.



exponential
longtithmic } No vibration.

Any vibrating system is a combination of -

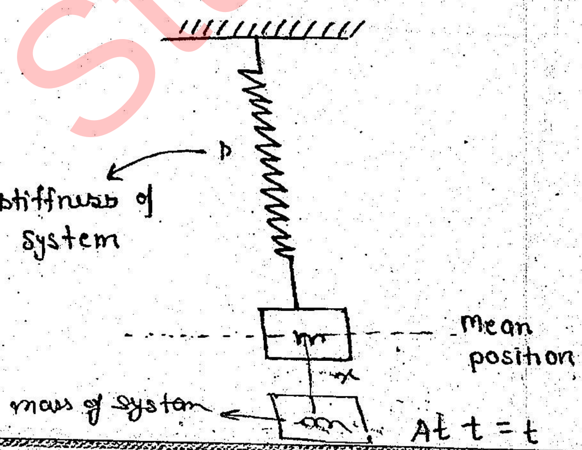
1) Mass : m
 - (K.E storing device)
2) Stiffness : s
 (P.E storing devices)
3) Damping
 (Energy loss due to Kinetic friction)
4) Fun ≠ 0

That vibrating system which is not having any rotat^n / Reciprocating parts
⇓
These vibrations are introduced due to some external disturbance given initially.

That vib. system which is having rot / reci parts
⇓
Running m/c.

Natural Vibration

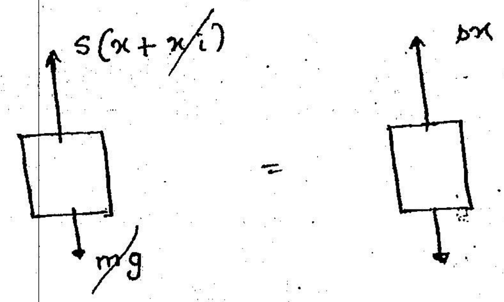
"The vibrations in which there is no Kinetic friction at all as well as there is no external force after the initial release of the system are known as natural vibrations"



$\boxed{mg = s x_i}$ eq^m equation

$x = f(t) = ??$

Newton's method -



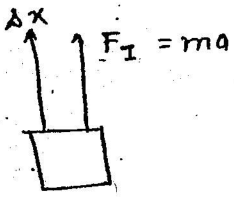
$$0 - sx = ma$$

$$\underline{ma + sx = 0}$$

D'Alembert Principle →

At t = t

system FBD →



DAP = $\boxed{ma + sx = 0}$

$$\cancel{ma} \quad m\ddot{x} + sx = 0$$

$$\boxed{\ddot{x} + \frac{s}{m}x = 0} \quad **$$ eq^n of natural system.

Its solution will be

$$x = R\sin\left(\sqrt{\frac{s}{m}}\,t + \phi\right)$$ R, ϕ → const.

R → Amplitude = const

sin (—) = vibrating with freq $\omega_n = \sqrt{\frac{s}{m}}$ rad/s

$$T_n = \frac{2\pi}{\omega_n} \text{ (sec)}$$

$$\boxed{f_n = \frac{\omega_n}{2\pi}} \text{ (Hertz)}$$

const

R, ϕ are formed by Initial condition :-

1) At t = 0 $\left\{ \begin{array}{l} x = x_0 \\ \dot{x} = 0 \end{array} \right\}$ R = ✓, ϕ = ✓

2) At t = 0 $\left\{ \begin{array}{l} x = 0 \\ \dot{x} = v_0 \end{array} \right\}$ R, ϕ = ✓

3) At t = 0 $\left\{ \begin{array}{l} x = x_0 \\ \dot{x} = v_0 \end{array} \right\}$ R, ϕ = ✓

Finally →

Eq^n. of Nat. vibration is –

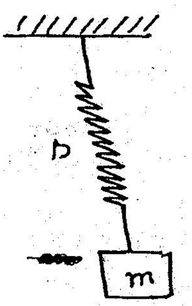
$$\boxed{\ddot{x} + (\omega_n)^2 x = 0}$$

Q → The natural vib of system is $5\ddot{x} + 3x = 0$, find $\omega_n = ?$

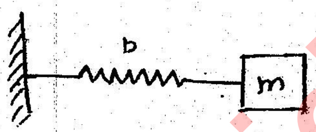
$$\ddot{x} + \frac{3}{5}x = 0$$

$$\omega_n^2 = \frac{3}{5} \quad ; \quad \omega_n = \sqrt{\frac{3}{5}} \text{ rad/s}$$

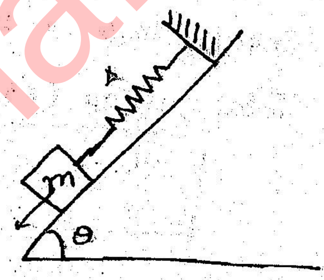
Note :-



$$\omega_n = \sqrt{\frac{p}{m}}$$



$$\omega_n = \sqrt{\frac{p}{m}}$$



$$mg \sin\theta = p x_i$$

$$\omega_n = \sqrt{\frac{p}{m}}$$

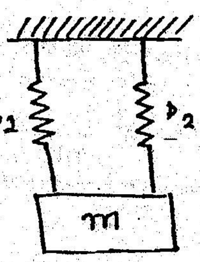
combination of spring :-

Series :-

$$\boxed{\frac{1}{p} = \frac{1}{p_1} + \frac{1}{p_2}}$$

$$\omega_n = \sqrt{\frac{p}{m}}$$

Parallel :-



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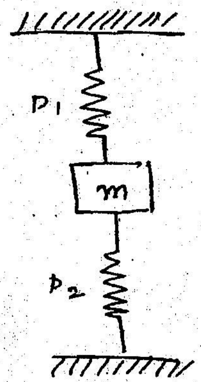
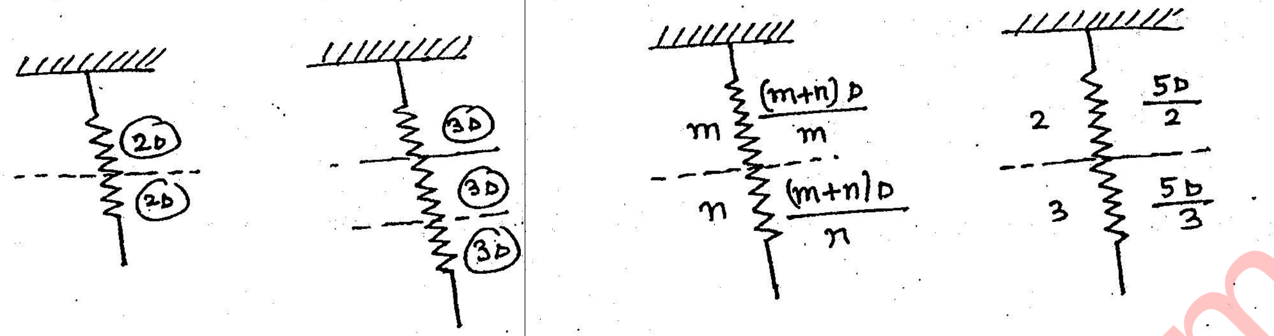


$$\boxed{p = p_1 + p_2}$$

$$\omega_n = \sqrt{\frac{p}{m}}$$

Cutting of spring :-



Energy Method :-

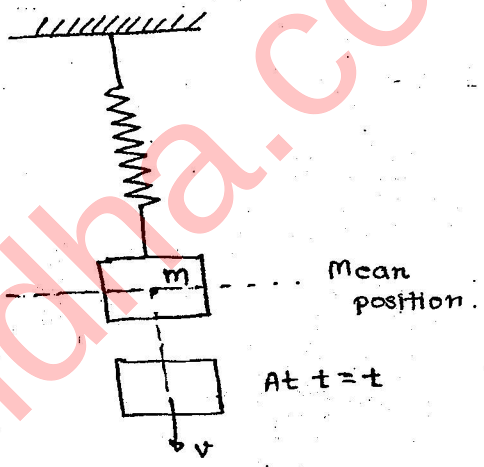
In natural vibration :-

Kinetic Energy = 0

Energy loss = 0

Total energy (E) = const.

$$\boxed{\frac{dE}{dt} = 0}$$



At t = t system energy →

$$\therefore \quad E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2$$

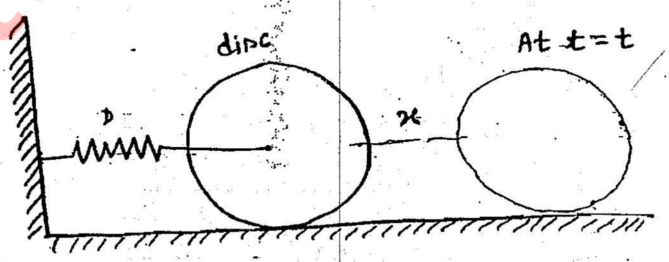
$$\frac{dE}{dt} = \frac{1}{\cancel{2}}k(\cancel{2}x)\frac{dx}{dt} + \frac{1}{\cancel{2}}m(\cancel{2}v)\cdot\frac{dv}{dt} = 0$$

$$ma + kx = 0$$

$$\boxed{\ddot{x} + \frac{k}{m}x = 0}$$

$$\omega_n = \sqrt{\frac{k}{m}} \text{ rad/s}$$

Q →



At t = t system energy –

$$E = \frac{1}{2}kx^2 + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$= \frac{1}{2} px^2 + \frac{1}{2} mv^2 + \frac{1}{2} \frac{mR^2}{2} \times \frac{v^2}{R^2}$$

$$= \frac{1}{2} px^2 + \frac{1}{2} \left(\frac{3m}{2}\right) v^2$$

$$\omega_n = \sqrt{\frac{p}{3m/2}} = \sqrt{\frac{2p}{3m}}$$

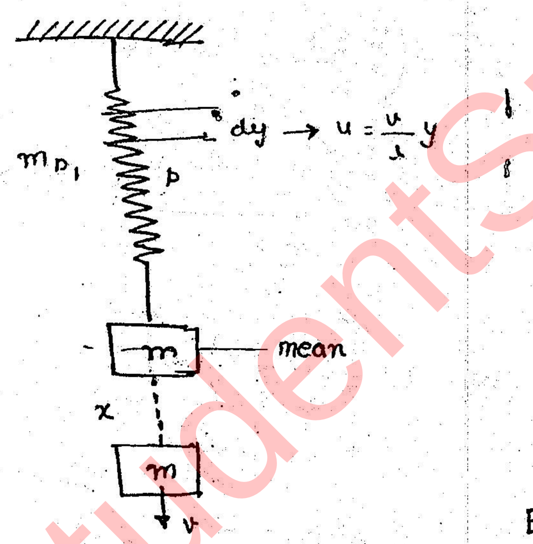
Ring, Hollow — $I = mR^2$

Disc, solid Cylinder — $I = \frac{mR^2}{2}$

Hollow sp — $I = \frac{2}{3} mR^2$

Solid sp — $I = \frac{2}{5} mR^2$

<u>Spring mass system</u> :-



$$K.E_{spring} = \int_0^l \frac{1}{2} \left(\frac{m_p\, dy}{l}\right) \left(\frac{v}{l} y\right)^2$$

$$= \frac{1}{2} \times \frac{m_p}{l} \cdot \frac{v^2}{l^2} \cdot \frac{l^3}{3}$$

$$= \frac{1}{6} m_p v^2$$

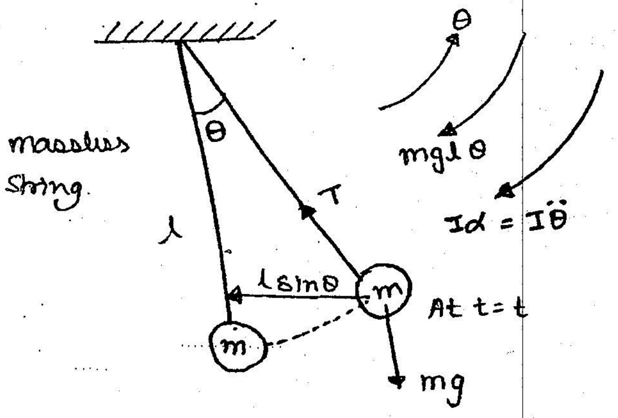
<u>At t = t</u>

$$E = \frac{1}{2} px^2 + \frac{1}{2} mv^2 + \frac{1}{6} m_p v^2$$

$$E = \frac{1}{2} px^2 + \frac{1}{2} \left(m + \frac{m_p}{3}\right) v^2$$

$$\omega_p = \sqrt{\frac{p}{m + \frac{m_p}{3}}}$$

Torque Method :- (very small oscillation)



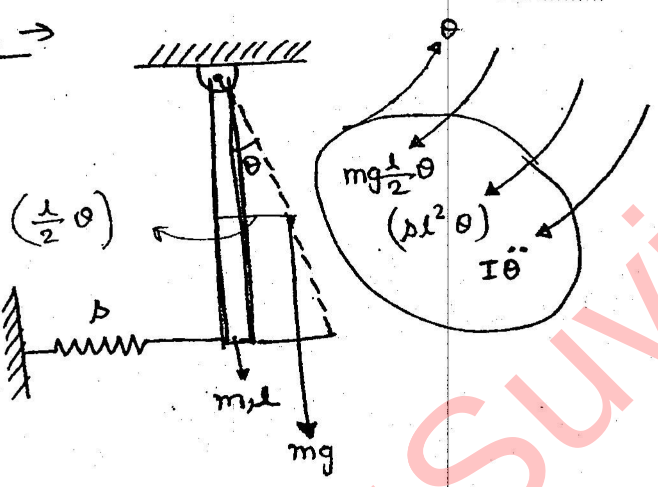
$$I\ddot{\theta} + (mgl)\theta = 0$$

$$(ml^2)\ddot{\theta} + (mgl)\theta = 0$$

$$\boxed{\ddot{\theta} + \left(\frac{g}{l}\right)\theta = 0}$$

$$\omega_n = \sqrt{\frac{g}{l}} \quad \text{rad/sec}$$

Q →



$$I = \frac{ml^2}{12} + m\left(\frac{l}{2}\right)^2$$

$$= \frac{ml^2}{12} + \frac{ml^2}{4}$$

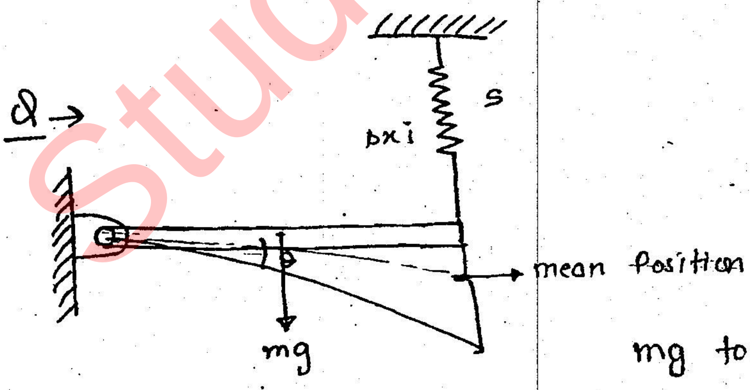
$$I = \frac{ml^2}{3}$$

$$I\ddot{\theta} + \left(\frac{mgl}{2} + sl^2\right)\theta = 0$$

$$\ddot{\theta} + \frac{\left(\frac{mgl}{2} + sl^2\right)}{\frac{ml^2}{3}}\theta = 0$$

$$\omega_n = \sqrt{\frac{\left(\frac{mgl}{2}\right) + sl^2}{\frac{ml^2}{3}}}$$

Q →



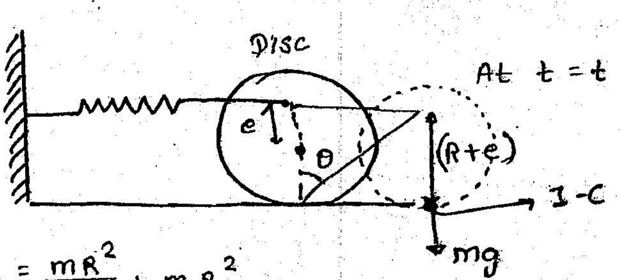
$$mg\,\frac{l}{2} = s x_i \times l$$

eq^n eq^m —

mg torque will be cancelled by $s x_i$ torque therefore mg torque will not be considered.

$$\omega_n = \sqrt{\frac{s l^2}{\frac{m l^3}{3}}} = \sqrt{\frac{3s}{m}} \text{ rad/s}$$

Q →



$s(R+e)^2\theta$

$I\ddot{\theta}$

$$I = \frac{mR^2}{2} + mR^2$$

$$I = \frac{3}{2} mR^2$$

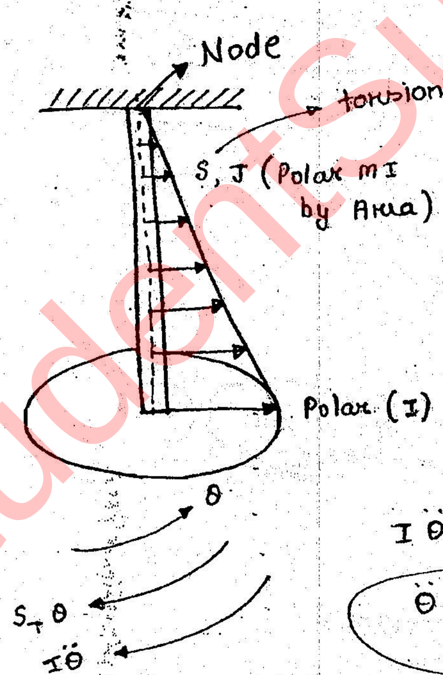
$$I\ddot{\theta} + s(R+e)^2\theta = 0$$

$$\frac{3}{2} mR^2\ddot{\theta} + s(R+e)^2\theta = 0$$

$$\ddot{\theta} + \frac{2s(R+e)^2}{3mR^2}\theta = 0$$

$$\boxed{\omega_n = \sqrt{\frac{2s(R+e)^2}{3mR^2}}}$$

Torsional Vibration



$$I\ddot{\theta} + S_T\theta = 0$$

$$\ddot{\theta} + \frac{S_T}{I}\theta = 0$$

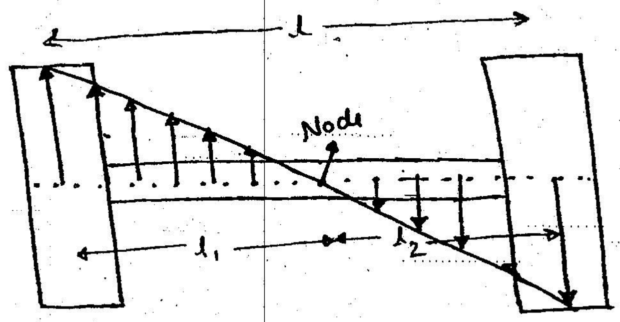
$$\omega_n = \sqrt{\frac{s_T}{I}}$$

Note :- If rod is also having mass (I_{rod}) →

Then,

$$\omega_n = \sqrt{\frac{k_T}{I + \frac{I_{rod}}{3}}}$$

Two rotor system :-



$$l_1 + l_2 = l \quad - (1)$$

$$\sqrt{\frac{k_{T_1}}{I_1}} = \sqrt{\frac{k_{T_2}}{I_2}}$$

$$\frac{k_{T_1}}{I_1} = \frac{k_{T_2}}{I_2}$$

$$\frac{\cancel{GJ}}{l_1 I_1} = \frac{\cancel{GJ}}{l_2 I_2} \Rightarrow \boxed{\frac{l_1}{l_2} = \frac{I_2}{I_1}} \quad - (2)$$

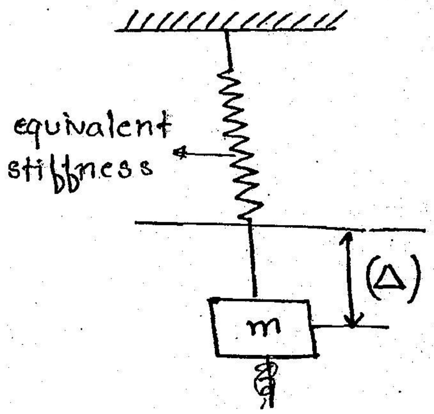
from (1) & (2) → l_1 → ✓ ; l_2 → ✓

$$\boxed{\begin{array}{l} \text{If no. of Rotor} = n \\ \text{No. of Node point} = n-1 \end{array}}$$

Method of Static Deflection of mass (Δ)

Rayleigh method :-

1) Only applied in natural vibration.
2) only applied in spring-mass system.
3) There should be only one mass in the system and that must be a point mass.

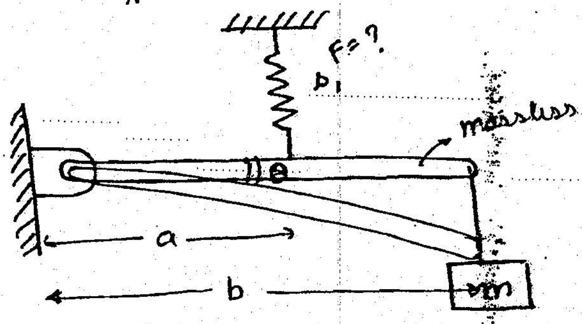


$$\Delta = \frac{mg}{s}$$

$$\sqrt{\frac{g}{\Delta}} = \sqrt{\frac{g}{\frac{mg}{s}}} = \sqrt{\frac{s}{m}} = \omega_n$$

$$\boxed{\omega_n = \sqrt{\frac{s}{m}} = \sqrt{\frac{g}{\Delta}}}$$

Q →



Moment →

$$mg\, b = F a$$

$$F = mg\,(b/a)$$

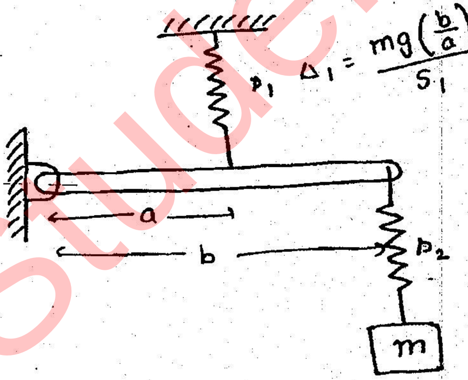
$$\frac{\Delta}{b} = \frac{\Delta_1}{a}$$

$$\Delta = \Delta_1 \frac{b}{a}$$

$$\boxed{\Delta = \frac{mg}{s}\left(\frac{b}{a}\right)^2}$$

$$\boxed{\omega_n = \sqrt{\frac{g}{\Delta}}} = \sqrt{\frac{S}{m}}$$

Q →

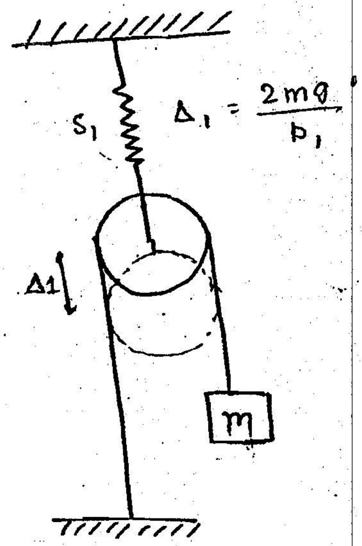


$$\Delta_1 = \frac{mg\left(\frac{b}{a}\right)}{s_1}$$

$$\Delta_2 = \frac{mg}{s_2}$$

$$\boxed{\Delta = \Delta_1 \times \frac{b}{2} + \Delta_2}$$ ✗

Q →

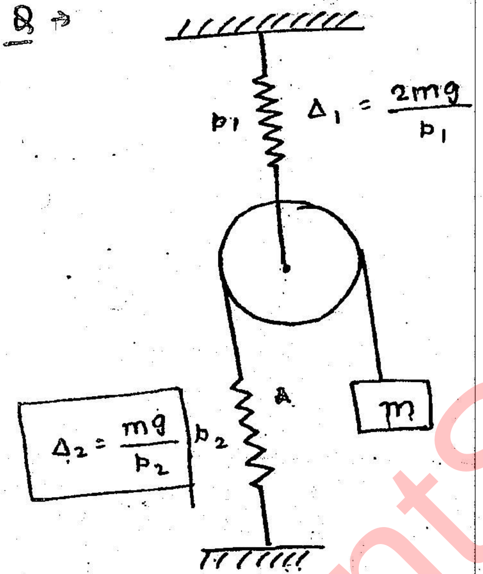


$\Delta = 2\Delta_1$

$= \cancel{4} \times \frac{2mg}{p_1}$

$\Delta = \frac{4mg}{p_1}$

Q →

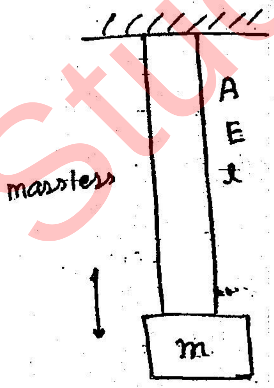


$\Delta = 2\Delta_1 + \Delta_2$

Longitudinal vibration of Beams :-

⇓

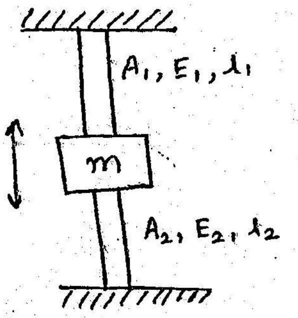
vibration along the length.



long. stiffness

$\boxed{s = \frac{AE}{l}}$

$\omega_n = \sqrt{\frac{s}{m}}$

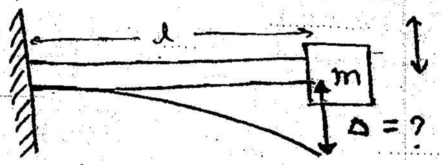


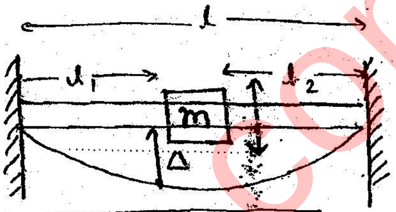
$$s_1 = \frac{A_1 E_1}{l_1} \qquad s_2 = \frac{A_2 E_2}{l_2}$$

$$s = s_1 + s_2$$

$$\omega_n = \sqrt{\frac{s}{m}}$$

Transverse vibration of Beam :-





$$\Delta = \frac{w l^3}{3EI}$$

$$\boxed{\Delta = \frac{w l_1^3 l_2^3}{3EI l^3}}$$

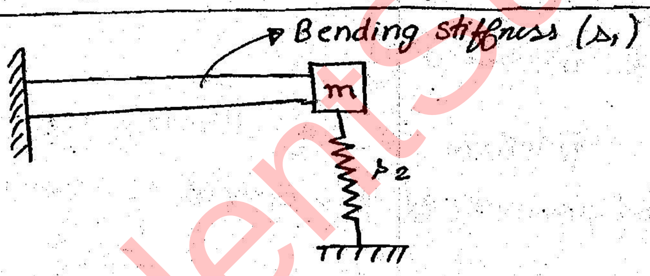
$w = mg$

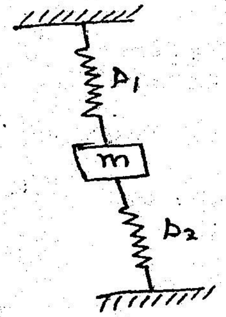
I = Area MOI of beam about bending axis.

$$\omega_n = \sqrt{\frac{g}{\Delta}} = \sqrt{\frac{s}{m}}$$

s → bending stiffness.

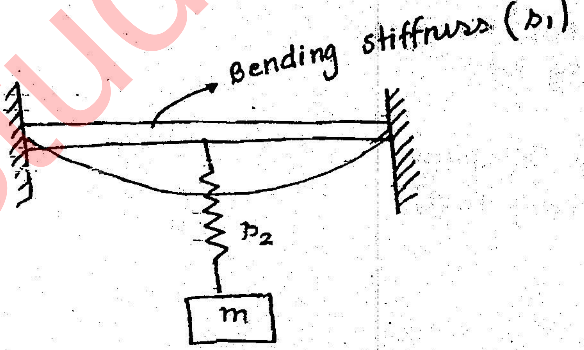
Q →

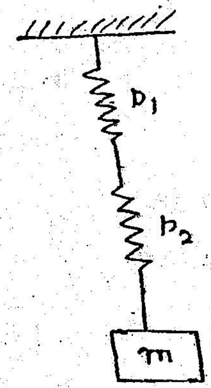




||

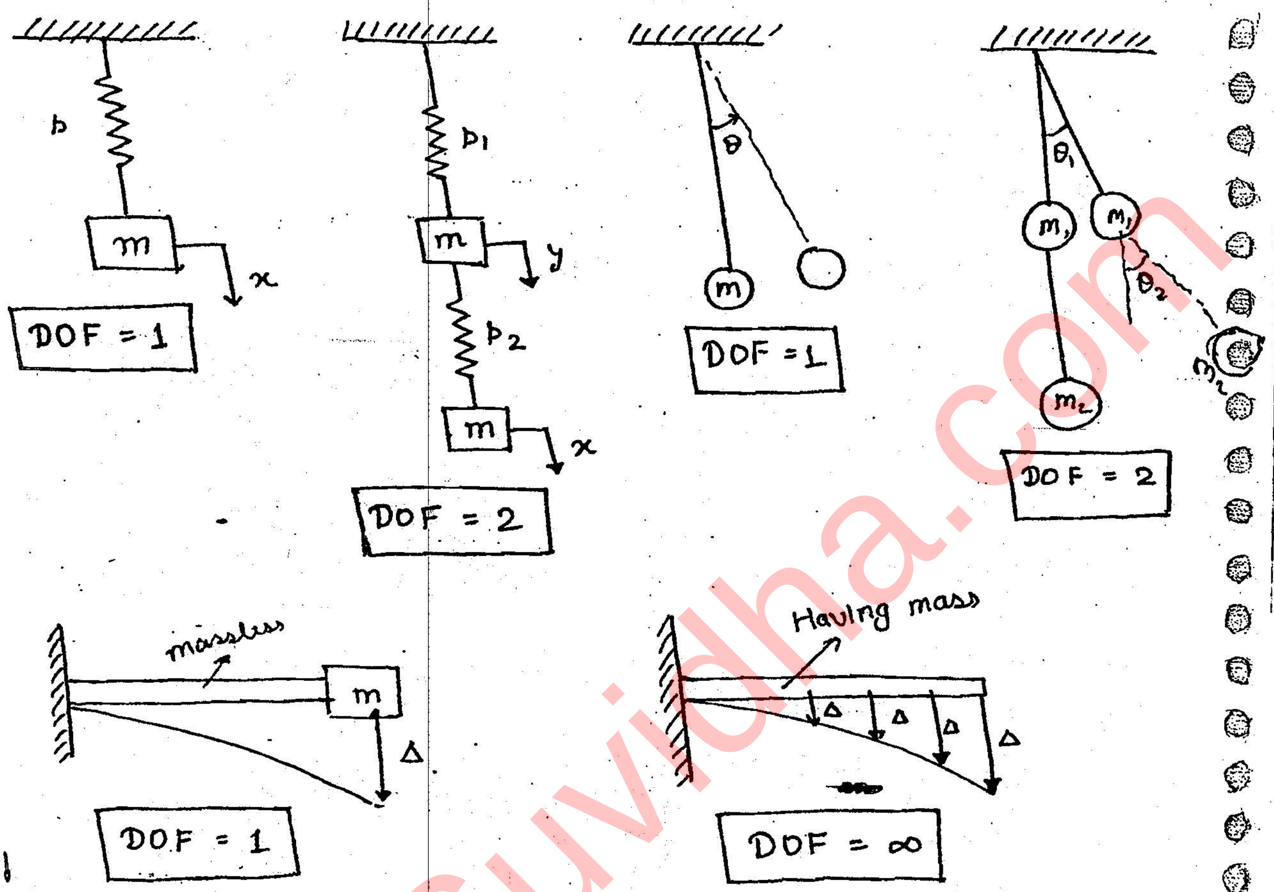
Q.





series

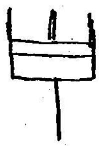
DOF of a vibrating system :-



Damped System :- (Kinetic friction ≠ 0)

Technical name of Kinetic friction to any vibrating system.

⇒ Damping and its is represented by the symbol ⇒ DAMPER

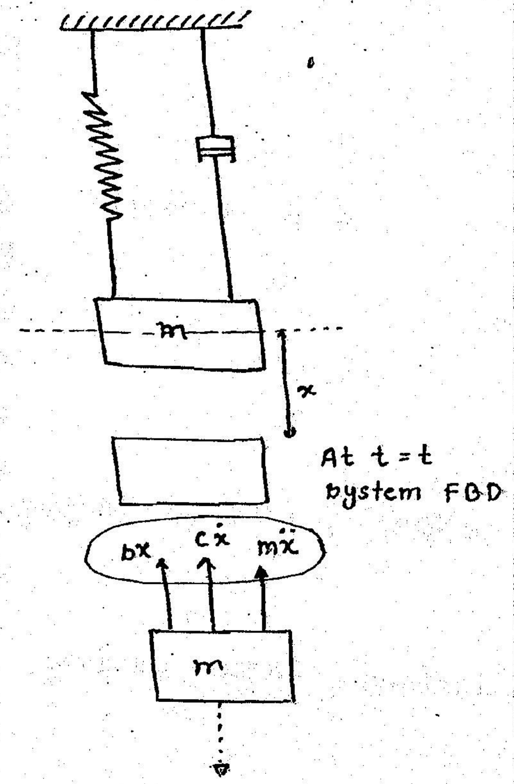


Damping in any system →
- Friction b/w dry surface. (colombian Damping) very high.
- Viscous Damping (very less)

Damping force in any direction $\propto \dot{x}$

$= c\dot{x}$

c → coeff. of damping = const for a system.



DAPs:

$$m\ddot{x} + c\dot{x} + px = 0$$

$$\boxed{\ddot{x} + \frac{c}{m}\dot{x} + (\omega_n^2)x = 0}$$

eq^n of damped system //

The solution of above eq^n is –

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t} \qquad (\alpha_1 \neq \alpha_2)$$

or

$$x = (A + Bt)e^{\alpha t} \qquad (\alpha_1 = \alpha_2 = \alpha)$$

where $\alpha_{1,2}$ → roots of auxillary eq^n –

$$\boxed{\alpha^2 + \left(\frac{c}{m}\right)\alpha + (\omega_n^2) = 0}$$

$$\alpha_{1,2} = \frac{-\frac{c}{m} \pm \sqrt{\left(\frac{c}{m}\right)^2 - 4\omega_n^2}}{2}$$

$$\boxed{\alpha_{1,2} = -\frac{c}{2m} \pm \sqrt{\left(\frac{c}{2m}\right)^2 - \omega_n^2}}$$

Responsible for loss of KE — Responsible for storing the energy.

where,

$$\frac{\left(\frac{c}{2m}\right)^2}{(\omega_n)^2} = \text{Degree of dampness.}$$

$$\boxed{\sqrt{\frac{\left(\frac{c}{2m}\right)^2}{(\omega_n)^2}} = \xi}$$ → Damping factor or Damping ratio.

$$\xi = \sqrt{\frac{\frac{c^2}{4m}}{\frac{p}{m}}} \Rightarrow \frac{c}{2\sqrt{pm}}$$

$$2\xi\omega_n = 2 \times \frac{c}{2\sqrt{pm}} \times \sqrt{\frac{p}{m}}$$

$$\boxed{2\xi\omega_n = \frac{c}{m}}$$

Finally, eq^n of damped system –

$$\boxed{\ddot{x} + (2\xi\omega_n)\dot{x} + (\omega_n)^2 x = 0}$$

The sol^n will be –

$$x = Ae^{\alpha_1 t} + Be^{\alpha_2 t} \qquad (\alpha_1 \neq \alpha_2)$$

A, B → const.

or

$$x = (A + Bt)e^{\alpha t} \qquad (\alpha_1 = \alpha_2 = \alpha)$$

where, $\alpha_{1,2} = \left(-\xi \pm \sqrt{\xi^2 - 1}\right)\omega_n$

if, $\xi > 1$ → overdamped system (over damping) / coulomb damping

$\xi = 1$ → critically damped system

$\xi < 1$ → underdamped system / under damping / viscous damping.

1) Overdamped system ($\xi > 1$) :-

The sol^n will be :

$$\boxed{x = Ae^{\left(-\xi + \sqrt{\xi^2 - 1}\right)\omega_n t} + Be^{\left(-\xi - \sqrt{\xi^2 - 1}\right)\omega_n t}}$$

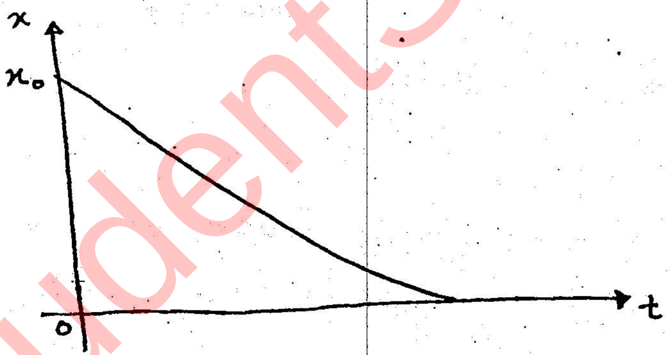
A, B = const.

No vibration

–ve exp. curve

→ Not periodic

→ Not Harmonic



2) Critically damped system ($\xi = 1$) :-

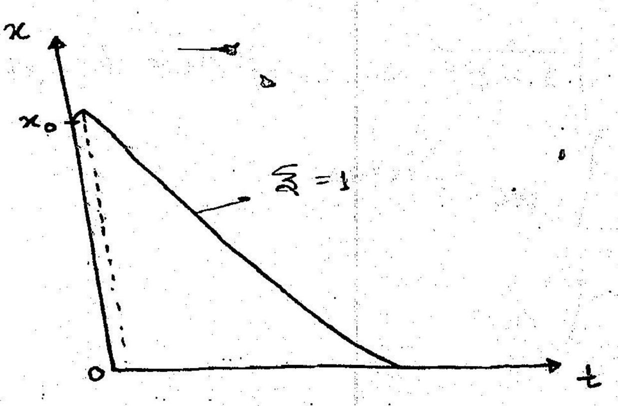
$$\alpha_1 = \alpha_2 = \alpha = -\omega_n$$

The solution will be :

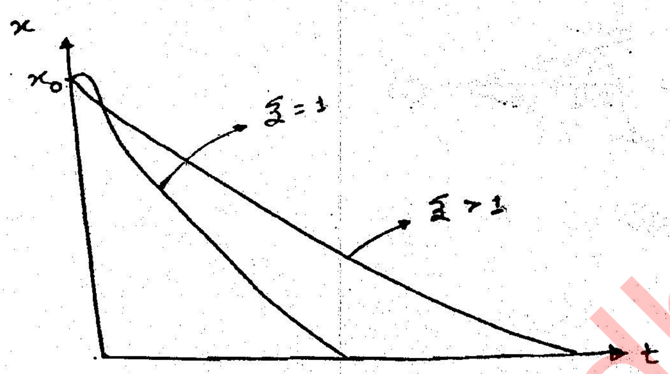
$$\boxed{x = (A + Bt)e^{-\omega_n t}}$$

No vibration

A, B → const.



Note :- Critical damping response is much fast than over damping.



for eg – door closer = over damped

AK-47 = critically damped.

3) Underdamped system ($\xi < 1$) :-

$$\alpha_{1,2} = (-\xi \omega_n \pm i \underbrace{\sqrt{1-\xi^2}\,\omega_n}_{\omega_d})$$

ω_d = const for system

$$\alpha_{1,2} = (-\xi \omega_n \pm i\omega_d)$$

The solution will be :-

$$x = A e^{\alpha_1 t} + B e^{\alpha_2 t}$$

$$x = A e^{(-\xi\omega_n + i\omega_d)t} + B e^{(-\xi\omega_n - i\omega_d)t}$$

$$= e^{-\xi\omega_n t}\left[\underbrace{(A+B)}_{X\sin\phi}\cos\omega_d t + \underbrace{i(A-B)}_{X\cos\phi}\sin\omega_d t\right]$$

$$= e^{-\xi\omega_n t}\left[X\sin(\omega_d t + \phi)\right]$$

$$x = X e^{-\xi\omega_n t}\sin(\omega_d t + \phi)$$

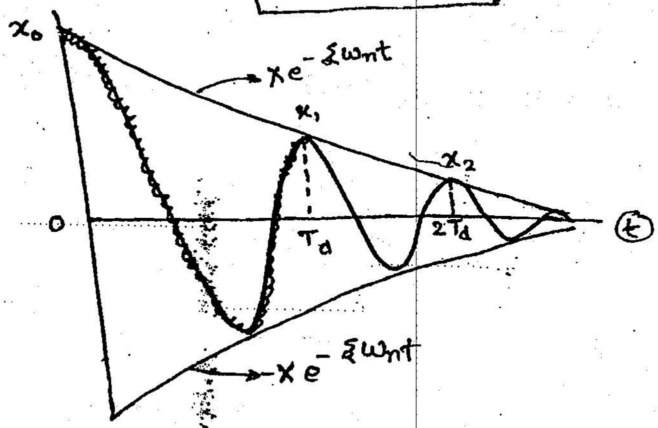
X, ϕ = const

Amplitude = ↓↓↓ f(n) → time

vib. with freq - $\omega_d = \sqrt{1-\xi^2}\cdot\omega_n$ // const. for system.

$$\boxed{T_d = \frac{2\pi}{\omega_d}} \text{ sec} = \text{const.}$$

$$\boxed{f_d = \frac{\omega_d}{2\pi}} \text{ Hz}$$



At $t = 0$

$$x_0 = X \sin\phi$$

At $t = T_d$:

$$x_1 = Xe^{-\xi\omega_n T_d}\sin\phi \qquad x_1 = Xe^{-\xi\omega_n T_d}\cdot\sin\phi$$

At $t = 2T_d$:

$$x_2 = Xe^{-\xi\omega_n(2T_d)}\sin\phi \qquad x_2 = Xe^{-\xi\omega_n(2T_d)}\cdot\sin\phi$$

⋮

Decrement Ratio ⇒

$$\frac{x_0}{x_1} = \frac{x_1}{x_2} = \frac{x_2}{x_3} \cdots\cdots = e^{\xi\omega_n T_d} = \text{const}$$

$$= e^{\delta}$$

Logarithmic decrement :- (δ)

$$\delta = \ln e^{\xi\omega_n T_d}$$

$$= \xi\omega_n T_d$$

$$= \frac{\xi \cancel{\omega_n} 2\pi}{\sqrt{1-\xi^2}\cdot\cancel{\omega_n}}$$

(*)

$$\boxed{\delta = \frac{2\pi\xi}{\sqrt{1-\xi^2}}}$$

Critical damping coeff :-

$$\frac{\cancel{2}\,\xi\,\cancel{\omega_n}}{\cancel{2}\times 1\times\cancel{\omega_n}} = \frac{c/\cancel{m}}{c_c/\cancel{m}}$$

$\xi = 1$ for critical dmp.

$c = c_c$

$$\boxed{\xi = \frac{c}{c_c}} = \frac{\text{Actual damping coeff.}}{\text{critical damping coeff.}}$$

Note :-

- The ratio of displacement of end of 3^{rd} cycle to start of 8^{th} cycle

$$= \frac{x_3}{x_7}$$

- The ratio of displacement of 3^{rd} cycle to 8^{th} cycle

$$= \frac{x_3}{x_8}$$

Q → 55 -

$m = 75$ kg

$T_d = \frac{35}{60}$ sec

$\omega_d = \frac{2\pi}{T_d} = ✓$

$\frac{x_1}{x_7} = 2.5$

$$\frac{x_1}{x_2}\cdot\frac{x_2}{x_3}\cdot\frac{x_3}{x_4}\cdot\frac{x_4}{x_5}\cdot\frac{x_5}{x_6}\cdot\frac{x_6}{x_7} = 2.5$$

$e^{6\delta} = 2.5$

$6\delta = \ln 2.5$

$\delta = ✓$

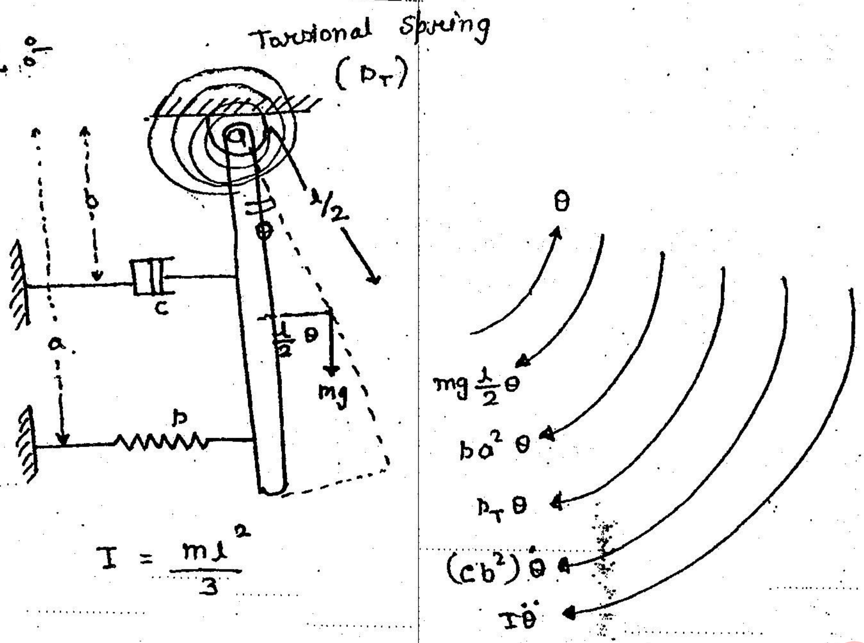
$\frac{6\times 2\pi\xi}{\sqrt{1-\xi^2}} = \ln 2.5$; $\xi = ✓$

$\omega_d = \sqrt{1-\xi^2}\,\omega_n = ?$

i) $\omega_n = \sqrt{\frac{s}{m}} \rightarrow ?$

ii) $2\xi\omega_n = \frac{c}{m} \rightarrow ?$; $\xi = \frac{c}{c_c} \rightarrow ?$

Note :-



$$I = \frac{ml^2}{3}$$

$$I\ddot{\theta} + (cb^2)\dot{\theta} + \left(\frac{mgl}{2} + ba^2 + b_T\right)\theta = 0 \quad \text{\# eq}^n \text{ of damped sys.}$$

$$m\ddot{x} + c\dot{x} + sx = 0 \quad \text{// linear motion.}$$

Q → The ~~damp~~ coeff in the vib. eq^n will be –

$= cb^2$

Q – $\omega_n = ?$

$$\omega_n = \sqrt{\frac{\left(\frac{mgl}{2} + ba^2 + b_T\right)}{I}}$$

Q – $\xi = ?$

$$2\xi\omega_n = \frac{cb^2}{I}$$

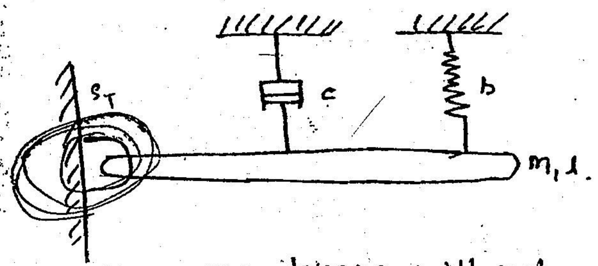
Q → $C_c = ?$

$$2\omega_n = \frac{C_c b^2}{I}$$

$\xi = 1,\ c = c_c$

Q → $\omega_d = ?$

$$\omega_d = \sqrt{1 - \xi^2}\ \omega_n$$



Here mg torque will not be considered.

$$I\ddot{\theta} + (cb^2)\dot{\theta} + (ba^2 + b_T)\theta = 0$$

Vibration causing unbalance forces in running mechanical m/c :-

m → machine mass which is under vibration.

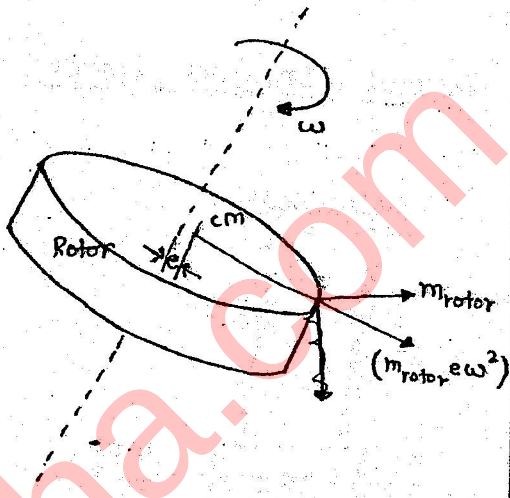
i) Rotating unbalance :-

At t = t

F_{un} in a particular direction -

$F_{un} = (m_{rotor} e\omega^2) \sin\theta$

$= (m_{rotor} e\omega^2) \sin \omega t$



$\boxed{F_{un} = F_0 \sin\omega t}$

F_0 = Max value

ω = forced freq (exitation freq).

ii) Reciprocating unbalance :-

At t = t

$F_{un} = (m_{reci}\, r\, \omega^2) \sin\theta$

$= (m_{reci}\, r\, \omega^2) \sin \omega t$

$\boxed{F_{un} = F_0 \sin \omega t}$

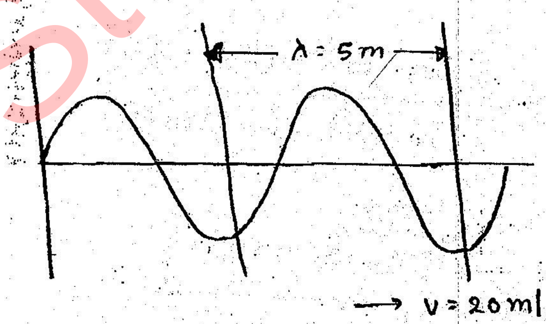
F_0 = max value

ω = forced freq. (speed).

r = crank radius

$= \frac{stroke}{2}$

wave form :-



$F_0 = 200\ N$

$v = f\lambda$

$20 = f \times 5$

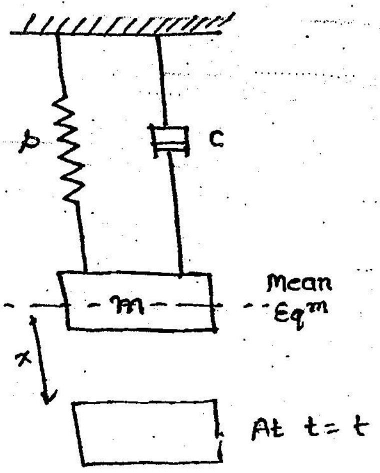
$f = 4\ Hz$

$\omega = 2\pi f = 8\pi\ rad/s$ *

Direct form :-

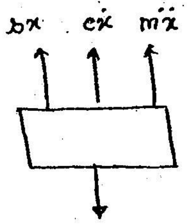
$$F_{un} = \underbrace{200}_{F_o} \cos \underbrace{3}_{\omega} t$$

Forced - Damped System :- Running m/c Analysis -



FBD

At $t = t$



$F_o \rightarrow$ max value

$\omega \rightarrow$ forced freq. (speed)

$$m\ddot{x} + c\dot{x} + px - F_o \sin \omega t = 0$$

$$\boxed{\ddot{x} + (2\xi\omega_n)\dot{x} + (\omega_n^2)x = \frac{F_o}{m} \sin \omega t}$$

Eq^n of forced - Damped system.

The sol^n will be -

$$x = CF + PI \quad **$$

CF $\rightarrow$ $\xi > 1$, $\xi = 1$] No vib ; $\xi < 1$] vib

After some time -

$$CF = 0$$

this time is very less -

$$\therefore \boxed{x = PI}$$

PI

$$PI = \frac{\left(\frac{F_o}{m}\right) \sin \omega t}{D^2 + (2\xi\omega_n)D + \omega_n^2}$$

$$\left\{ \begin{array}{l} D = \text{operator} \\ D^2 = -\omega^2 \end{array} \right\}$$

$$= \frac{\left(\frac{F_o}{m}\right) \sin \omega t}{(\omega_n^2 - \omega^2) + (2\xi\omega_n)D} \times \frac{\{(\omega_n^2 - \omega^2) - (2\xi\omega_n)D\}}{\{(\omega_n^2 - \omega^2) - (2\xi\omega_n)D\}}$$

$$= \frac{\frac{F_o}{m}\left[\overbrace{(\omega_n^2 - \omega^2)}^{R\cos\phi} \sin \omega t - \overbrace{2\xi\omega_n\omega}^{R\sin\phi} \cos \omega t\right]}{(\omega_n^2 - \omega^2)^2 - (-2\xi\omega_n\omega)^2}$$