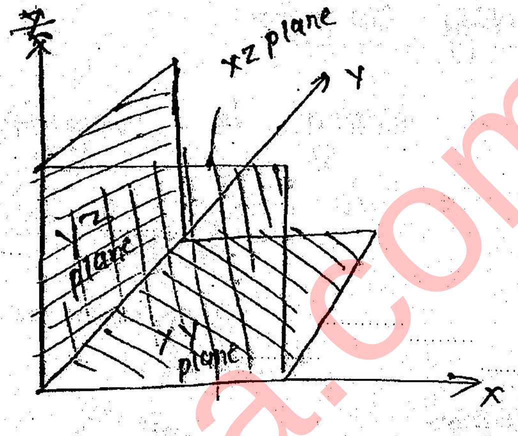
Chapter No:⇒ # * GYROSCOPE *

3-D Space co-ordinate system ⇒

Angular velocity $(\vec{\omega})$



Vector quantity
- Mag. → Rate of change of Angular Displacement
- Dirn → Pridicted by Right Hand thumb rule.

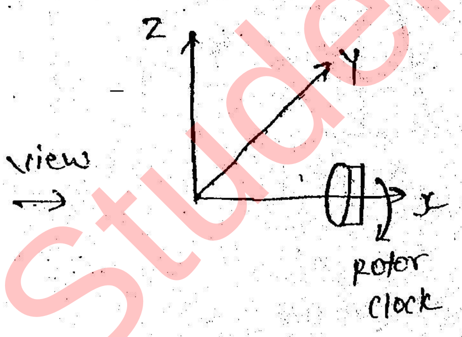
Dirn of ω Also known as Axis of spin

1 → Place right hand in such a way that fingers are $\perp^{ar}$ to the thumb.

2 → Rotate the fingers in the sense of rotation, then the dirn of thumb will be the dirn of Agular velocity.

For Exq ⇒

1.

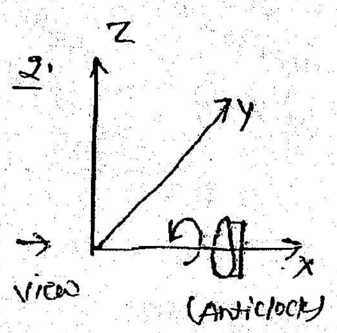


view →   O —ω→ X

Axis of spin → OX

Plane of ~~axis~~ spin → YZ

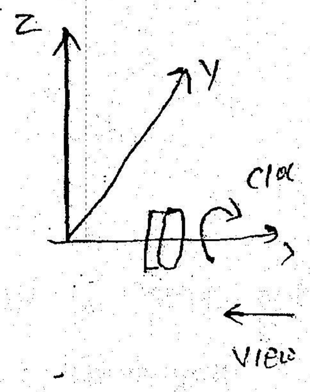
2.



→ view   X ←$\vec{\omega}$— O

Axis of spin → OX

Plane of spin → YZ



X ←ω— O ← ω

Axis of spin → OX

Plane of spin → YZ

Angular Acce^n (α) ⇒

$$\vec{\alpha} = \frac{d\vec{\omega}}{dt}$$

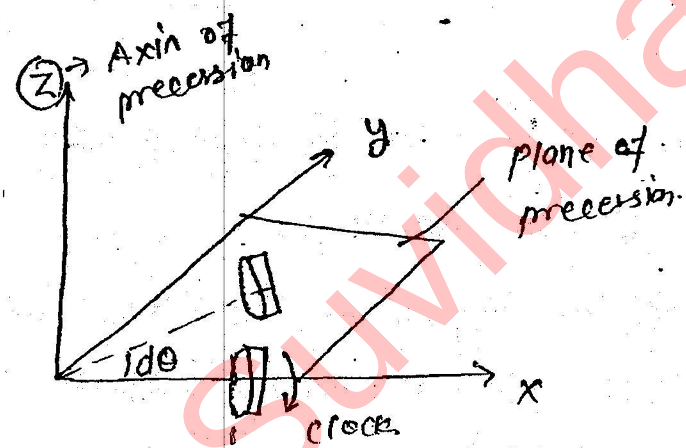
α may come ⇒

- Due to change in magnitude of $\vec{\omega}$.
- Due to the change in Dir^n of $\vec{\omega}$.

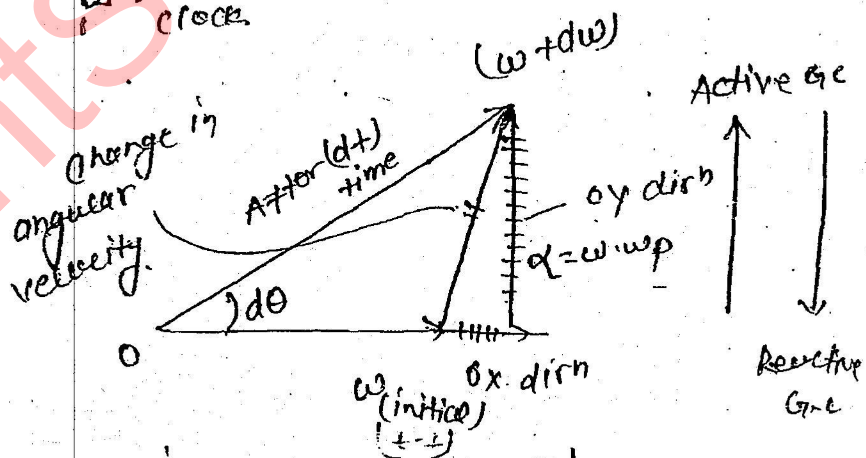
Phenomenum → (precession)

For exam ⇒

view →



view →



Ang Acce^n because of change in magnitude of $\vec{\omega}$ $\Rightarrow \dfrac{(\omega + d\omega)\cos d\theta - \omega}{dt}$

$$\Rightarrow \frac{(\omega + d\omega) - \omega}{dt}$$

$$\Rightarrow \left(\frac{d\omega}{dt}\right) \quad (\text{ox dir}^n)$$

Angular Acceleration because of the change in dir^n of $\vec{\omega}$

$$\Rightarrow \frac{(\omega+d\omega)\sin\theta - 0}{dt} = \frac{(\omega+d\omega)\,d\theta}{dt}$$

$$\Rightarrow \frac{\omega\cdot d\theta + \cancel{d\omega\cdot d\theta}^{\,0}}{dt} = \omega\cdot\left(\frac{d\theta}{dt}\right)$$

$$= \omega\cdot\omega_p$$

(cy dir^n)

$\omega_p \rightarrow$ Ang. velocity of precession

$\left(\omega_p = \frac{d\theta}{dt}\right)$

Total ang. Acce^n

$$\boxed{\vec{\alpha} = \left(\frac{d\omega}{dt}\right)\hat{i} + \underbrace{(\omega\cdot\omega_p)}_{\text{Gyroscopic Acce}^r}\hat{j}}$$

If only dir^n of $\vec{\omega}$ is changing, magnitude is not changing

Then $\underline{\underline{\alpha}} = \underbrace{(\omega\cdot\omega_p)}_{\text{Gyroscopic Acce}^n}$ (cy dir^n)

There fore to have a precession (ie to change the dir^n of $\vec{\omega}$)

In order to provide the $\alpha = \omega\cdot\omega_p$ (cy dir^n),

An external torque is required to be applied to the sys

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That external requirment of torque is known as

<u>Action Gyroscopic couple</u>.

Then:

$$\boxed{\alpha = (\omega \cdot \omega_p) \;\; (\text{oy dir}^n)}$$

⇓

Gyroscopic Acceleration

$C = I\alpha$

$\boxed{C = I\omega \cdot \omega_p}$ (oy dir^n)

Simillar torque will be applied by the system to the external agency in Reverse dir^n.

and that torque is known as Reactive Gyroscopic couple.

$C = I\omega \cdot \omega_p$ (oy dir^n)

And this torque will also be experienced by the system itself because that external agency is the part of system.

After effect of Gyroscopic action on the system

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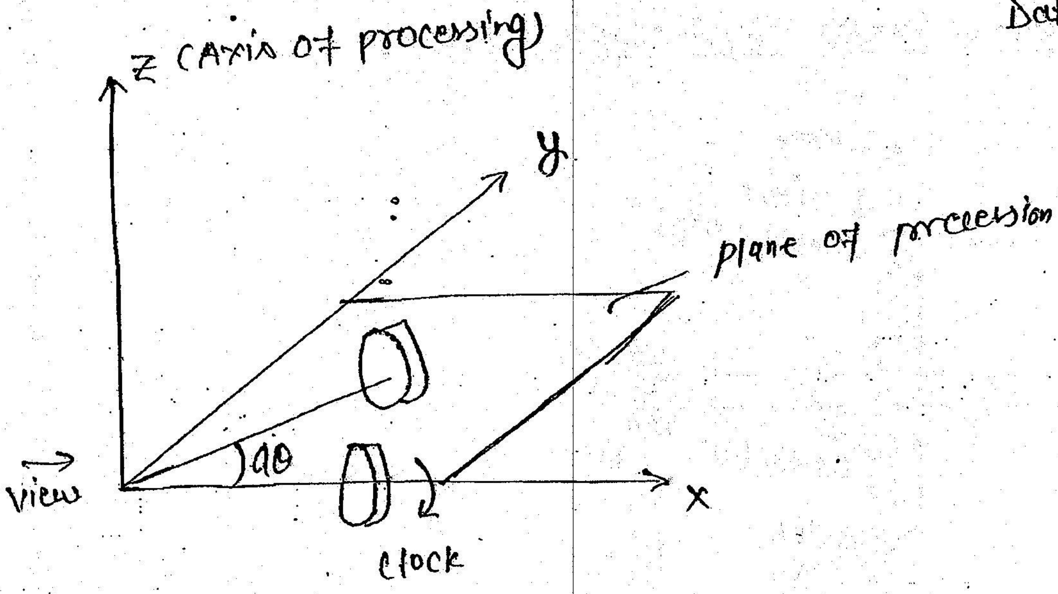
Reactive Gyroscopic couple

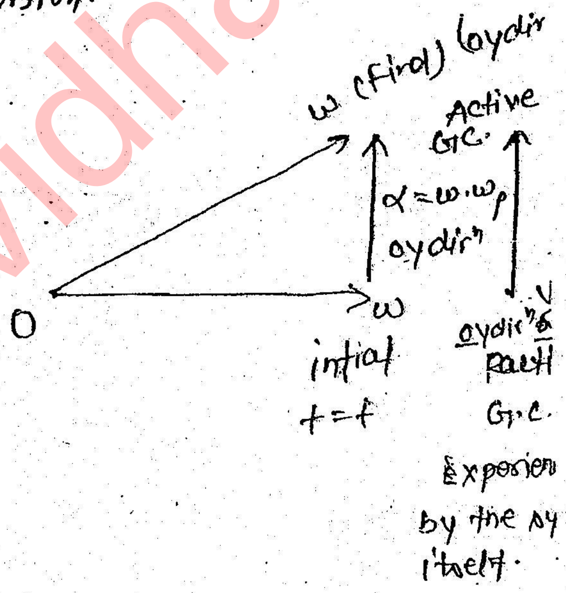
Date - 17/Jun/201



ω → Ang. velocity of rotor.

ω_p → Ang. velocity of precession.

$$\boxed{\omega_p = \frac{d\theta}{dt}}$$



OX - Axis of spin

YZ - plane of spin

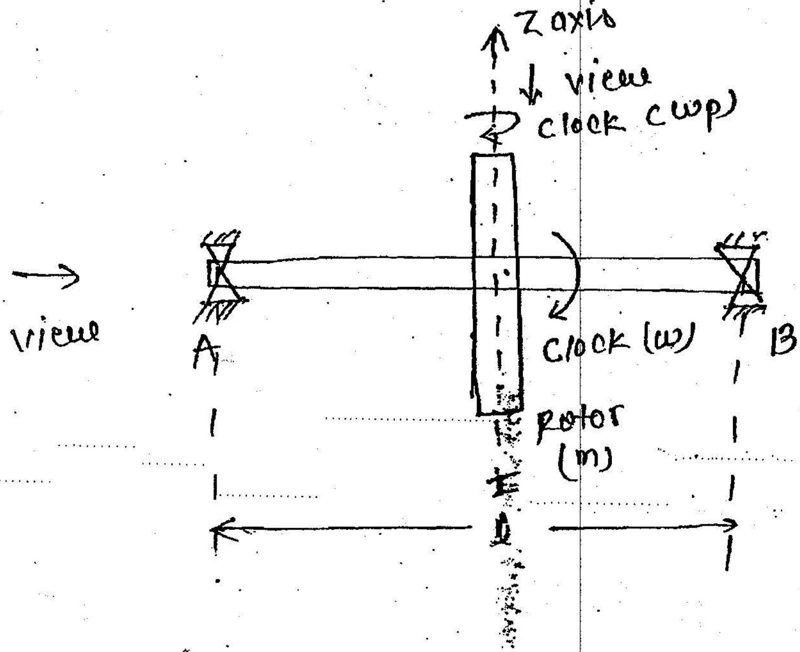
OZ - Axis of precession

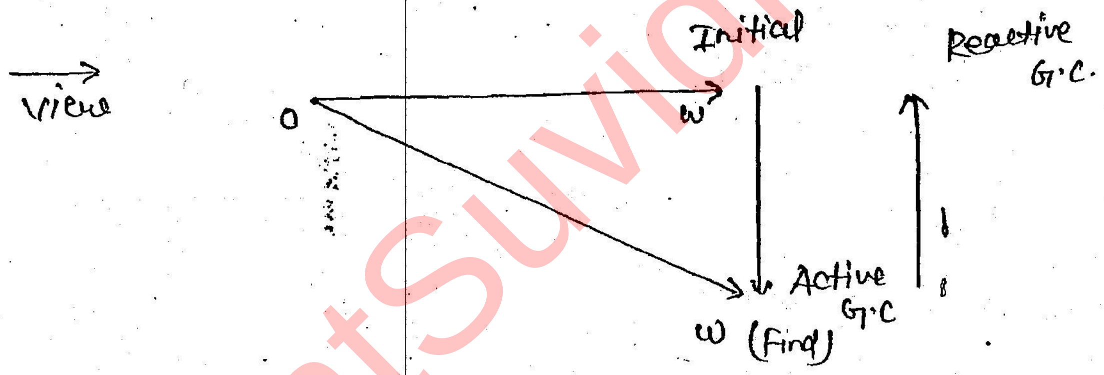
XY - plane of precession

OY - Axis of Gyroscopic couple

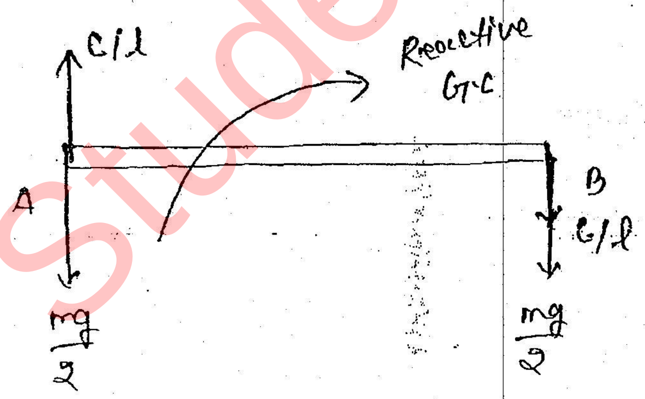
XZ - plane of Gyroscopic couple.

Bearings Reactions under Gyroscopic effect ⇒





Force on shaft



Bearing Reaction Rxn

$$R_A = -\left(\frac{c}{l} - \frac{mg}{2}\right) \text{ (downward)}$$

$$R_B = \left(\frac{c}{l} + \frac{mg}{2}\right) \text{ (upword)}$$

$$F_A = \left(\frac{c}{l} - \frac{mg}{2}\right) \text{ (upword)}$$

$$F_B = \left(\frac{c}{l} + \frac{mg}{2}\right) \text{ (downword)}$$

Qu≥16 $d = 0.2m$ $r = 0.1m$

$m = 10kg$

$I = \frac{mr^2}{2}$ (known)

$l = 0.150m$

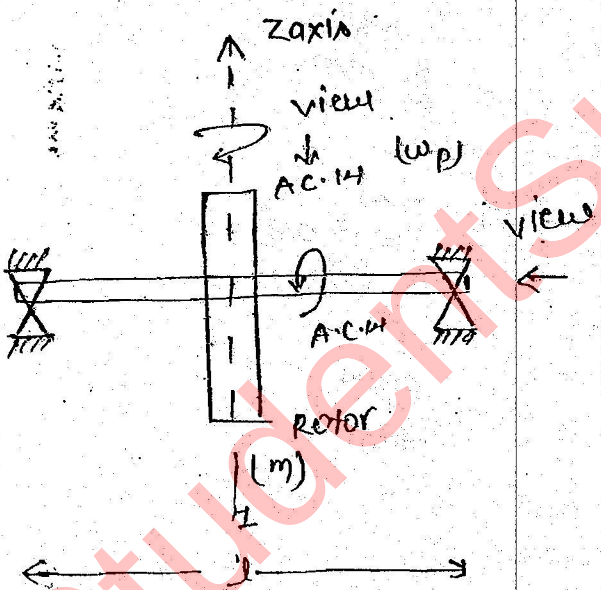
$N = 2000$ rpm $\omega = \frac{2\pi N}{60}$ rad/s.

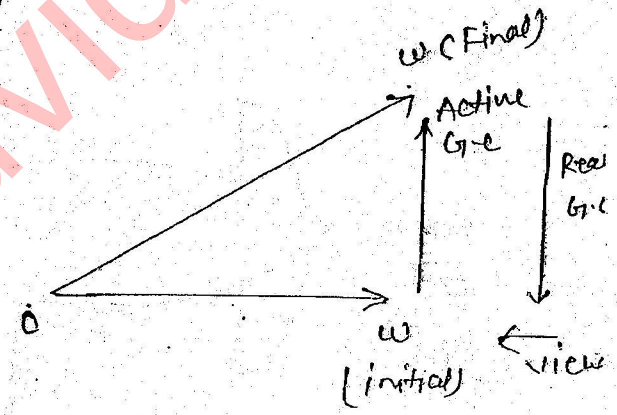
$N_p = 50$ rpm $\omega_p = \frac{2\pi \times 50}{60}$

$C = I \cdot \omega \cdot \omega_p = 54.8103$ N-m

$\frac{C}{l} = \frac{54.8103}{0.150} = 365.4553$ N

$mg = 10 \times 9.81 = 98.1$ N

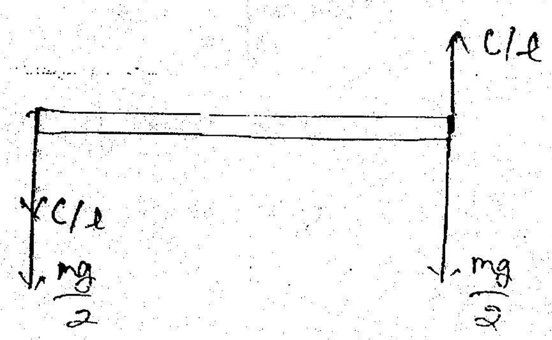




Force on shaft

$F_B = \left(\frac{C}{l} - \frac{mg}{2}\right)$ upward

$F_A = \left(\frac{C}{l} + \frac{mg}{2}\right)$ down word.



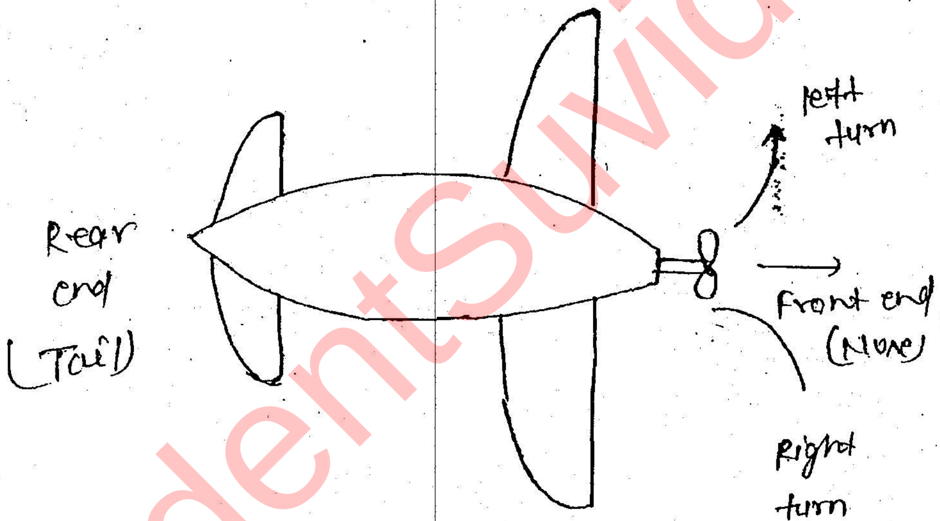
Bearing Reaction

$$R_B = \left(\frac{C}{\ell} - \frac{mg}{2} \right) \quad \text{(downword)}$$

$$R_A = \left(\frac{C}{\ell} + \frac{mg}{2} \right) \quad \text{(upword)} \quad = 414$$

Gyroscopic effect on turnning of Aircraft ⇒

Front end of the aircraft is known as Nose and the Rear end of the aircraft is known as Tail.



$$(\omega_p)_{Turnning} = \frac{V}{R}$$

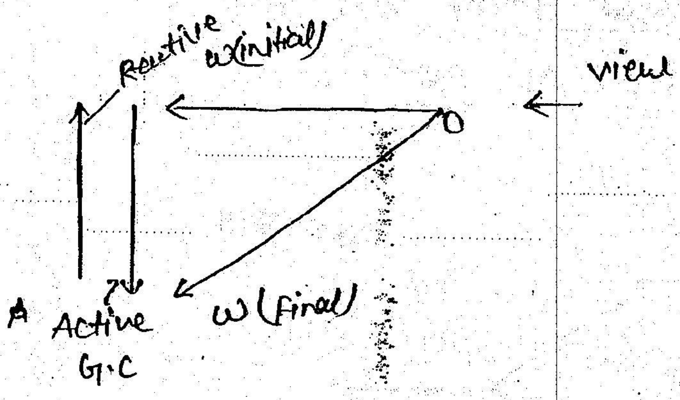
V → velocity of turn

R → Radi of Turn.

viewed from Nose (from end)

(Engine Rotate clockwise)

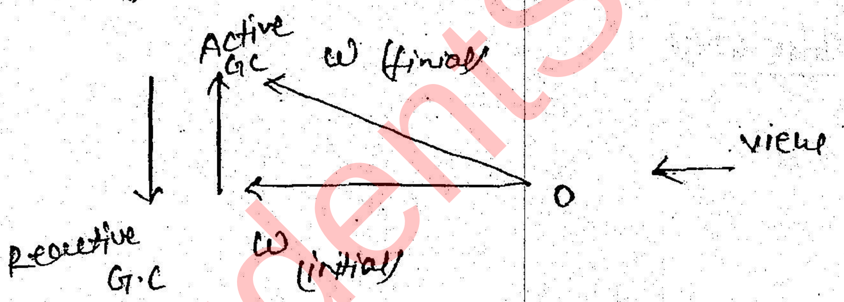
1- Left turn



Effect

Nose will go down & Tail will go up.

2- Right turn ⇒



Effect ⇒ Nose will go up & Tail will go down.

Gyroscopic effect on ship during turning ⇒

Front end of the ships is known as Bow and the Rear end of the ships is known as Stern.

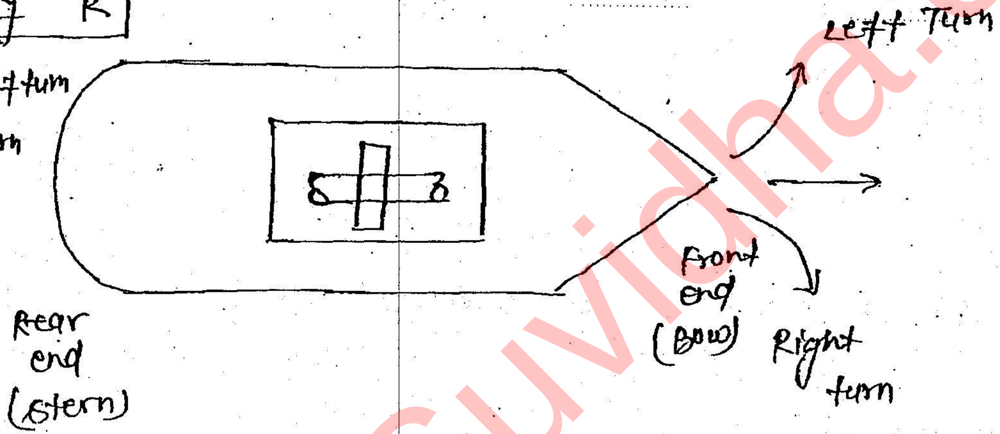
Looking from ~~stren~~ stern side the R.H.S of the ship portion is known as Star Board and the L.H.S portion of ship is known as port.

$$\boxed{(\omega_p)_{turning} = \frac{V}{R}}$$

V ⇒ velocity of turn

R ⇒ radi of turn

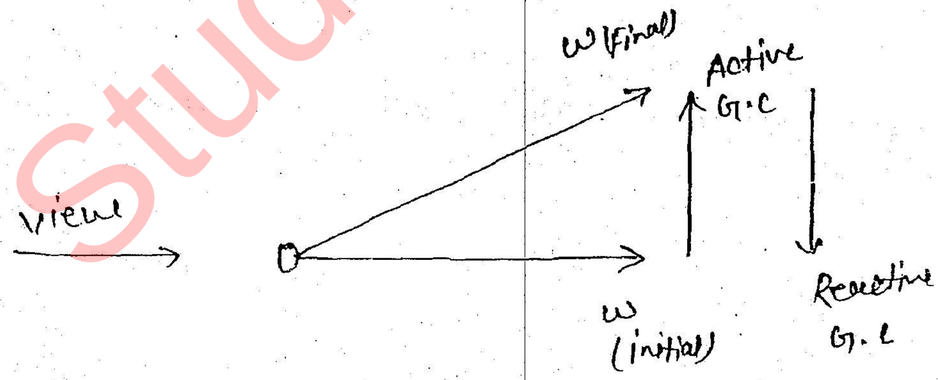
view →



Viewed from stern (Rear End)

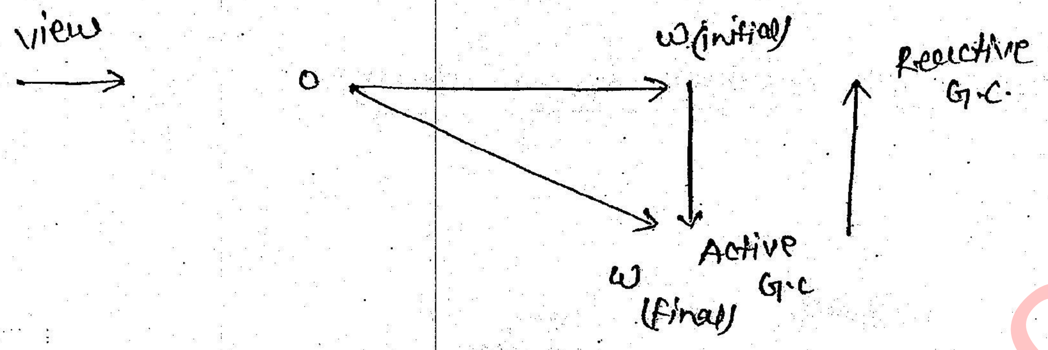
Engine Rotate clockwise

① Left turn ⇒



Effect → Bow will go up and stern will go down.

(2) Right turn ⇒

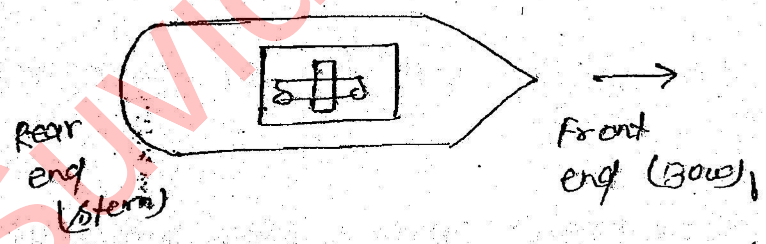


Effect ⇒ Bow will go down & stern will go up.

Gyroscopic effect on ships during pitching ⇒

$\omega \rightarrow$ speed of engine

$\omega_p \rightarrow$ For pitching
↓
oscillation



If $T \rightarrow$ Time period of oscillation of pitching.

$$\boxed{(\omega_p)_{pitching} = \frac{2\pi}{T}}$$

Transverse axis – pitching
Longitudinal axis – Rolling

$$x = A \sin \omega_n t$$

$$\theta = \theta_0 \sin \omega_n t$$

$$\omega_p = \frac{d\theta}{dt} = (\theta_0 \cdot \omega_n) \cdot \cos(\omega_n t)$$

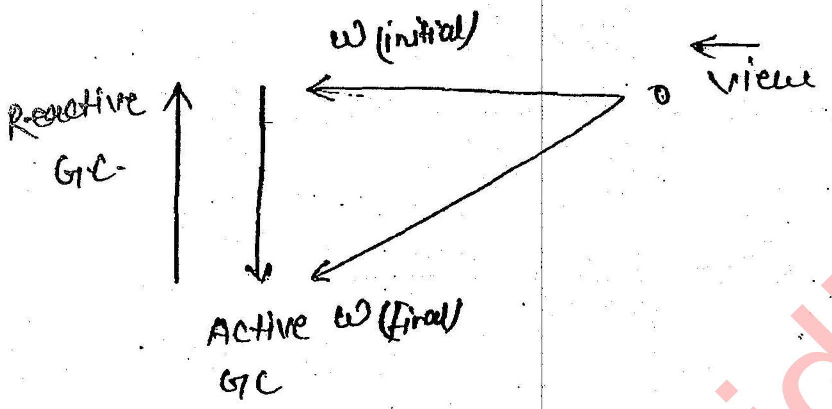
$$\boxed{(\omega_p)_{max\,(pitching)} = \theta_0 \cdot \omega_n}$$
(rad)

Angular Amplitude.

Viewed from Bow :⇒

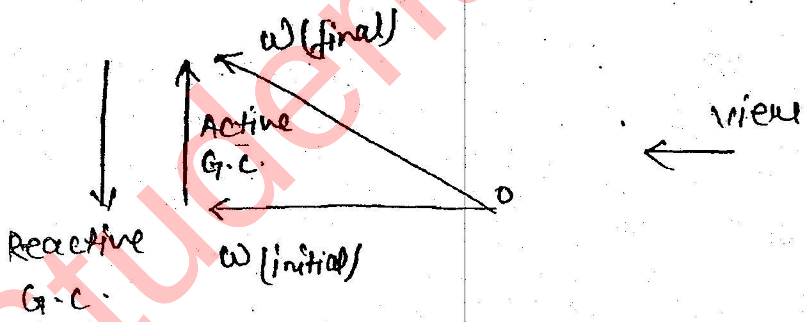
Engine Rotate clockwise.

1⇒ Bow moves up (stern moves down)



Effect ⇒ Ship will try to turn toword left (port) side.

2. Bow moves down (stern moves up)



Effect ⇒ Ship will try to turn towords Right (star board side)

Gyroscopic effect on ships under Rolling ⇒

During rolling, The direction of Angular velocity is not changing

$$\Rightarrow \omega_p = 0$$

$$\Rightarrow I \cdot \omega \cdot \omega_p = 0$$

Ship will not experience any Gyroscopic effect during Rolling.

Effect (Gyroscopic) on a 4-wheeler during Turning ⇒

$I_E \rightarrow$ M.I. of Engine.

$I_W \rightarrow$ M.I. of wheel.

$\omega_E \rightarrow$ speed of engine

$\omega_W \rightarrow$ speed of wheel

$\omega_p \rightarrow \frac{V}{R}$ (Turning)

$\longrightarrow$ same for wheel and engine.

Gyroscopic couple ⇒

Engine $\quad C_E = I_E \cdot \omega_E \cdot \omega_p$

Wheel: $\quad C_W = 4 \cdot I_W \cdot \omega_W \cdot \omega_p$.

Total Gy. couple

$$C = C_E \pm C_W$$

$$\boxed{C = (I_E \cdot \omega_E \pm 4 I_W \cdot \omega_W) \cdot \omega_p}$$

If $l \rightarrow$ distance b/w inner and outer wheel.

~~for each on wheel~~

For on each Inner wheel. ~~2 outer wheels.~~ $= \frac{C}{2l}$ (up word)

Force on each outer wheel $= \frac{C}{2l}$ (downword)

Gy. effect during Turning.

Pb-17

solve

$m = 3500\ kg$

$k = 0.45m$

$I = mk^2$

$I = 708.75\ kg\text{-}m^2$

$N = 3000 rpm$

$\omega = \frac{2\pi \times 300\cancel{0}}{6\cancel{0}}$ $\omega = 100\pi = 314.15922\ rad/s$

(a) $(\omega_p)_{Turning} = \frac{v}{R} = \frac{\left(36 \times \frac{5}{18}\right)}{100} = 0.1\ rad/s.$

$C = I \cdot \omega \cdot (\omega_p)_{Turning}$

$= 22266.033\ N\text{-}m$

(5) $\omega_n = \frac{2\pi}{T} = \frac{2\pi}{40} = 0.157$ rad/s.

$\theta_0 = 6^\circ = \frac{6\pi}{100} = \frac{\pi}{30}$ rad

$(\omega_p)_{\text{pitching (max)}} = \theta_0 \cdot \omega_n$

$= \frac{\pi}{30} \times 0.157$

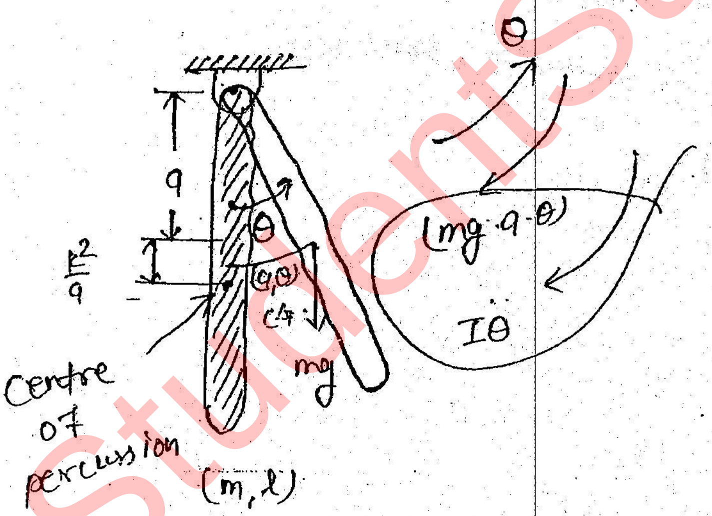
$= 0.01644$ rad/s.

$C = I \cdot \omega \cdot (\omega_p)_{\text{pitching}}$

$= 3659.0460$ N-m //

Date 18/Jun/2016

Compound pendulum ⇒



K = Radius of Gyration (About CG Axis)

$I\ddot{\theta} + (mg \cdot a)\theta = 0$

$(\cancel{m}k^2 + \cancel{m}a^2)\ddot{\theta} + \cancel{m}g \cdot a \cdot \theta = 0$

$\ddot{\theta} + \left(\frac{ag}{k^2+a^2}\right)\theta = 0$

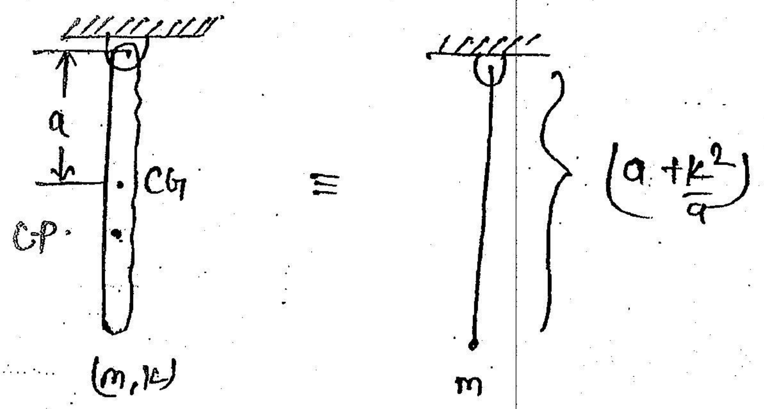
$\ddot{\theta} + \left(\frac{g}{\left(a + \frac{k^2}{a}\right)}\right)\theta = 0$

$\omega_n = \sqrt{\frac{g}{\left(a + \frac{k^2}{a}\right)}}$

$T = 2\pi \sqrt{\frac{a + \frac{k^2}{a}}{g}} = 2\pi \sqrt{\frac{l_e}{g}}$

Here -

$$l_e = \left(a + \frac{k^2}{a}\right)$$

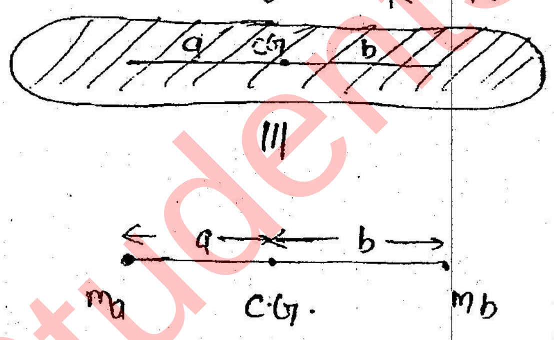


Dynamically equivalent system ⇒

The system is going to be said as dynamically equivalent if.

1) Total mass of the system is same
2) centre of gravity must be same.
3) Moment of inertia must be same.

Body (m). k → Rad. of Gyration about C.G.



$$m_a + m_b = m \quad \text{— (1)}$$

$$m_a \cdot a = m_b \cdot b \quad \text{— (2)}$$

$$m_a \cdot a^2 + m_b \cdot b^2 = m k^2 \quad \text{— (3)}$$

By eq^n ① & ②

$$m_a + m_a \cdot \frac{a}{b} = m$$

$$m_a \cdot \frac{(a+b)}{b} = m$$

$$\boxed{m_a = \frac{m \cdot b}{(a+b)}} \qquad \boxed{m_b = \frac{m \cdot a}{(a+b)}}$$

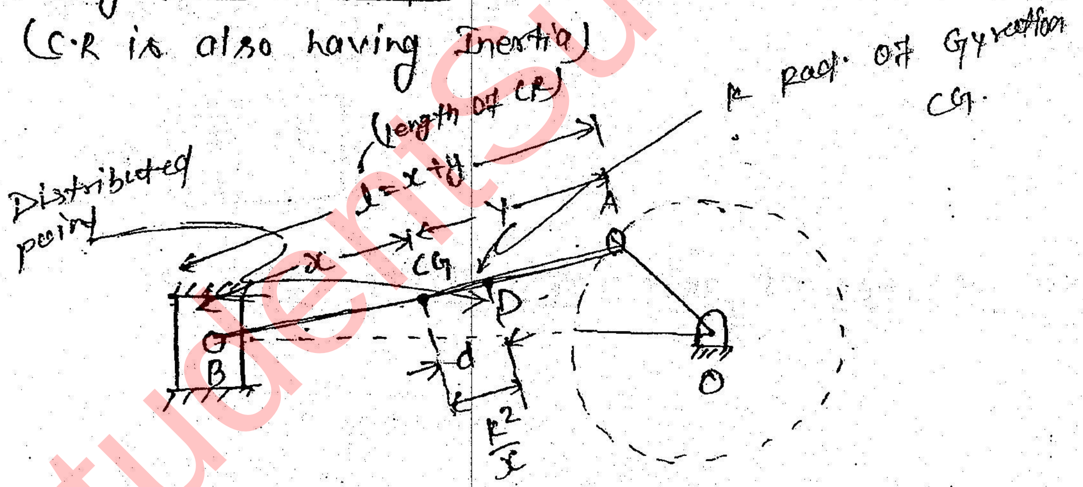
$$\frac{\cancel{m} \cdot b}{(a+b)} \cdot a^2 + \frac{\cancel{m} \cdot a}{(a+b)} b^2 = \cancel{m} k^2$$

$$ab \frac{\cancel{(a+b)}}{\cancel{(a+b)}} = k^2$$

$$\boxed{k^2 = ab} \qquad b = \frac{k^2}{a}$$

Correcting couple (Torque)

(C.R is also having Inertia)



For C.R. to be

Dynamically equivalent system mass must be distributed at B & D

But in the analysis it is convenient to distribute the mass of CR and B & A instead of B & D.
Therefor there is a correction in Torque.

Correcting Torque (ΔT)

$m = I\alpha$
$= mk^2\alpha$

$$\Delta T = \underbrace{m \cdot \alpha_c (x \cdot y)}_{\text{we are taking this}} - \underbrace{m\alpha_c (x \cdot d)}_{\text{Actual}}$$

$$= m\alpha_c \cdot x [y - d]$$

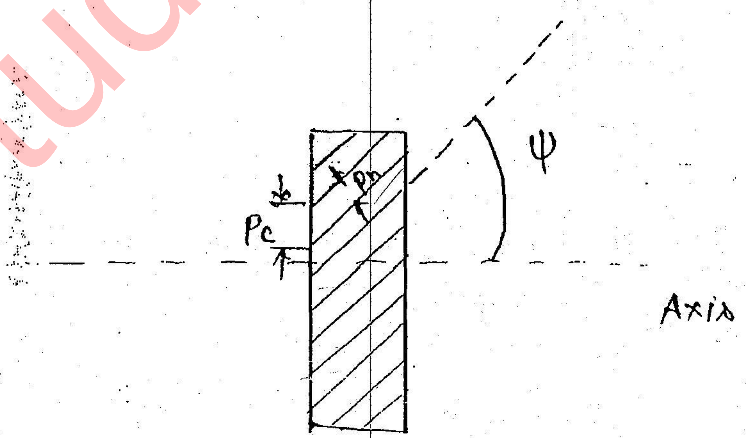
$$= m\alpha_c \cdot x [(x+y) - (x+d)]$$

$$\boxed{\Delta T = m\alpha_c \cdot x [l - L]} \; **$$

$\boxed{L = x + d}$ **

$\boxed{d = \frac{k^2}{x}}$ **

Terminology of Helical gear or spiral Gear ⇒



ψ = spiral angle (Helix angle)

P_c = circular pitch

P_n = Normal pitch

$$\boxed{P_n = P_c \cos\psi}$$

For two mating Gears :-

$$* \boxed{P_{n_1} = P_{n_2}}$$

$P_c = \pi \cdot m$

$$\boxed{P_n = \pi \cdot m_n}$$

$$\boxed{m_n = m \cdot \cos\psi}$$

For two mating Gear -

$$\boxed{m_{n_1} = m_{n_2}}$$

$\theta \rightarrow$ Angle b/w two shaft

$1 \rightarrow$ Driver

$2 \rightarrow$ Driven

$\theta = (\psi_1 + \psi_2)$ (same Handed Gears)

$\theta = (\psi_1 - \psi_2)$ (opposite Handed Gears)

Note:- If in the problem, it is not given that which hand gear in contact then always take same hand.

$$** \boxed{\theta = \psi_1 + \psi_2}$$

Centre distance $\Rightarrow$

$$r_1 + r_2$$

$$= \frac{m_1 T_1}{2} + \frac{m_2 T_2}{2}$$

$$= \frac{m_n}{\cos\psi_1} \cdot \frac{T_1}{2} + \frac{m_n}{\cos\psi_2} \cdot \frac{T_2}{2}$$

$$\boxed{= \frac{m_n}{2}\left[\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right]}$$

<u>velocity Ratio (V.R)</u> $\Rightarrow$

$$V_{normal} = V_1 \cos\psi_1 = V_2 \cos\psi_2$$

$$\frac{V_2}{V_1} = \frac{\cos\psi_1}{\cos\psi_2}$$

$$V.R = \frac{\omega_2}{\omega_1} = \frac{\frac{V_2}{r_2}}{\frac{V_1}{r_1}} = \frac{V_2}{V_1} \cdot \frac{r_1}{r_2}$$

$$VR = \frac{V_2}{V_1} \cdot \frac{\frac{m_1 T_1}{\cancel{2}}}{\frac{m_2 T_2}{\cancel{2}}}$$

$$= \frac{V_2}{V_1} \cdot \frac{m_1}{m_2} \cdot \frac{T_1}{T_2}$$

$$= \frac{V_2}{V_1} \cdot \frac{\frac{\cancel{m_n}}{\cos\psi_1}}{\frac{\cancel{m_n}}{\cos\psi_2}} \cdot \frac{T_1}{T_2}$$

$$= \frac{\cancel{\cos\psi_1}}{\cancel{\cos\psi_2}} \cdot \frac{\cancel{\cos\psi_2}}{\cancel{\cos\psi_1}} \cdot \frac{T_1}{T_2}$$

$$\boxed{VR = \frac{T_1}{T_2}}$$

Efficiency ⇒

$$\eta = \frac{F_2 \cdot V_2}{F_1 \cdot V_1}$$

$$\boxed{\eta = \frac{\cos(\psi_2+\phi)}{\cos(\psi_1-\phi)} \cdot \frac{\cos\psi_1}{\cos\psi_2}}$$

ϕ = pressure angle

if μ = coefficient of friction

$\phi = \tan^{-1}\mu$

Max efficiency

$$\boxed{\eta_{max} = \frac{1+\cos(\theta+\phi)}{1+\cos(\theta-\phi)}}$$

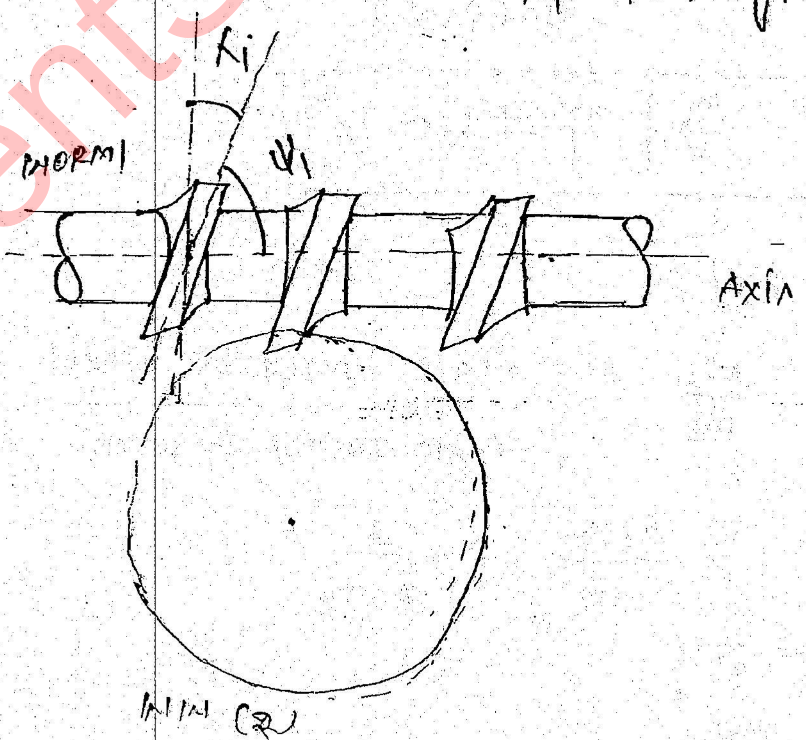
* $\theta = \psi_1 + \psi_2$
* $\psi_1 = \frac{(\theta+\phi)}{2}$

Terminology of worm wheel ⇒

(worm: or spiral)

1- WORM
2- worm wheel

λ_1 = lead angle



$\theta = 90°$

$\lambda_1 + \psi_1 = 90°$

$\boxed{\psi_1 = (90° - \lambda_1)}$ *

$\theta = \psi_1 + \psi_2$

$90° = (90° - \lambda_1) + \psi_2$

$\boxed{\psi_2 = \lambda_1}$ *

$$\tan\lambda_1 = \frac{\text{lead}}{\pi d_1}$$

$$\boxed{\tan\lambda_1 = \frac{l}{\pi d_1}}$$

l = lead = p (single start)

= $2p$ (Double start)

= $3p$ (Triple start)

Centre distance ⇒

$$= \frac{m_n}{2}\left[\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right]$$

$$= \frac{m_2 \cancel{\cos\psi_2}}{2} \cdot \frac{1}{\cancel{\cos\psi_2}}\left[\frac{T_1\cos\psi_2}{\cos\psi_1} + T_2\right]$$

$$= \frac{m_2}{2}\left[\frac{T_1\cos\lambda_1}{\cos(90-\lambda_1)} + T_2\right]$$

$$\boxed{= \frac{m_2}{2}\left[T_1\cot\lambda_1 + T_2\right]}$$

Velocity Ratio ⇒

$$VR = \frac{\omega_2}{\omega_1} = \frac{\text{Angle turned by wheel}}{\text{Angle turned by worm}}$$

$$= \frac{l/r_2}{2\pi} = \frac{l}{2\pi r_2}$$

$$\boxed{VR = \frac{l}{\pi d_2}}$$

Efficiency (η)

$$\eta = \frac{\cos(\lambda_1+\phi)}{\cos(90^\circ-(\lambda_1+\phi))} \cdot \frac{\cos(90^\circ-\lambda_1)}{\cos\lambda_1}$$

$$\boxed{\eta = \frac{\tan\lambda_1}{\tan(\lambda_1+\phi)}}$$ ***

$$\boxed{\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}}$$ ***