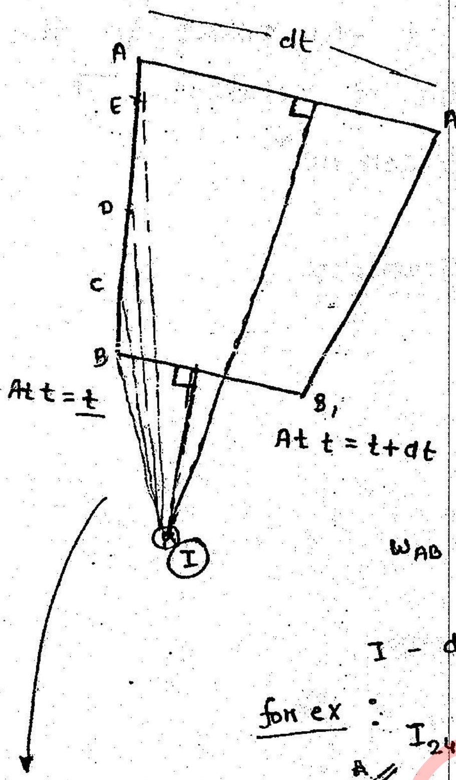
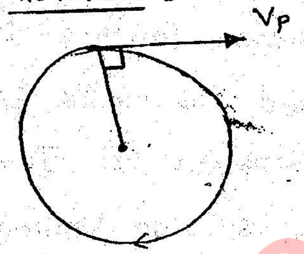
Velocity Analysis

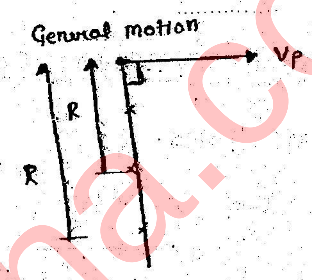
Instantaneous centre method App :-

(Sir Arnhold Keneddy).

I-centre of Rotation :-







$$\omega_{AB} = \frac{V_A}{AI} = \frac{V_B}{BI} = \frac{V_C}{CI} = \frac{V_D}{DI} = \dots\dots$$

I - defined for relative motion b/w two links.

for ex : I_{24} → I-centre for the relative motion b/w link - 2 and link 4

A // I_{42}

In reality -

$\left.\begin{matrix} AA_1 \\ BB_1 \end{matrix}\right\} \rightarrow 0$ (differential)

The link AB at this instant is in general motion.

In general when the link moves to IC of rotation keeps on changing.

Locus of IC of rotation for the relative motion b/w the link ⇒ Centrode.

Locus of I-axis of rotation for the relative motion b/w the link. ⇒ Axode.

Motion	centrode	Axode
General motion	curve	Curved surface
Pure translation	st. line	Plane surface
pure rotation	point	st. line.

" In general the motion of the link in mechanism is neither translation nor pure rotation. It is a combination of translation and rotation which we normally say the link is in general motion but any link at any instant can be assumed to be in pure rotation w.r.t the point in the space known as Instantaneous centre of rotation. This centre is also known as the virtual centre."

No. of Instantaneous centre in a mechanism :-

If no. of links : l

No. of I.C $= {}^{l}C_2$

$$\boxed{= \frac{l(l-1)}{2}}$$

for exm :-

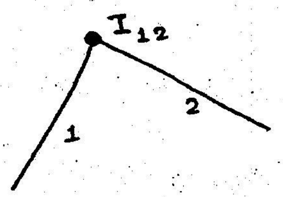
$l = 5$

IC $= 10$

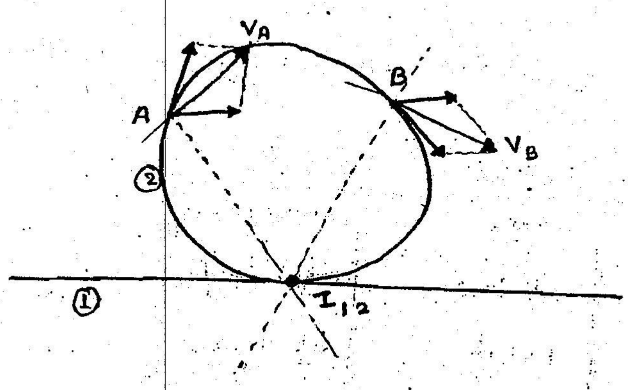
I_{12}	I_{13}	I_{14}	I_{15}
	I_{23}	I_{24}	I_{25}
		I_{34}	I_{35}
			I_{45}

Basic I-centre in a Mechanism :-

1) Turning Pair :-

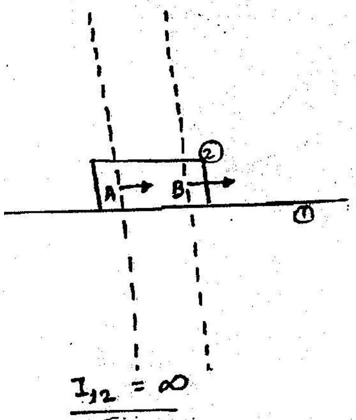


2) Rolling Pair :-



3) Sliding Pair :

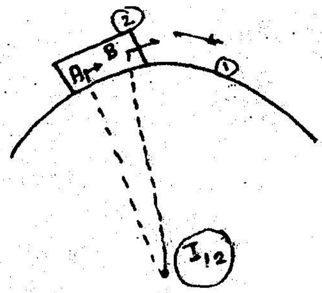
on plane surface



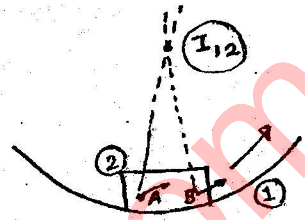
I-c will go at ∞, but in the direction $\perp$ to the sliding surface.

curved surface

convex

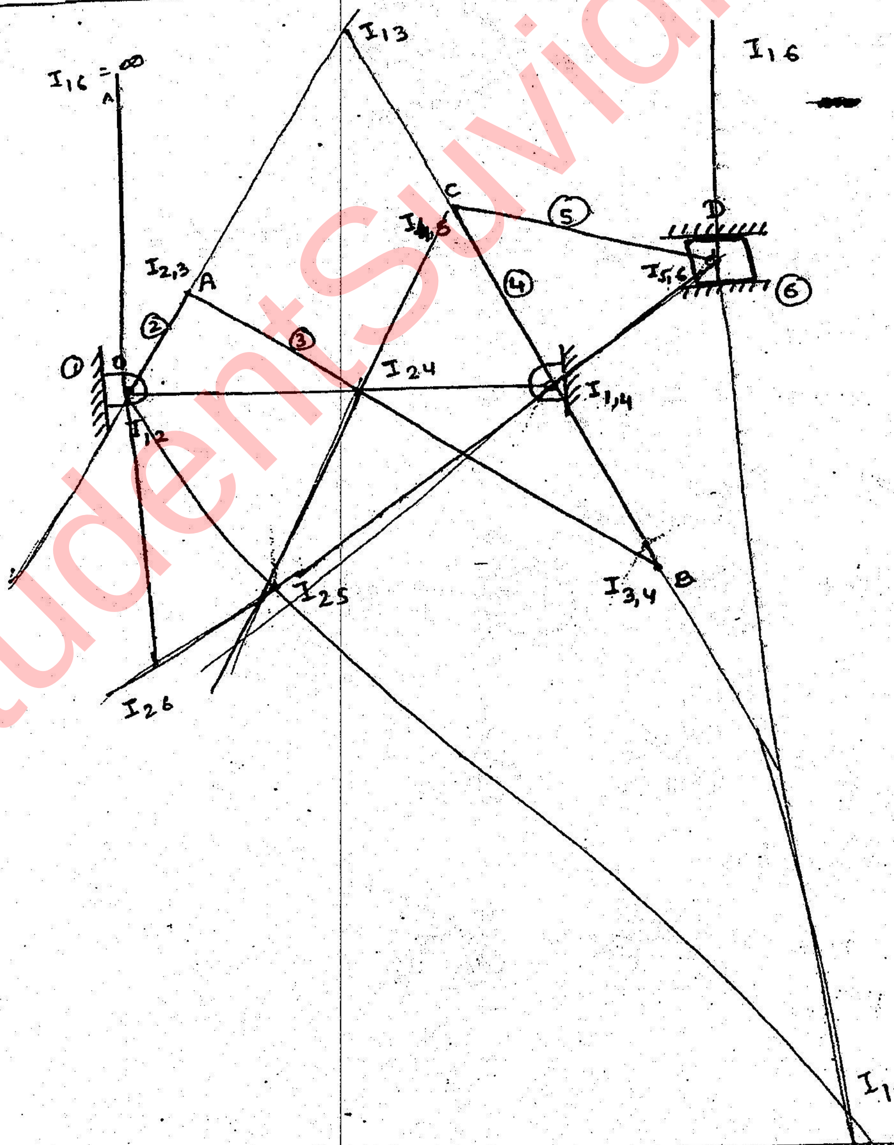


concave



Q27 →

Graphical



$l = 6$

$IC = 15$

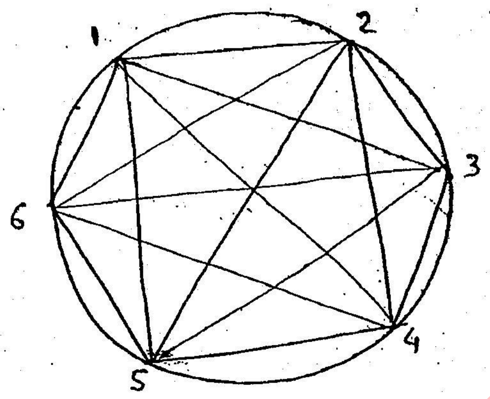
I_{12} I_{13} I_{14} I_{15} I_{16}

I_{23} I_{24} I_{25} I_{26}

I_{34} I_{35} I_{36}

I_{45} I_{46}

I_{56}



I_{13} < 12, 23 ; 14, 43

I_{15} < 14, 45 ; 16, 65

I_{24} < 23, 34 ; 21, 14

25 < 24, 45 ; 21, 15

I_{26} < 25, 56 ; 21, 16

I_{35} < 34, 45 ; 32, 25 ; 31, 15 — Same

I_{46} < 41, 16 ; 42, 26 ; 43, 36 ; 45, 56

I_{36} < 35, 56 ; 32, 26 ; 31, ~~16~~

Given :-

$N_{OA} = 120$ rpm $\qquad \omega_{OA} = \frac{2\pi \times 120}{60} = 4\pi$ rad/s

$V_A = (OA)\,\omega_{OA}$

$= 0.2 \times 4\pi = 2.5132$ m/s

link 3 (A, B) :- $I_{1,3} \rightarrow$ fixed

$\omega_3 = \omega_{AB} = \frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B} \rightarrow$ Graph

link 4 (B, C) :- $I_{1,4}$

$\omega_4 = \omega_{BC} = \frac{V_B}{I_{14}B} = \frac{V_C}{I_{14}C}$

link 5 (C, D) : I_{15}

$\omega_5 = \omega_{CD} = \frac{V_C}{I_{15}C} = \frac{V_D}{I_{15}D}$

Kennedy's Theorum :-

For the relative motion b/w the number of links in a mechanism, any three links, their three IC must lie in st. line.

Theorem of Angular Velocities :-

to find the velocity of I-C.

Any I.C

I_{mn}

can be treated on link m | can also be treated on link n

$$V_{I_{mn}} = \boxed{\omega_m (I_{mn} I_{1m}) = \omega_n (I_{mn} I_{1n})} = V_{I_{mn}}$$

This theorum is applied at I_{mn}

But total IC in use -

$$\left.\begin{matrix} I_{mn} \\ I_{1m} \\ I_{1n} \end{matrix}\right] \begin{matrix} \text{link } 1 \\ \text{link } m \\ \text{link } n \end{matrix} \Bigg]$$

If I_{1m}, I_{1n} lie at the same side of I_{mn} ⇒ Direction same.

If otherwise ⇒ Direction opposite.

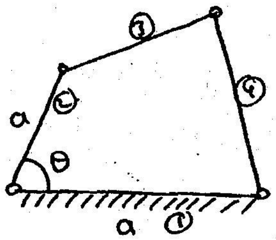
Q, ω_2 → given (clock).

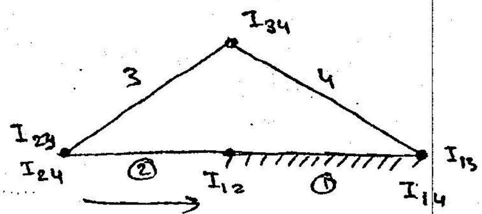
25 : $\omega_2 (I_{25} I_{12}) = \overset{?}{\omega_5} (I_{25} I_{15})$

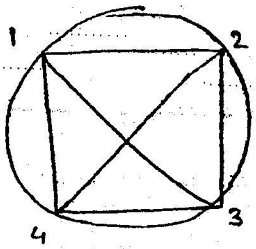
24 : $\omega_2 (I_{24} I_{12}) = \underset{?}{\omega_4} (I_{24} I_{14})$

45 : $\omega_4 (I_{45} I_{14}) = \underset{?}{\omega_5} (I_{45} I_{15})$

Q →







when θ = 180°

ω_2 = 5 rad/s (clock)

ω_5 = ?

$13 \begin{cases} 12, 23 \\ 14, 43 \end{cases}$

$24 \begin{cases} 23, 34 \\ 21, 14 \end{cases}$

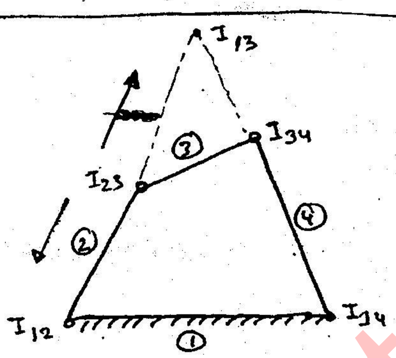
23 →

$$\omega_2 (I_{23} I_{12}) = \omega_3 (I_{23} I_{13})$$

$$5 (a) = \omega_3 (2a)$$

$$\omega_3 = \frac{5}{2} = \underline{\underline{2.5 \text{ rad/s}}} \text{ (clock)}$$

Q →



ω_2 = 5 rad/s (cw)

ω_3 = 14 rad/s

Find the ang. velocity of Link 2 wrt. Link 3.

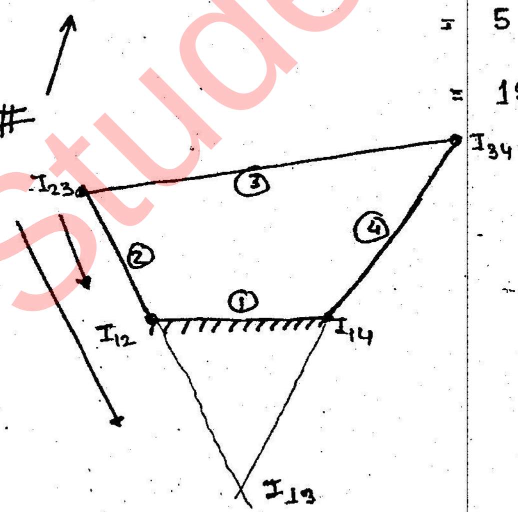
$$\rightarrow \vec{\omega}_{2,3} = (\vec{\omega}_2 - \vec{\omega}_3)$$

$$= +5 - (-14)$$

$$= 5 + 14$$

$$= 19 \text{ rad/s (cw)}$$

#

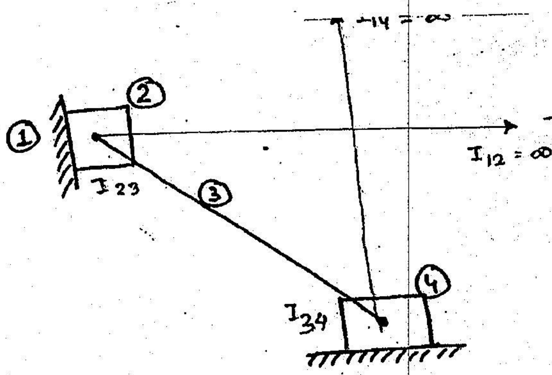


$$\vec{\omega}_{23} = (+5) - (+14)$$

$$= -9$$

$$= \underline{\underline{9 \text{ rad/s}}} = \text{(ACW)}$$

Q →

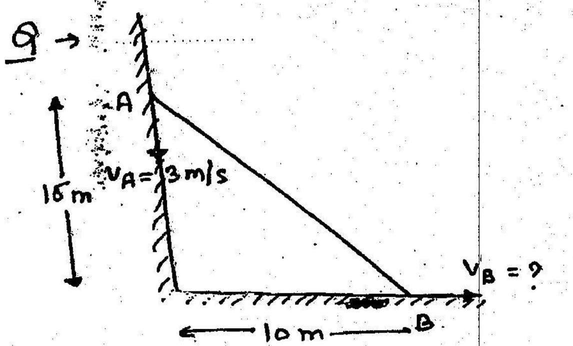


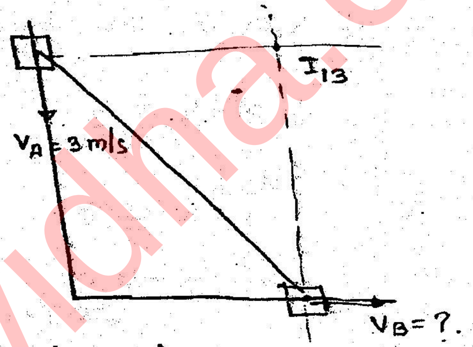
$I_{24} = ?$

$$24 \rightarrow \begin{cases} 23,\ 34 \\ \underbrace{21,\ 14}_{} \ \infty\text{ly displaced line} \end{cases}$$

will lie at ∞

Q →





ladder (link 3)

(I_{13})

$$\omega_3 = \omega_{AB} = \frac{V_A}{I_{13}A} = \frac{V_B}{I_{13}B}$$

$\Downarrow$

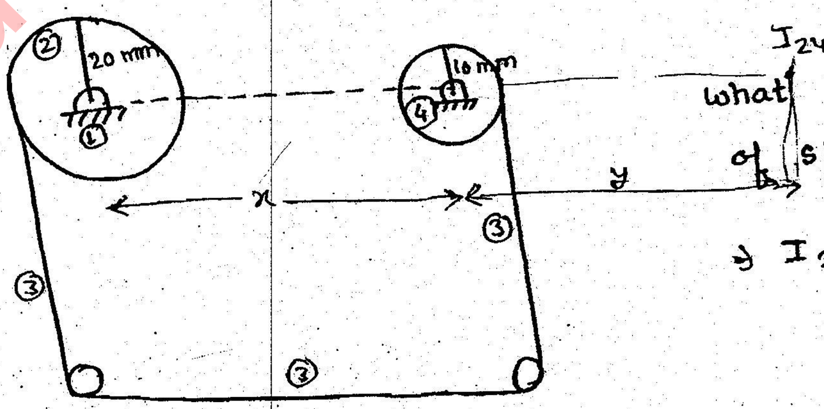
ω on whole length will be same.

$V = r\omega$

$$\frac{3}{10} = \frac{V_B}{15}$$

$$V_B = \frac{9}{2} = 4.5 \text{ m/s}$$

Q →



What is the IC of spools?

→ $I_{24} = ?$

let $v \rightarrow$ velocity of tape.

(24) : $\omega_2 (I_{24} I_{12}) = \omega_4 (I_{24} I_{14})$

$$\frac{V}{0.02}(x+y) = \frac{V}{0.01}(y)$$

$$\frac{1}{2}(x+y) = y$$

$$x+y = 2y$$

$$\boxed{y = x}$$

Note :- if both of the spools are of same size –

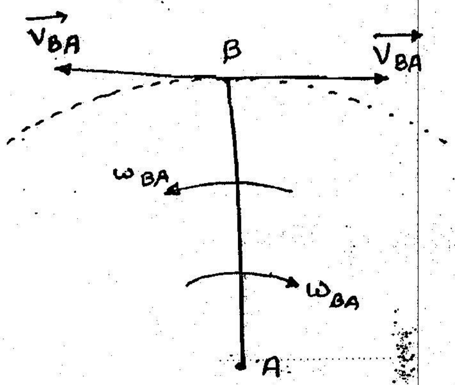
$$\Rightarrow x+y = y$$

Here

$$x \neq 0$$

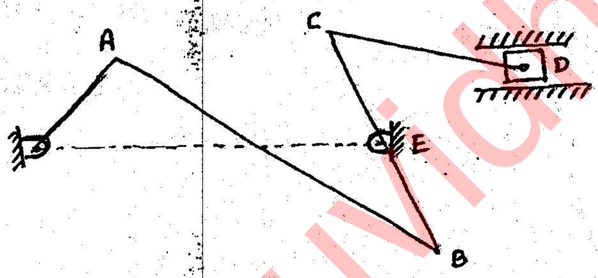
$$\therefore y \rightarrow \infty$$

Relative velocity Method



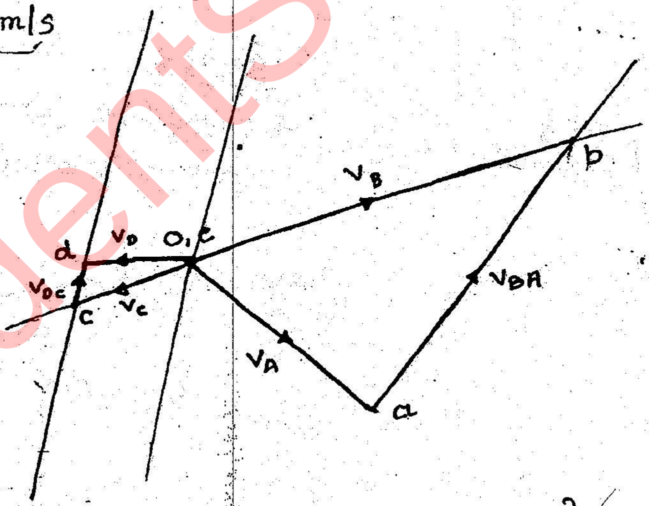
The velocity of point B w.r.t point B will be in the direction ⊥ to the link AB.

Q27 **Graphical** :-



velocity diagram :-

$V_A = 2.5132$ m/s
Scale



V_B = Abs. velocity

V_{BA} = relative velocity.

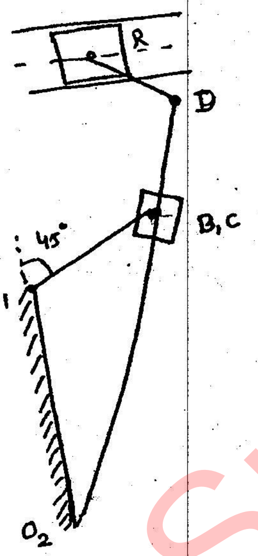
$$\omega_{AB} = \frac{V_{AB}}{AB} \text{ rad/s } (AC)$$

$$\omega_{BC} = \frac{V_{BC}}{BC} \text{ rad/s } (AC)$$

$$\omega_{CD} = \frac{V_{CD}}{CD} \text{ rad/s } (AC)$$

Point	wrt	Procedure
A	O	line $\perp^r$ to link OA
B	A	line $\perp^r$ to link AB
B	E	line $\perp^r$ to link BE
C	$\frac{BC}{BE} = \frac{bc}{be}$	
D	C	line $\perp^r$ to link CD
D	Fixed	line $\parallel$ to the motion of slider

Q- IES- 2006



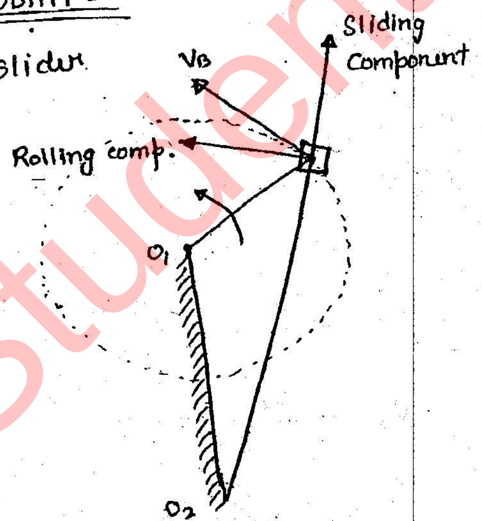
If Crank O_1B = 40 rpm (ACW)

Given : $V_B = (O_1B)\,\omega_{O_1B}$

$= \frac{.25 \times 2\pi \times 40}{6}$ m/s

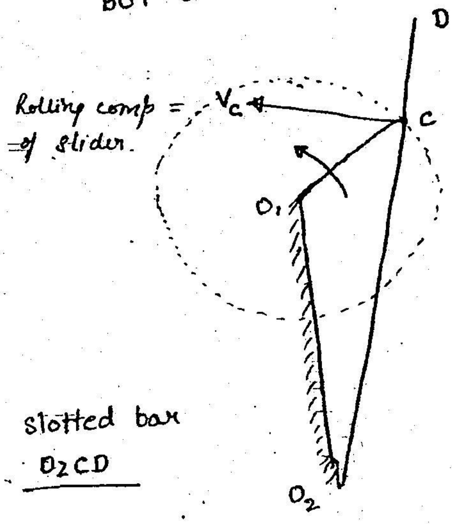
point B

→ slider

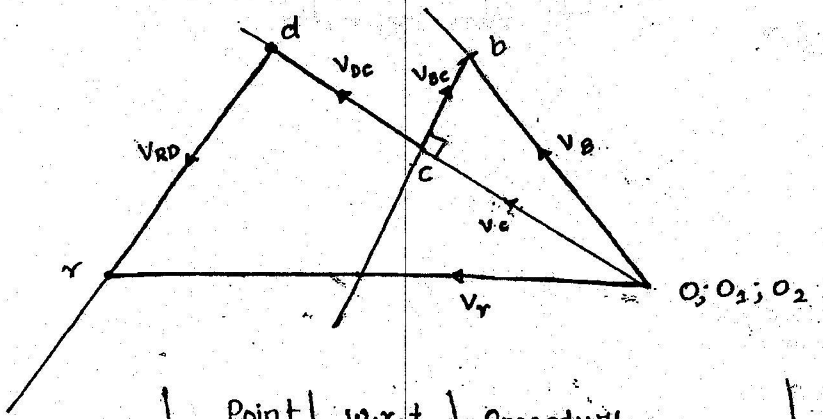


point C

→ coincident point of slider B But on slotted bar



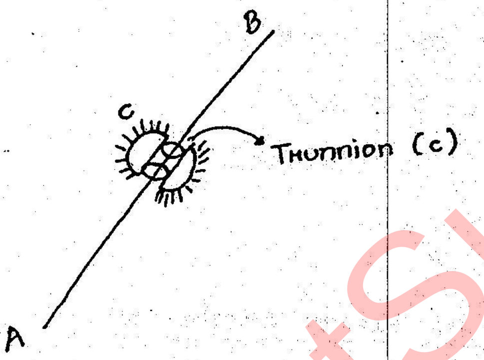
slotted bar O_2CD



Point	w.r.t	Procedure
C	O_2	line $\perp^r$ to link O_2C
C	B	line ∥ to link O_2C

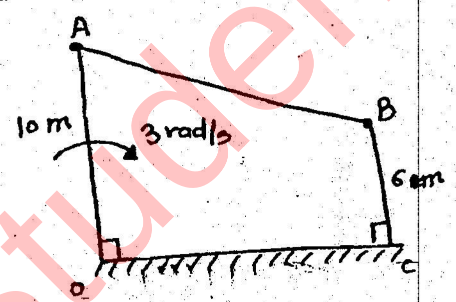
Swivel Trunnion :-

Trunnion joint

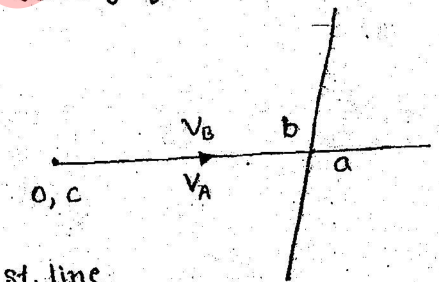


Point	wrt	Procedure
C	A	line $\perp^r$ to link AC
C	Fixed	line ∥ to link AC.

Q →



velocity form -



st. line

Q) The velocity diagram for the given conf. at this instant will be ?

st. line.

Q) $V_B = ?$

$V_B = V_A$

$= 10 \times 3 = 30$ m/s

Q) $\vec{V}_{BA} = ?$

zero

∵ lie at same point

Q → The motion of link AB at this momomunt will be –
↳ Pure translation.
→ General motion

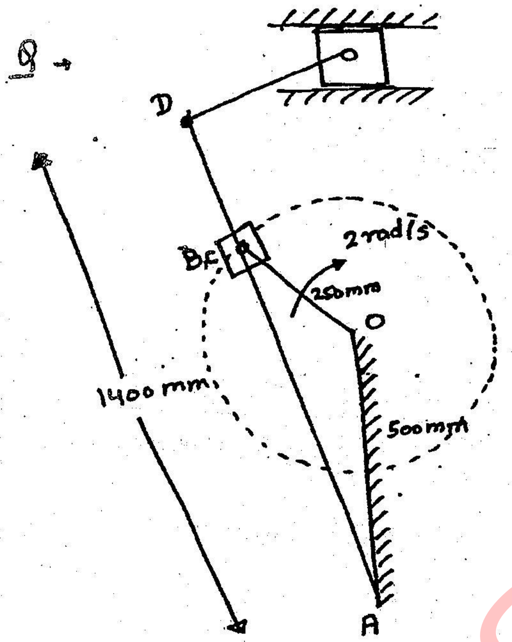
Q → $\omega_{BC} = ?$

$V_A = V_B$

$6 \times \omega_{BC} = 10 \times 3$

$\omega_{BC} = 5 \text{ rad/s}$

Q →



find the Ang velocity of slotted bar when the velocity of RAM is maximum.

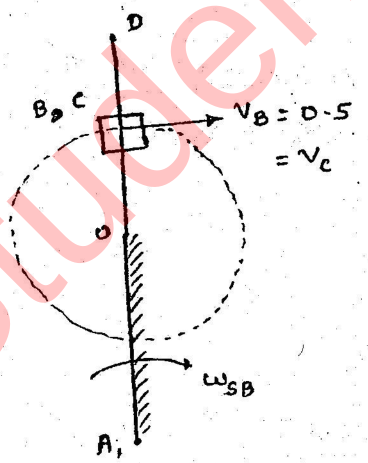
→ Slider velocity –

$V_B = (OB)\,\omega_{OB}$

$= 0.250 \times 2$

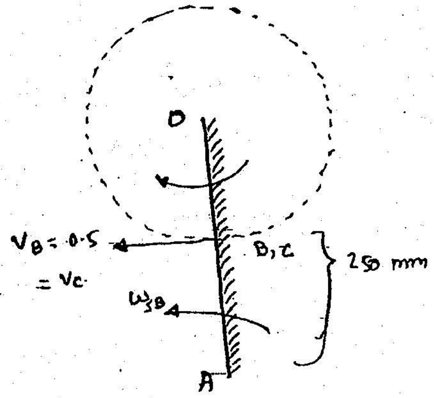
$= 0.5 \text{ m/s}$
(uniform)

In forward stroke :-
cutting –



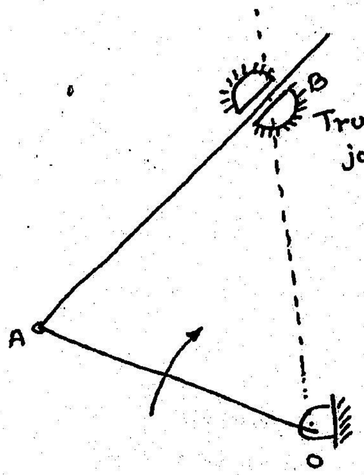
$\omega_{SB} = \dfrac{V_c}{0.75} = \dfrac{0.5}{.75} = \dfrac{2}{3} \text{ rad/s}$

In Return stroke :-



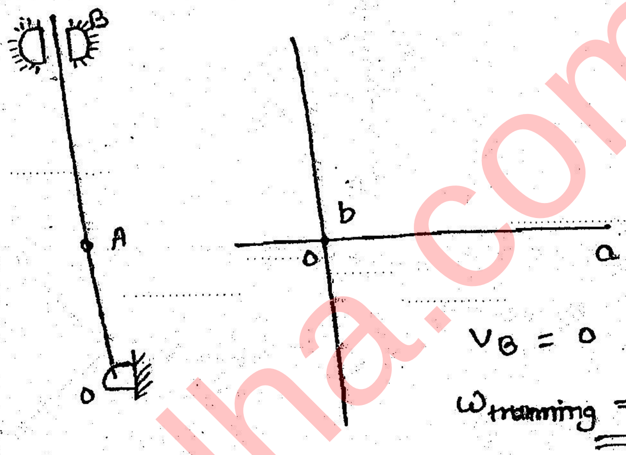
$\omega_{SB} = \dfrac{V_c}{0.250} \rightarrow \dfrac{.5}{.25} = 2 \text{ rad/s}$

Q -



Find the ang. velocity of swivel trunnion when the link OA and AB become vertical .

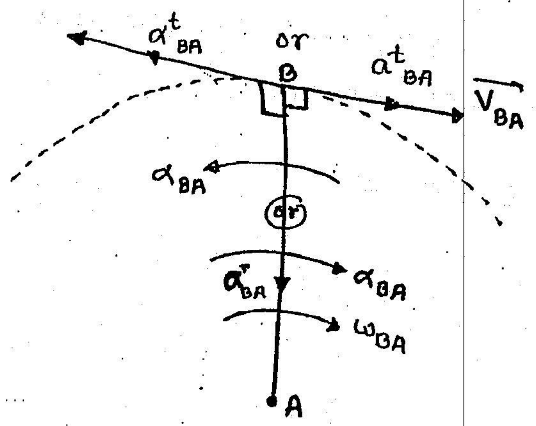
At that moment →



$V_B = 0$

$\omega_{trunnion} = 0$

Acceleration Analysis



In circular motion (this acc $\neq 0$)

b'coz of the change in dirⁿ of velocity

$$\vec{a}_{BA} = \frac{d}{dt}\vec{V}_{BA}$$

$$a_{BA} = \frac{V_{BA}^2}{BA} = BA \times \omega_{BA}^2$$

known for all the link

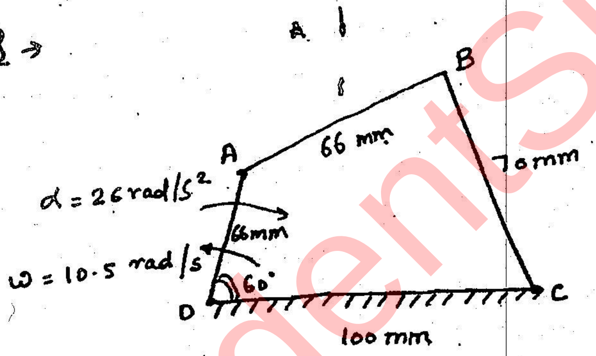
$$a^t_{BA} = (BA)\cdot\alpha_{BA}$$

b'coz of the change in magnitude of velocity

may or may not be zero

only known for input link. ($\perp^r$ to radial dir)

Q →



$a_B = ?$

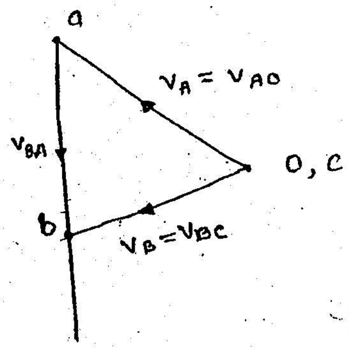
$a_{BA} = ?$

$\alpha_{BA} = ?$

$\alpha_{BC} = ?$

$V_A = 0.066 \times 10.5$ ✓ Given

Scale of velocity Diag. -



$$a^r_{AO} = \frac{V_{AO}^2}{AO}$$

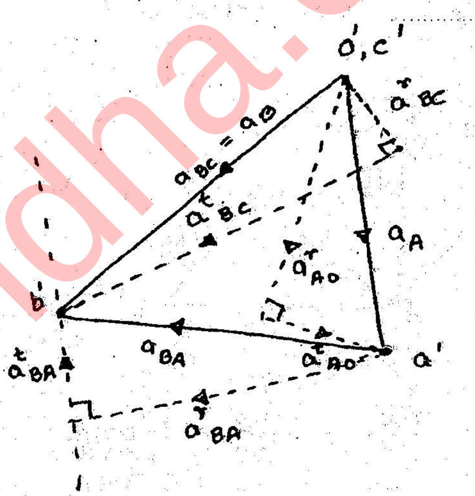
$$a^r_{BA} = \frac{V_{BA}^2}{BA}$$

$$a^r_{BC} = \frac{V_{BC}^2}{BC}$$

$$a^t_{AO} = (AO)\,\alpha_{AO}$$

Known → Scale of Accⁿ Dgm.

Point	wrt	Procedure
A	O	$a^r_{AO} = \frac{V_{AO}^2}{AO}$ (known) $A \to O$ $a^t_{AO} = (AO)\alpha_{AO}$ (known) ($\perp^r$ to radius)
B	A	$a^r_{BA} = \frac{V_{BA}^2}{BA}$ (known) $(B \to A)$ $a^t_{BA} = (BA)\alpha_{BA}$ (known) ? ($\perp^r$ to rad.)
B	C	$a^r_{BC} = \frac{V_{BC}^2}{BC}$ (known) $(B \to C)$ $a^t_{BC} = (BC)\alpha_{BC}$ (known) ? ($\perp^r$ to rod)

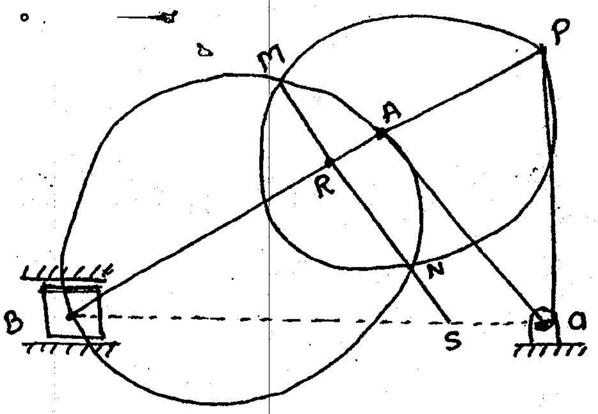


Klein's Construction :-

If It is only applied in Basic single slider - crank mechanism when $\alpha_{crank} = 0$

$\omega_{crank} \to$ given

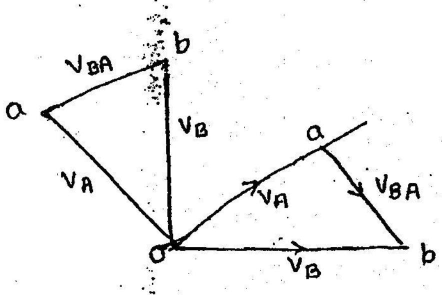
Graphical :



Velocity Analysis →

Δ OAP ≡ velocity Δ

velocity Diag. (Rough)

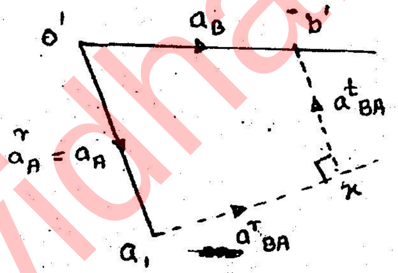


$$\frac{V_A}{OA} = \frac{V_B}{OP} = \frac{V_{BA}}{AP} = \underbrace{\omega_{crank}}_{Given}$$

Acceleration Analysis →

▱ OARS → Acc^n ▱

Acc diag – (Rough)



$$\frac{a_A}{OA} = \frac{a^t_{BA}}{RS} = \frac{a_B}{OS} = \frac{a^r_{BA}}{AR}$$

$$= (\omega_{crank})^2$$

Note :-

- if point O and P coincide ⇒ OP = 0 ⇒ $V_B = 0$
  → Piston at extreme end
- if point O and S coincide ⇒ OS = 0 ⇒ $a_B = 0$ ⇒ V_B = max
  → Slider at mid position
- if point A and R coincide ⇒ AR = 0 ⇒ $a^r_{BA} = 0$
  → Pure translation.
- if points R and S coincide –
  ⇒ RS = 0 ⇒ $a^t_{BA} = 0$
  $(BA)\,\alpha_{BA} = 0$; $\alpha_{BA} = 0$

  ω_{BA} = const ⇒ connecting rod is having const. Ang. velocity.

Coriolis Acceleration (a^c) :-

It is always associated with the slider when the slider is sliding on the rotating body.

The magnitude of this Accⁿ –

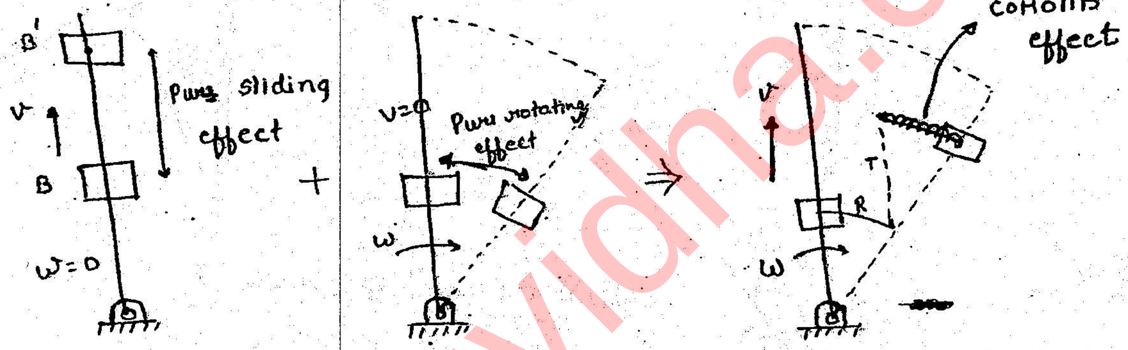
$$\boxed{a^c = 2 v \omega}$$

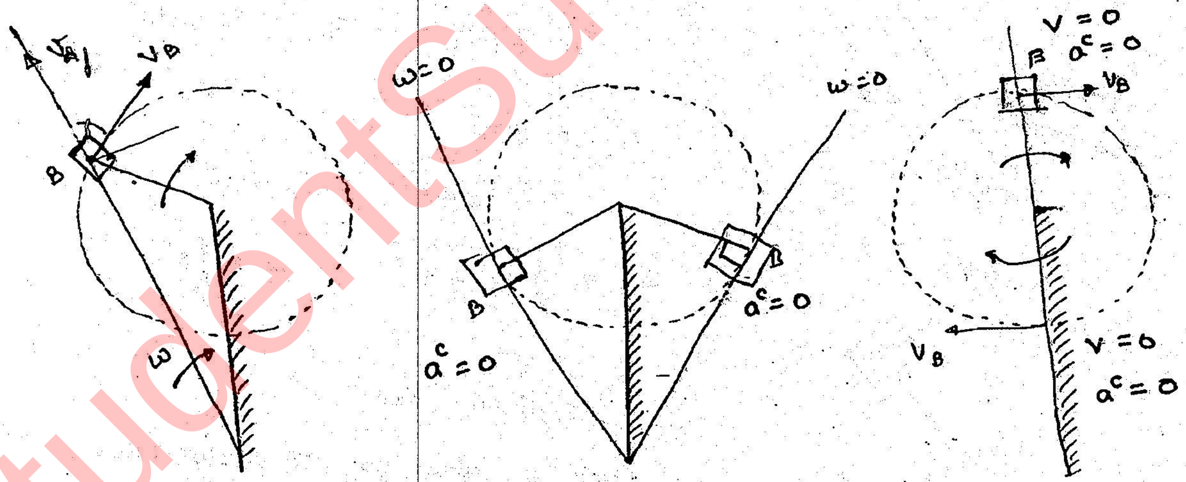
where,

$v \longrightarrow$ sliding velocity of slider.

$\omega \longrightarrow$ Angular velocity of Body on which slider is sliding.

eg :-



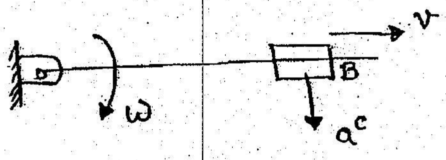


The direction of a^c :-

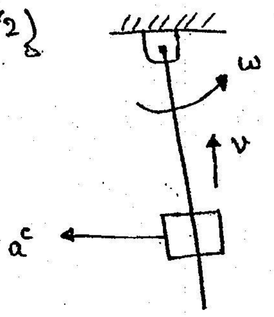
i) take the sense of ω

ii) Rotate the $\vec{v}$ in that sense by $90°$.

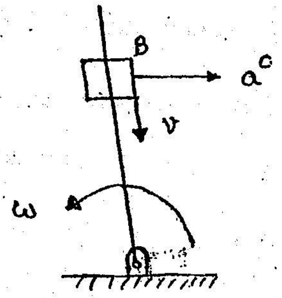
for eg :- i)



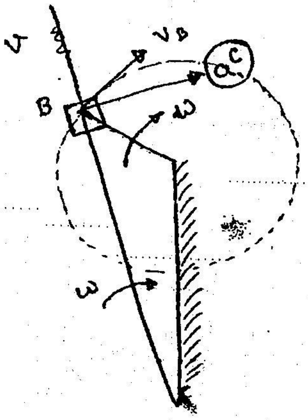
2)



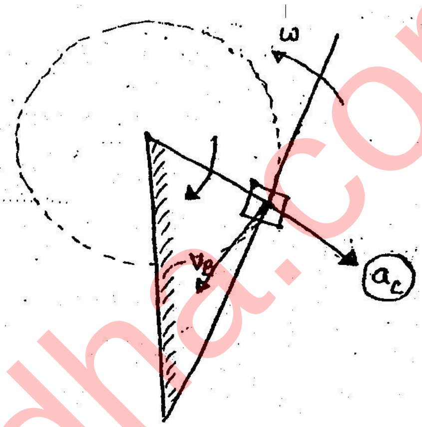
3)



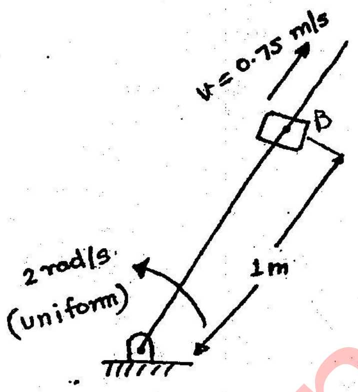
4)

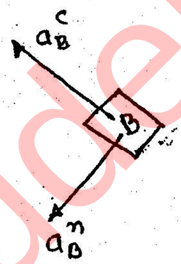


5)



Q →





find the accⁿ of slider ?

$$a_B^c = 2 v \omega$$

$$= 2 \times 0.75 \times 2 = 3 \ m/s^2$$

$$a_B^r = r\omega^2$$

$$= (1)(2)^2 = 4 \ m/s^2$$

$$a_B = \sqrt{3^2 + 4^2} = \underline{\underline{5 \ m/s^2}}$$

if α is given then
acc is taken as its vector sum.