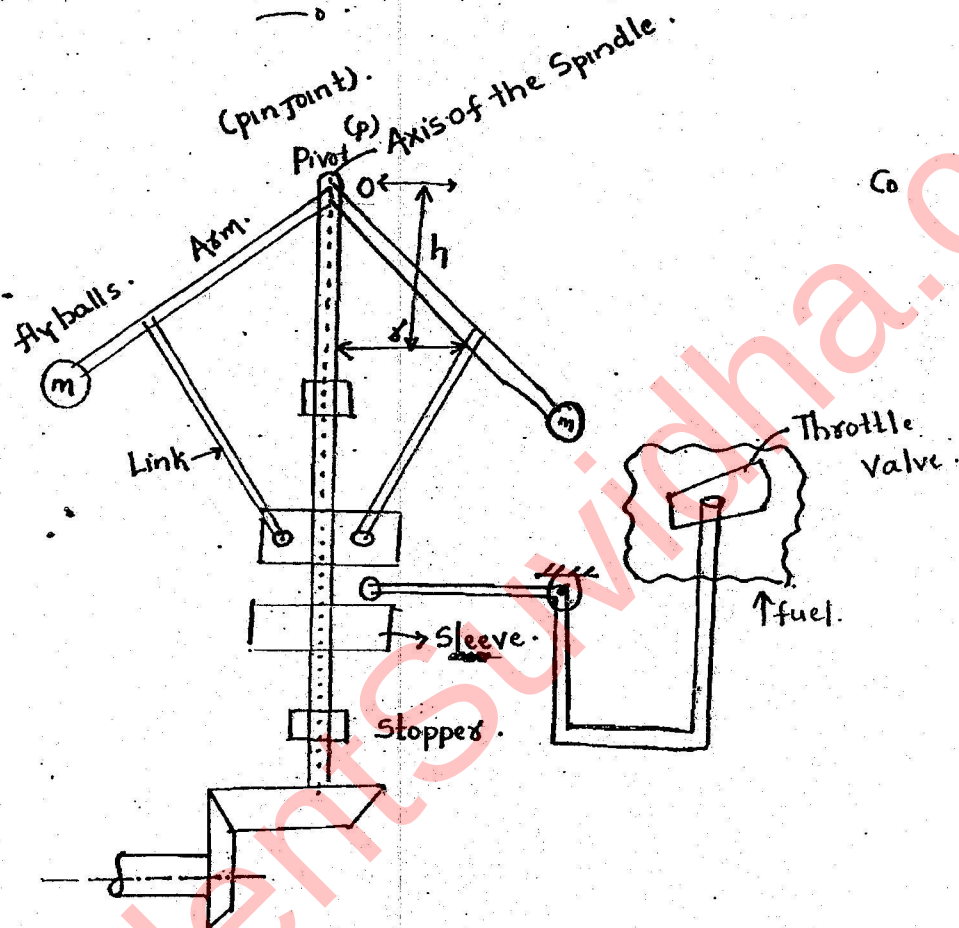


Governer

Basic Concept of Centrifugal Governer



When the Sleeve of the governer moves down .

→ Throttle Will open More .

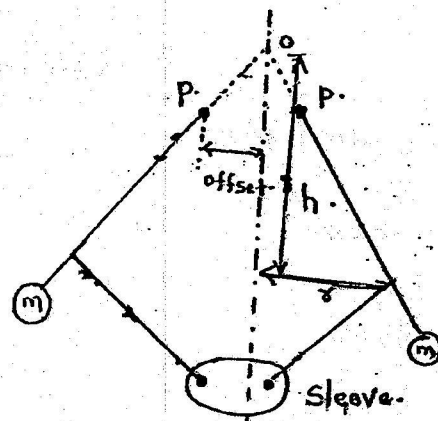
When the Sleeve of Governer hits the bottom Stopper
the throttle valve will open full .

So Engine develops Max. power

StudentSuvidha.com

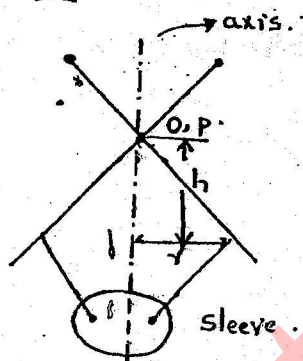
3-diff. arrangements of Centrifugal Governor

Open arm type



P - point of pivot of ARM
 O - point of inter sect
 of arm (TPoint)
 Arm and links
 are joint by
 Turning pair
 (Radius of
 Rotation of
 Arms and links
 - Radius of Gov)

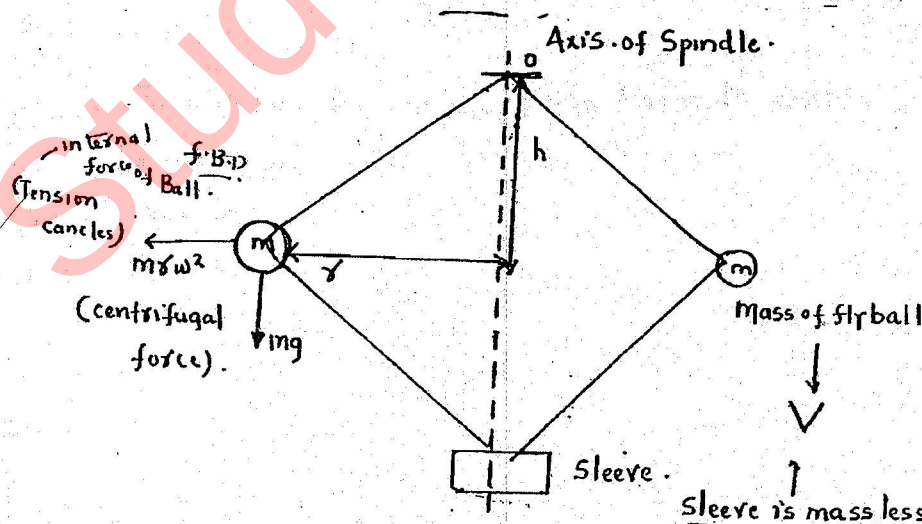
Crossed arm type



r - Radius of Governor
 h - height of Governor

used to represent
 position of Governor / sleeve

26. Pendulum Type Governor (Watt Governor)



Rotational Eqm
 Net Moment
 $= 0$
 No parameter
 is changing

Rotation Eqn of Governed

F.B.D

Taking Moment abt point 'p'.

(We have to see one
Side only
- Symmetric).

$$\text{Net Moment} = 0.$$

$$(m\omega^2) \cdot h = mg \cdot r.$$

$$\omega^2 = g/h.$$

$$N^2 = \left(\frac{60}{2\pi}\right)^2 \cdot g/h$$

$$\boxed{N^2 = \frac{895}{h}}$$

Eqn. Eqn of Watt Governed.

Governed at 208rpm (mitse gear)

Engine has to run @ 208rpm (designed)

So connected to Watt Governed

$$h_1 = \frac{895}{(20)^2} : \rightarrow \text{Then Load } \uparrow = 158\text{rpm}.$$

$\Downarrow$

$$h_2 = \frac{895}{(15)^2} \quad (\uparrow)$$

Sleeve Moment $\propto (h_2 - h_1)$. $\rightarrow$ fuel rate $\uparrow$
made equal
to 208rpm.

Engine : 2 (80 rpm)

$$h_1 = \frac{895}{(80)^2} \text{ --- Load } (\uparrow) = \underline{75 \text{ rpm}} (\downarrow)$$

$$h_2 = \frac{895}{(75)^2}$$

Sleeve Moment $\propto (h_2 - h_1)$

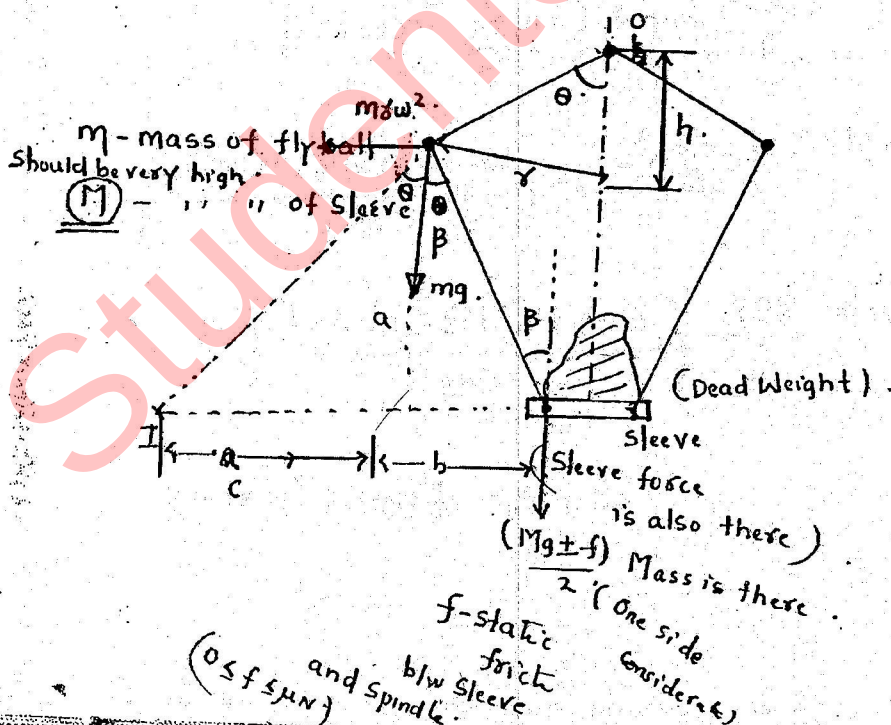
Very very less diff.

**

Practically Watt Governor has become insensitive beyond 60 rpm (Slow Speed running engine) [failed Governor]

Dead load Type Governor (ported Governor)

Basic reason for the failing of Watt Governor is that the sleeve is made weight less.



$M \gg m$. (Mass of Sleeve $>$ mass of flyball).

Rot Eq. m of Governor

F.B.D

Taking moment about I.

NET moment = 0.

$$(m\delta\omega^2) \cdot a = mg \cdot c + \left(\frac{mg \pm f}{2}\right) (b+c).$$

$\div$ by a

$$(m\delta\omega^2) = mg \cdot \left(\frac{c}{a}\right) + \left(\frac{mg \pm f}{2}\right) \cdot \left(\frac{b}{a} + \frac{c}{a}\right).$$

$$(m\delta\omega^2) = mg \tan \theta + \left(\frac{mg \pm f}{2}\right) (\tan \beta + \tan \theta)$$

$$k = \frac{\tan \beta}{\tan \theta}$$

$$(m\delta\omega^2) = \tan \theta \left[mg + \left(\frac{mg \pm f}{2}\right) (1+k) \right]$$

$$= \frac{\delta}{h} \left[mg + \left(\frac{mg \pm f}{2}\right) (1+k) \right]$$

$$\omega^2 = \left(\frac{2\pi N}{60}\right)^2 = \frac{g}{h} \left[\frac{2mg + (mg \pm f)(1+k)}{2mg} \right]$$

$$N^2 = \frac{895}{h} \left[\frac{2mg + (mg \pm f)(1+k)}{2mg} \right]$$

Eqn. eqn of Porter Governor.

Rot. Eqm

Sleeve is not Moving

But tendency is there.

consider at.

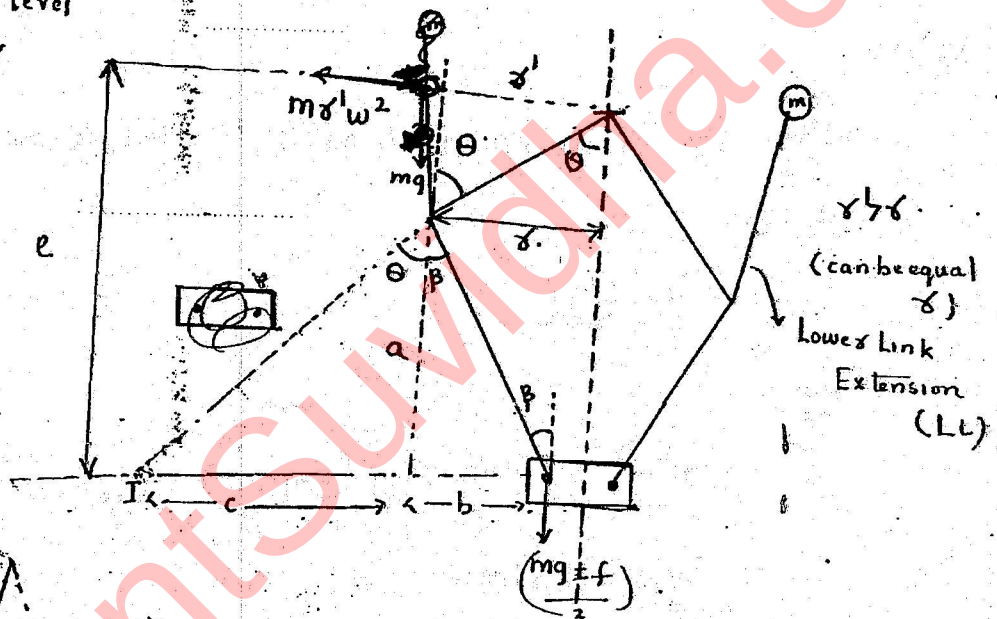
for
Sleeve rpm
Should be
high

$$h = \frac{895}{N^2} \left[\frac{1 + (Mg \pm f)(1+k)}{2mg} \right]$$



Proell Governor

he raised the level
of the fly
ball



≠ one link

∴ it's Geometrical
Parameters will not be
changed during movement.

Here

$$\delta' > \delta$$

only when the Extension of LH is || to the Governor
axis

Then $\delta' = \delta$

otherwise

always $\delta' > \delta$

Prot. Eqm. Eqn of Governed

F.B.D

Taking momen abt I

**

$$(m\dot{\gamma}'\omega^2) \cdot e = mg(c + \gamma - \gamma') + \frac{(mg \pm f)}{2} (b+c)$$

Eqm. Eqn of posell Governed

Note:-

When the Extension of LH is $\parallel$ to the Governed axis.

Then $\underline{\underline{\gamma' = \gamma}}$

Then:

$$(m\dot{\gamma}'\omega^2) \cdot e = mg \cdot c + \frac{(mg \pm f)}{2} (b+c)$$

$\div$ by e and $\times$ by a

$$(m\dot{\gamma}'\omega^2) \cdot a = mg \left(\frac{a}{e}\right) \left[mg \cdot c + \frac{(mg \pm f)}{2} (b+c) \right]$$

$$N^2 = \frac{895}{h} \left(\frac{a}{e}\right) \left[\frac{2mg + (Mg \pm f)(1+k)}{2mg} \right]$$

desired
Speed
@ desired
posiⁿ

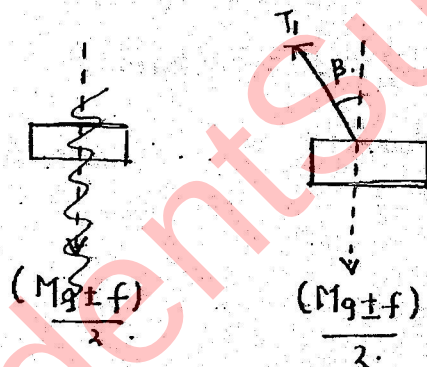
$$h = \frac{895}{N^2} \left(\frac{a}{e}\right) \left[1 + \frac{(Mg \pm f)(1+k)}{2mg} \right]$$

Here $\frac{a}{e} > 1$

for the Same performance in the proell Gov. as earlier obtained in the porter Governor. The Mass. of flyball req. in the Porter proell Gov. will be less as Compared to that required in the porter Gov. Hence overall inertia of the Gov. is reduced while getting the same performance. Hence it is highly Modify Gov. as Compared to porter Gov.

Tensions in arms and Links

Sleeve Eqm

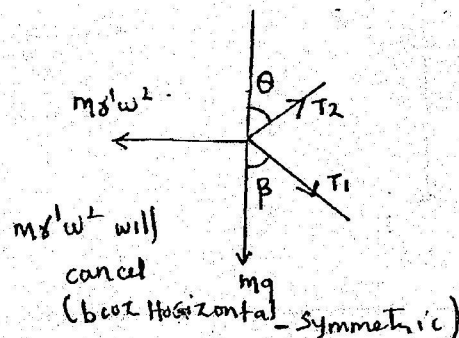


Horizontal forces Eqm.

$$T_1 \cos \beta = \frac{(Mg \pm f)}{2}$$

$$T_1 = \frac{(Mg \pm f)/2}{\cos \beta}$$

Taking pairs of Arms and Links



$$T_2 \cos \theta = mg + T_1 \cos \beta$$

$$T_2 = \frac{mg + T_1 \cos \beta}{\cos \theta}$$

Turning pair of arms and links

Because of the change in load conditions

Sleeve displaced up.

$\delta \uparrow$ $\left. \begin{array}{l} \omega \uparrow \rightarrow \text{Stability} \\ \omega \downarrow - \text{Stability decreases} \end{array} \right\} \text{Unstability}$

$\Rightarrow \omega = \text{Const.}$ Isochronous Gov. (ZERO time) infinite ~~time~~ Response Sensitivity (BEST)

Q.26) Arm $L = 250 \text{ mm}$.

$m = 15 \text{ kg}$.

$LL = 100 \text{ mm}$.

Min $\delta = \delta = 125 \text{ mm}$.

Max $\delta = 175 \text{ mm}$.

$M = 75 \text{ kg}$.

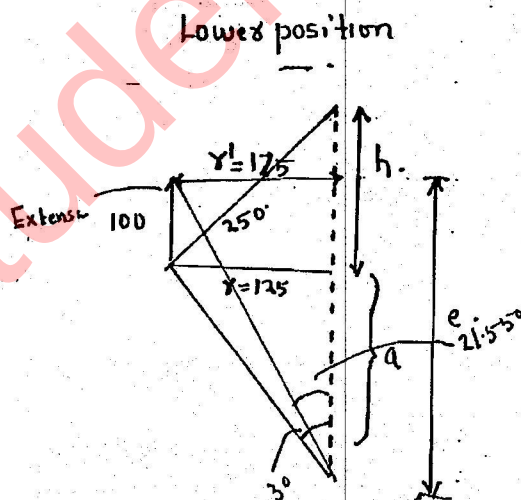
$\delta' = \delta$

Det. the Range of Eq. speeds.

Assume $f = 0$ (Static friction).

$(N_{\text{max}} - N_{\text{min}}) = ? =$

S. Units.



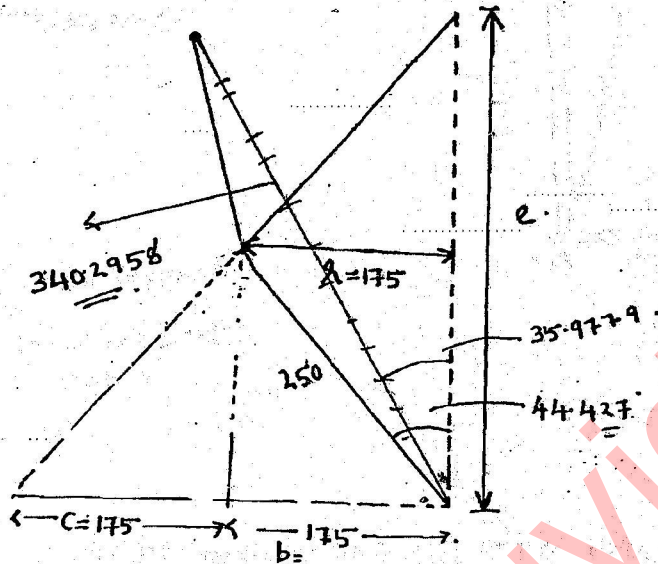
$$a = h = 216.5063$$

$$e = a + 100$$

$$= 316.5063$$

$$N^2 = \frac{895}{h} \left(\frac{a}{e} \right) \left[\frac{2mg + Mg(1+t)}{2mg} \right]$$

upper position



$$x' = (340.2958) \sin(35.97) = 199.9146$$

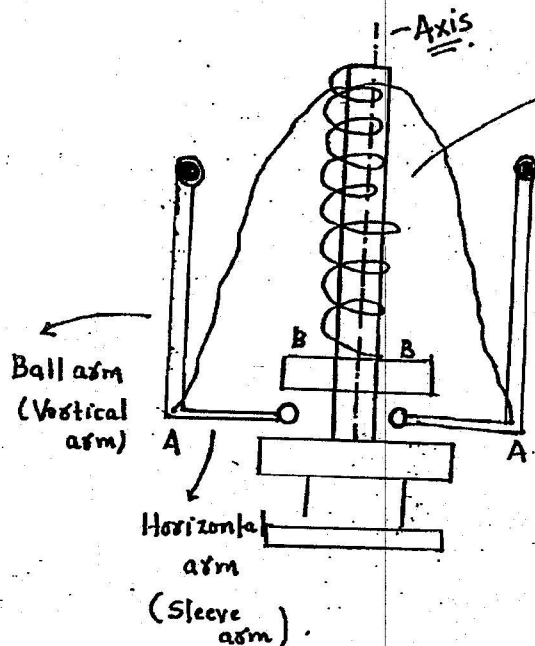
$$c = (340.2958) \cos(35.97) = 275.3822$$

$$(m a' \omega^2 e) = mg(x + c - x') + \frac{mg}{2}(b + c)$$

$$= 0 = ?$$

$$N = ?$$

* Spring Controlled Type (Hartnell Governor)

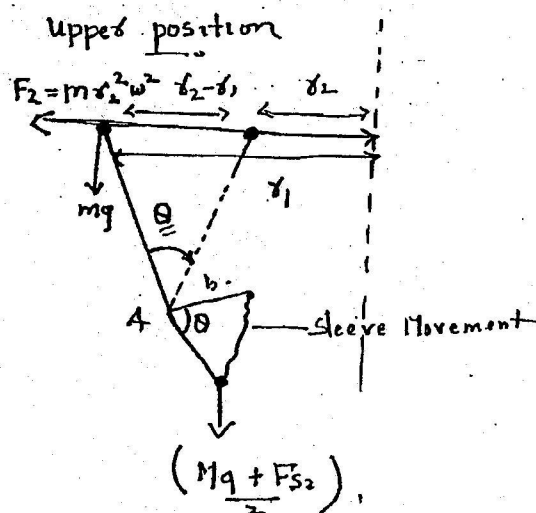
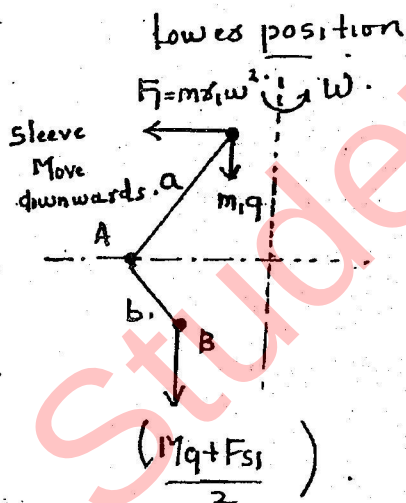


Central Spring Stiffness = 75

Always be in compression.

only one radius
- Ball radius of rotation.

If Mass of the sleeve is not given in the pb. and is required in calculation; then $M = 70$



Rot. Eqm. Eqn of Gov

F.B.D

Taking Moment abt A. (Neglection of obliquity).

mg (moment Neglected).

$$F_1 \cdot a = \left(\frac{Mg + F_{s1}}{2} \right) \cdot b \quad \text{--- (i)}$$

$$F_2 \cdot a = \left(\frac{Mg + F_{s2}}{2} \right) \cdot b \quad \text{--- (ii)}$$

$$\underbrace{(F_2 - F_1)}_{\text{known}} \cdot \frac{2a}{b} = \underbrace{(F_{s2} - F_{s1})}_{\text{unknown}}$$

Objective formulae

1. Total Sleeve Moment $= b\theta = \frac{b(\delta_2 - \delta_1)}{a}$

2. Additional Compression in Spring

$$= 2(x_2 - x_1) = \frac{b(\delta_2 - \delta_1)}{a}$$

3. $(F_{s2} - F_{s1}) = \frac{b(\delta_2 - \delta_1)}{a} \times S.$

$$(F_2 - F_1) = \frac{2a}{b} = \frac{b(\delta_2 - \delta_1)}{a} \times S.$$

$$S = 2 \left(\frac{F_2 - F_1}{\delta_2 - \delta_1} \right) \cdot \left(\frac{a}{b} \right)^2$$

Initial Compression

$$x_1 = \frac{Fs_1}{s} = 11$$

Final Compression

$$x_2 = \frac{Fs_2}{s} = 11$$

Q.27) Given

$$\frac{b(x_2 - x_1)}{a} = 0.03 \quad \text{--- (i)}$$

$$\left. \begin{array}{l} m = 1.5 \text{ kg} \\ M = 0 \end{array} \right\} \quad \left. \begin{array}{l} b = 0.065 \text{ m} \\ a = 0.075 \text{ m} \end{array} \right\}$$

$$\frac{x_1 + x_2}{2} = 0.0105 \text{ m} \quad \text{--- (2)}$$

By (i) and (ii)

$$x_1 = ?$$

$$x_2 = ?$$

$$N_1 = 430 \text{ rpm}$$

$$\omega_1 = \frac{2\pi \times 430}{60}$$

$$N_2 = 415 \text{ rpm}$$

$$\omega = \frac{2\pi \times 415}{60}$$

$$F_1 = m x_1 \omega_1^2$$

$$F_2 = m x_2 \omega_2^2$$

$$s = \frac{2(F_2 - F_1)}{(x_2 - x_1)} \left(\frac{a}{b} \right)^2 = 1$$

by Moment

$$\text{Eqn. } F_1 \cdot a = \left(\frac{Mg + Fs_1}{2} \right) \cdot b$$

$$x_1 = \frac{Fs_1}{s} = 11$$

ii) $N_2 = 440 \text{ r.p.m.}$

$$\omega_2 = \frac{2\pi \times 440}{60} \Rightarrow F_2 = m \omega_2^2 \cdot$$

$$S = \frac{2(F_2 - F_1)}{(\omega_2 - \omega_1)} \left(\frac{a}{b} \right)^2$$

$$\underline{\underline{x_1 = \frac{F_{s1}}{S} = 11}}$$

EFFORT OF GOVERNOR

11. The mean force acting on the sleeve to change its equilibrium position for the fractional change in speed of Governor is known as Effort of the Governor.

$\omega \rightarrow$ Speed of gov. at a particular position.

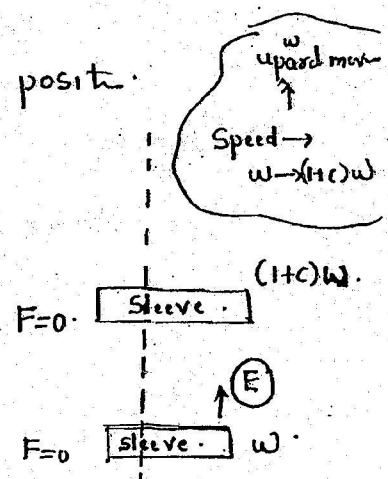
Let $c \rightarrow$ fractional change in Speed.

New Speed of sleeve $\Rightarrow (1 \pm c)\omega$

Mean force act on the sleeve.

$$= \left(\frac{0 + E}{2} \right) = \frac{E}{2} \Rightarrow \text{Effort}$$

Effort of the
Governor



(Avg. force)
Effort
value

Porter Governor

$$\frac{E}{2} = \frac{C}{(1+k)} [2mg + Mg(1+k)]$$

frictional force
Neglected

Hastnell Governor

$$\frac{E}{2} = C[Mg + Fs]$$

Power of the Governor (unit is not Watt, Joule (WD))

"The Work done on the Sleeve to change it's.

Eqm. position for the fractional change in Speed of Governor is known as power of Governor."

$$P = \frac{E}{2} \times \text{Sleeve displacement}$$

for Example

Porter Governor

All arms equal & no effect ^{offset}

So, at any moment $\theta = \phi$ $(a = h)$

So, $k=1$

$$\frac{E}{2} = \frac{C}{1+k} [2mg + Mg(2)]$$

$$= Cg[m+M]$$

$$\underline{\underline{W}} \quad h = \frac{g}{\omega^2} \left[\frac{2mg + Mg(1+i)}{2mg} \right]$$

$$\underline{\underline{(1+c)W}} = h_1 = \frac{g}{(1+c)^2 \omega^2} \left[\frac{2mg + Mg(2)}{2mg} \right]$$

$$\underline{\underline{h > h_1}}$$

$$\text{Sleeve Movement} = 2(h - h_1)$$

$$= 2h \left(1 - \frac{h_1}{h} \right) = 2h \left(1 - \frac{1}{(1+c)^2} \right)$$

$$(1+c)^2 = 1 + 2c + c^2$$

$$\underline{\underline{1+2c}}$$

$$= \frac{2h \cdot 2c}{(1+2c)}$$

power

$$\text{Power of Governor} = \frac{Cg(m+M) \cdot 2h \cdot 2c}{(1+2c)}$$

$$P = gh(m+M) \frac{4c^2}{1+2c}$$

The effect of Static friction on the performance of Centrifugal Governor

for eg:-

Potter's Governor

$$\text{Eqm. Eqn.} = \frac{895}{h} \left[\frac{2mg + [Mg \pm f] [1+k]}{2mg} \right]$$

Where $f \rightarrow$ Static friction

$$0 \leq f \leq \mu N$$

Because of static friction at one eqm position

So, ∞ eqm speeds are there

(No change in height)

Co-efficient of Insensitiveness

Calculated at Same (h)

$$= \frac{(1+f) N_{\max} - (1-f) N_{\min}}{N(f=0)}$$

Speed is changing but sleeve is not moving.

To make the insensitive zone to be closed to zero.

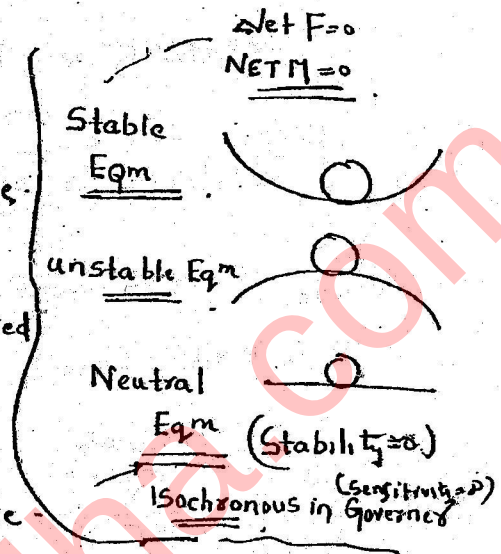
$$(f)_{\max} = \mu N \rightarrow \text{close to zero.}$$

Properties of the Governor

Stability of Governor

A governor is said to be in stable Eqm. if the restoring forces are becoming more dominant as compared to disturbing forces.

It is one of the Most desirable property of Governor.



$\begin{matrix} \delta \uparrow & N \uparrow \\ \delta \downarrow & N \downarrow \end{matrix} \left\{ \begin{array}{l} \text{Stability} \end{array} \right.$

$\begin{matrix} \delta \uparrow \\ \delta \downarrow \end{matrix} \left\{ \begin{array}{l} N = \text{constant} \end{array} \right\} \text{Isochronism}$

$\begin{matrix} \delta \uparrow & N \downarrow \\ \delta \downarrow & N \uparrow \end{matrix} \left\{ \begin{array}{l} \text{unstable} \end{array} \right.$

Sensitivity of Governor

The Extent of Sleeve movement for a change in the Speed of Governor is known as Sensitivity of Governor.

In other Words for the Same Sleeve movement the Governor in which less change in Speed is req. will be more Sensitive.

for Same Sleeve Movement

$$\text{Sensitivity} \propto \frac{1}{(N_1 - N_2)}$$

$$\boxed{\text{Sensitivity} = \frac{N}{N_1 - N_2}}$$

Where

$N \rightarrow$ mean Speed

$$= \frac{N_1 + N_2}{2}$$

It is also highly desirable property of Governor
like Stability [like in porter Governor]

Note

Stability & the Sensitivity both are the highly desirable properties of the Governor but both are inversely proportional to each other.

$$\text{Stability} \propto \frac{1}{\text{Sensitivity}}$$

$\therefore$ in design of the Governor both the parameters should be well optimised.

if $\frac{N}{N_1 - N_2}$ will be

not given in opt

then $\frac{N_1 - N_2}{2}$ will be correct

$$\text{Sensitivity} = \frac{N_1 - N_2}{N}$$

Sensitivity of Gov. by considering performance of Engine

Hunting of the Governed

An Excessively fast to and fro motion of the sleeve b/w the stoppers is known as hunting". As a result a big noise vibration and a very high level of fluctuation of speed will be observed in the system. and After few movement the stoppers will be broken and the system will be out of control of Governed.

It is an Extreme tough situation with over Sensitive Governness.

eg:-

Engine
→ Gov. (over-Sensitive)

When the loads on Engine ↑ slightly.

- Speed will be ↓ slightly.
- Sleeve hits the bottom stopper with very high speed.
- Throttle valve fully open
- Speed ↑ highly.
- Sleeve hits the upper stopper with more fastly.
- Throttle valve fully closed.
- Speed ↓ highly.

$\Rightarrow$ Again sleeve hits the bottom stopper and this phenomena will be further repeated.

* Isochronous Gov $= (f=0)$

A governor is said to be isochronous. Excluding friction if it's Eqm. Speed is constant at all radii of rotation.

$$\begin{array}{c} \gamma \uparrow \\ \gamma \downarrow \end{array} \quad \begin{array}{c} N \\ \omega = \text{const} \end{array}$$

$$\text{friction } (f) = 0$$

** Hartnell Governor can be Made isochronous Governor.

$$(\text{Stability})_{\text{isochronous}} = 0$$

$$(\text{Sensitivity})_{\text{isochronous}} = \infty$$

$$(\text{Hunting})_{\text{isochronous}} = 0 \quad (\text{No time for change, Zero time})$$

Hartnell Governor can be Made Isochronous $(f=0)$.

$$(m_1 \omega_1^2) \cdot a = \left(\frac{m_1 g + F_{s1}}{2} \right) b - (ci)$$

$$(m_2 \omega_2^2) \cdot a = \left(\frac{m_2 g + F_{s2}}{2} \right) b - (ci)$$

$$\textcircled{1}/\textcircled{2} \cdot \left(\frac{\gamma_1}{\gamma_2}\right) \left(\frac{\omega_1}{\omega_2}\right)^2 = \left(\frac{Mg + F_{s1}}{Mg + F_{s2}}\right)$$

for isochronism,

$$\underline{\omega_1 = \omega_2}$$

$$\boxed{\frac{\gamma_1}{\gamma_2} = \frac{(Mg + F_{s1})}{(Mg + F_{s2})}}$$

→ Condition of
isochronism

Isochronous Gov. are not for practical use just because :
of static friction. ($f_{max} \neq 0$; practically)

If isochronous Gov. are used — bcoz. of static friction

$$f_{max} \rightarrow 0 \text{ (close to zero)}$$

⇒ Neglecting change in speed is there. (Governor is not working like isochronous)

$$(\text{Hunt})_{\text{level}} = \underline{\underline{\infty}}$$

* Controlling force diagram

(performance characteristics curve of Governor)

In all the Governors fly balls are rotating; ∴ the force

which is controlling the fly balls in rotation is

i.e., Centripetal force acting along radius towards

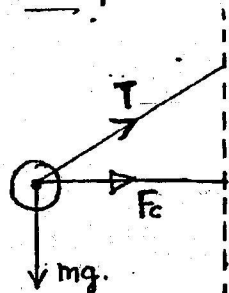
Centre is known as. Controlling force on the fly ~~balls~~ ball.

Controlling force diagram (in Watt Gov).

(performance characteristics of curve of Governor)

Controlling force diagram

In Watt Gov



(Static friction controls ^f when brake turn a turn)

In watt Governed, the Controlling force is Weight.

$$r = \frac{Mg}{\omega^2}$$

In Porter / parallel Gov

$$\underline{mg, Mg} \Rightarrow \text{Controlling force is } \underline{mg + Mg}$$

In Spring Controlled Gov

$$mg + Mg + \textcircled{F_s} \text{ --- dominant force.}$$

$$C.F. = F_c = \frac{mv^2}{r} = \underline{\underline{m\omega^2 r}}$$

How F_c is changed by
 $(F_c - r) \Rightarrow$ the radius of rotation of fly ball $\Rightarrow$ Controlling force diagram

in Watt / Porter / parallel Governed

Curve.

In Spring Controlled Governed

St. line

$$F_c = m \omega^2 r$$

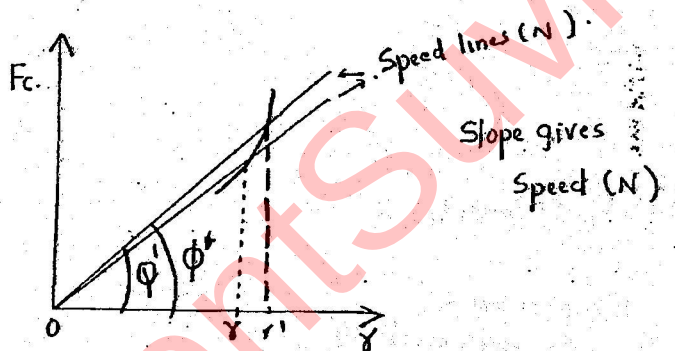
$$\omega^2 = \frac{1}{m} \frac{F_c}{r}$$

$$N^2 = \left(\frac{60}{2\pi} \right)^2 \cdot \frac{1}{m} \frac{F_c}{r}$$

if once Governed is made.

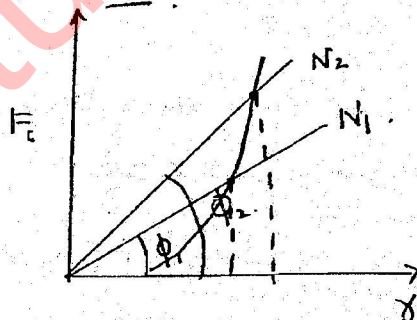
$$N = \frac{60}{2\pi\sqrt{m}} \sqrt{\frac{F_c}{r}} \rightarrow \tan \phi$$

Const.



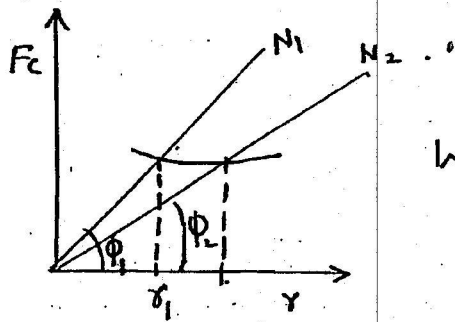
Watt / Porter / parallel Gov

i) checking stability



When $r \uparrow$, $N \uparrow$ Stable

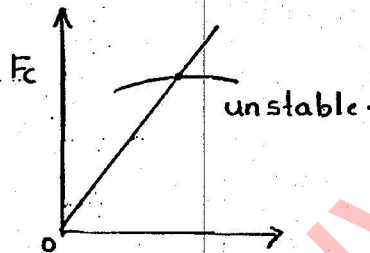
4.1.



When $\delta \uparrow$, $N \downarrow$] unstable

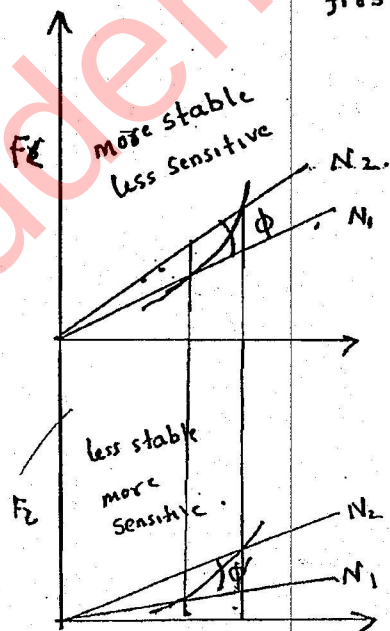
for stability

The slope of Controlling force diagram must be More than that of Speed. line at ^{and} Every point.



2) Comparison of stability / Sensitivity

first check for sensitivity



$$N = f(\phi)$$

$$\Delta\phi \uparrow N \uparrow$$

for Same Sleeve Movement.

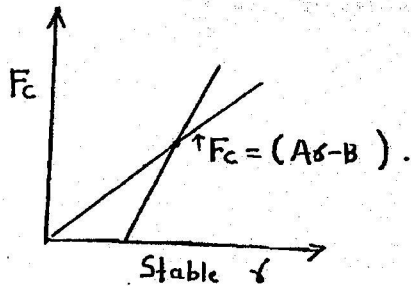
less change in $\Delta\phi' < \Delta\phi$.

Speed is

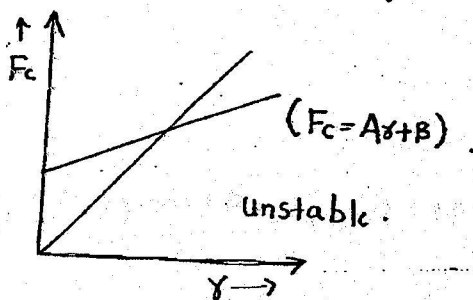
$$\text{more sensitive } (N_2' - N_1') < (N_2 - N_1)$$

== Given 2 more Sensitive

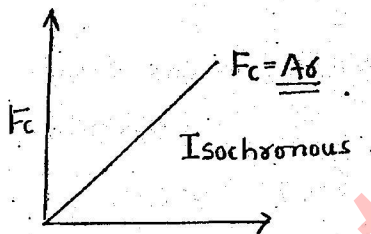
Spring Controlled (St-line)



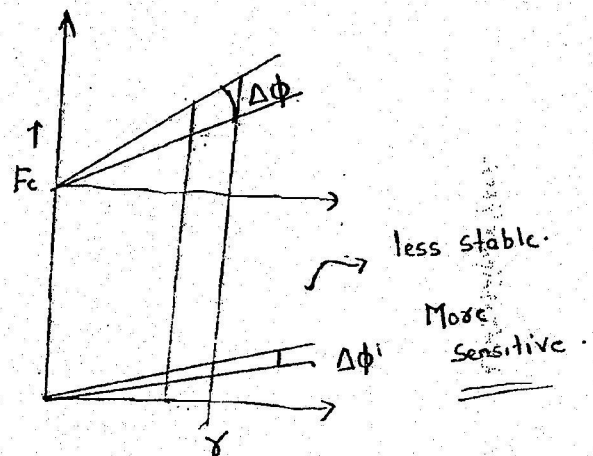
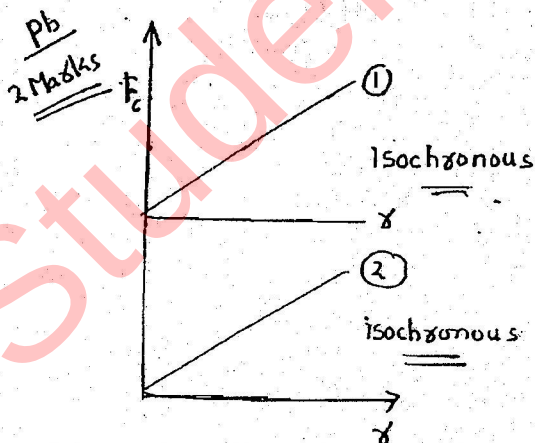
Slope of Controlling force diagram is more than Slope of Speed line. Hence it is stable.



Slope of Controlling force diagram is less than slope of Speed line. So unstable.



When the Controlling force dia. force dia. passes through origin. is known as Isochronous.



Real Controlling force dia. for a Governor.

— [Effect of static frict.]

in Reality

Practically

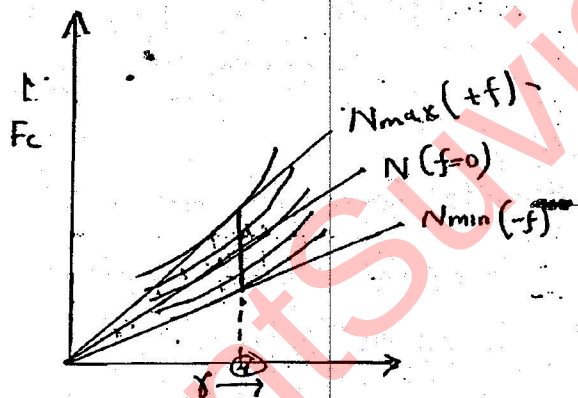
$f \rightarrow$ (close to zero) $\neq 0$.

At one eqm position.

$\Downarrow$

∞ eqm Speedo-lie.

$\Rightarrow \infty$ Controlling forces are there at one posn.

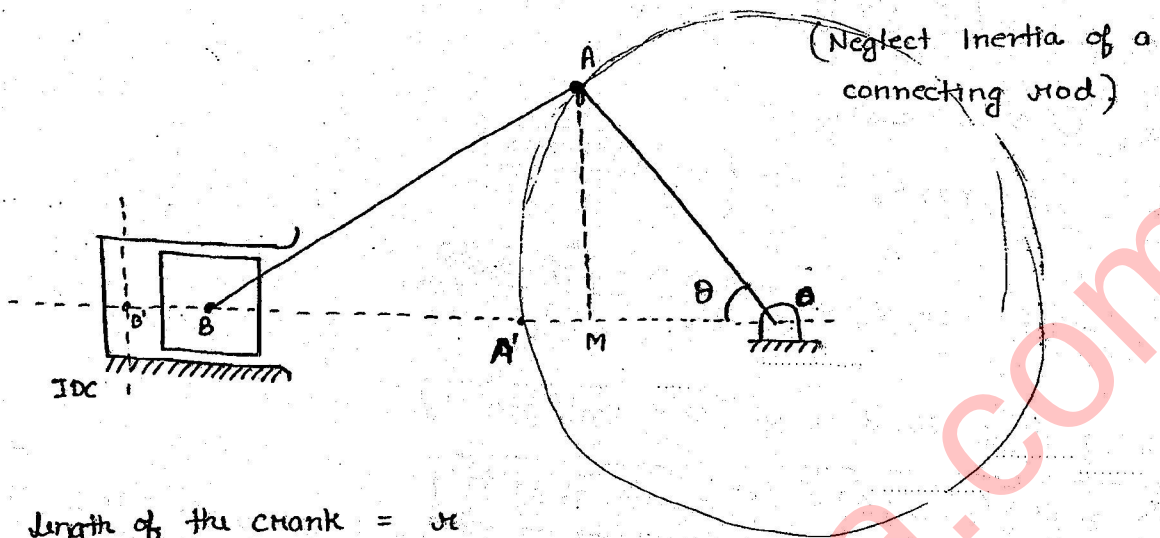


co-efficient of insensitiveness.
(detent)

$$\frac{N_{\max} - N_{\min}}{N(f=0)}$$

At same position

Kinematics Analysis of Single slider crank mechanism :-



Length of the crank = r

Length of connecting rod = l

$\theta \rightarrow$ Angle of crank from line of stroke (IDC)

$n \rightarrow$ obliquity ratio = $\frac{l}{r}$ (> 1)

Displacement of a piston (x) = $B'O - B'O$

$$= (l+r) - (MO + BM)$$

$$= (l+r) - (r \cos \theta + l \cos \beta)$$

$$= r(1 - \cos \theta) + l(1 - \cos \beta)$$

In $\triangle AMO$ -

$$AM = r \sin \theta$$

In $\triangle AMB$ -

$$AM = l \sin \beta$$

$$r \sin \theta = l \sin \beta$$

$$\sin \beta = \frac{r \sin \theta}{l}$$

$$= \frac{\sin \theta}{n}$$

$$\sin^2 \beta + \cos^2 \beta = 1$$

$$\cos^2 \beta = 1 - \sin^2 \beta$$

$$= 1 - \frac{\sin^2 \theta}{n^2}$$

$$\Rightarrow \frac{n^2 - \sin^2 \theta}{n^2}$$

$$\cos \beta = \frac{\sqrt{n^2 - \sin^2 \theta}}{n}$$

$$\cos \beta = \frac{\sqrt{n^2 - \sin^2 \theta}}{n}$$

Putting $\cos \beta$ in value of x -

$$x = r(1 - \cos \theta) + nr \left(1 - \frac{\sqrt{n^2 - \sin^2 \theta}}{n} \right)$$

$$= r(1 - \cos \theta) + nr \left(\frac{n - \sqrt{n^2 - \sin^2 \theta}}{n} \right)$$

$$x = r(1 - \cos \theta) + r(n - \sqrt{n^2 - \sin^2 \theta})$$

$$* \quad x = r \left[(1 - \cos \theta) + (n - \sqrt{n^2 - \sin^2 \theta}) \right]$$

$$n^2 \gg \sin^2 \theta$$

$$x = r(1 - \cos \theta)$$

Velocity of the piston :-

$$v = \frac{dx}{dt}$$

$$= \frac{dx}{d\theta} \cdot \frac{d\theta}{dt}$$

$$= \omega \left[\frac{d}{d\theta} \left[r \left\{ (1 - \cos \theta) + (n - \sqrt{n^2 - \sin^2 \theta}) \right\} \right] \right]$$

$$= \omega r \left[(0 + \sin \theta) + 0 - \frac{1}{2\sqrt{n^2 - \sin^2 \theta}} (-2 \sin \theta \cos \theta) \right]$$

$$v = \omega r \left[\sin \theta + \frac{\sin 2\theta}{2\sqrt{n^2 - \sin^2 \theta}} \right]$$

$$n^2 \gg \sin^2 \theta$$

$$* \quad v = \omega r \left[\sin \theta + \frac{\sin 2\theta}{2n} \right]$$

Acceleration of the piston :-

$$a = \frac{dv}{dt} \Rightarrow \frac{dv}{d\theta} \times \frac{d\theta}{dt}$$

$$= \omega \frac{d}{d\theta} \left[r \omega \left\{ \sin \theta + \frac{\sin 2\theta}{2n} \right\} \right]$$

$$= r \omega^2 \left(\cos \theta + \frac{1}{2n} \times 2 \cos 2\theta \right)$$

$$a = r \omega^2 \left(\cos \theta + \frac{\cos 2\theta}{n} \right)$$

Connecting rod :-

• Angular velocity :-

$$\omega_{cr} = \frac{d\beta}{dt}$$

$$\omega_{cr} = \frac{\omega \cos \theta}{\sqrt{n^2 - \sin^2 \theta}}$$

$$\omega_{cr} = \frac{\omega \cos \theta}{n}$$

$$\therefore \sin \beta = \frac{\sin \theta}{n}$$

differentiate wrt t

$$\cos \beta \frac{d\beta}{dt} = \frac{1}{n} \left[\cos \theta \frac{d\theta}{dt} \right]$$

$$\frac{d\beta}{dt} = \frac{1}{n \cos \beta} \left[\omega \cos \theta \right]$$

$$\frac{d\beta}{dt} = \frac{\omega \cos \theta}{\sqrt{n^2 - \sin^2 \theta}}$$

$$\cos \beta = \frac{\sqrt{n^2 - \sin^2 \theta}}{n}$$

• Angular acceleration :-

$$\alpha_{cr} = \frac{d\omega_{cr}}{dt}$$

$$= \frac{d}{dt} \left(\frac{\omega \cos \theta}{n} \right)$$

$$= \frac{\omega}{n} \frac{d}{dt} (\cos \theta)$$

$$= \frac{\omega}{n} (-\sin \theta) \cdot \frac{d\theta}{dt}$$

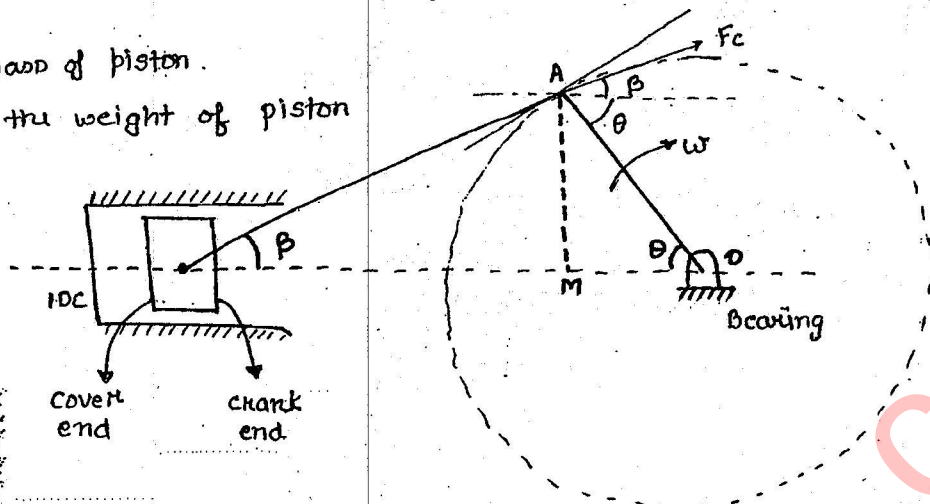
→ Negative acc.
→ retardation

$$\alpha_{cr} = - \frac{\omega^2}{n} \sin \theta$$

• Effective Driving force to Drive the piston (Piston Effort) :-

$m \rightarrow$ mass of piston.

Neglect the weight of piston



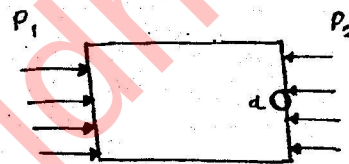
force analysis - from cover end to crank end.

- i) Gas pressure force
- ii) Kinetic friction force.
- iii) Inertia force.

i) Gas pressure force.

$$F_p = F_1 - F_2$$

$$= P_1 A_1 - P_2 A_2$$



$$A_1 = \frac{\pi}{4} d^2$$

$$A_2 = \frac{\pi}{4} (d^2 - d_c^2)$$

ii) friction force -

$$0 \leq \theta \leq \pi = -F_k$$

$$\pi \leq \theta \leq 2\pi = +F_k$$

iii) Inertia force -

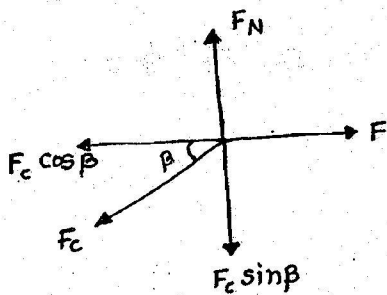
$$F_I = m \omega^2 \left[\cos \theta + \frac{\cos 2\theta}{n} \right]$$

D'Alembert Principal :-

The body is in static eq^m under the action of external forces and fictious force. This fictious force is also called inertia force. It is a reversed effective force.

● Piston effort (F) :-

$$F = F_p \pm f_k - F_I$$



Horizontal eq^m →

$$F_c \cos \beta = F$$

$$F_c = \frac{F}{\cos \beta}$$

connecting rod force.

vertical eq^m →

$$F_c \sin \beta + mg = F_N$$

$$F_c \sin \beta = F_N$$

$$F_N = \frac{F}{\cos \beta} \times \sin \beta$$

$$F_N = F \tan \beta$$

F_N = normal thrust to cylinder wall.

Radial thrust on crank shaft bearing -

$$F_R = F_c \cos (\theta + \beta)$$

$$F_R = \frac{F}{\cos \beta} \cos (\theta + \beta)$$

crank effort -

$$F_T = F_c \sin (\theta + \beta)$$

$$F_T = \frac{F}{\cos \beta} \sin (\theta + \beta)$$

Tangential force.

Turning moment of crank shaft (T) →

$$T = F_T \times r$$

$$T = \frac{F_R}{\cos \beta} \sin (\theta + \beta)$$

$$T \propto f(\theta)$$

$$\theta \propto f(t)$$

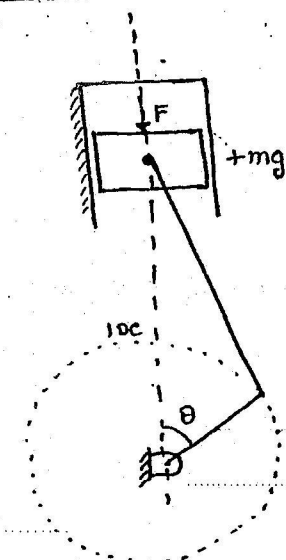
$$T \propto f(t)$$

$$Id \propto f(t)$$

$$a \propto f(t)$$

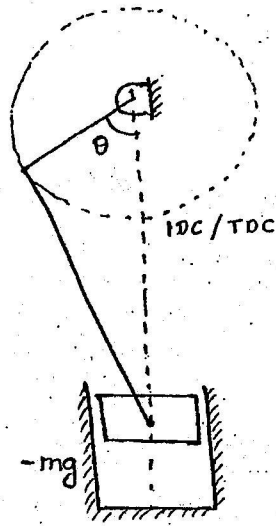
fluctuation
with
jerks

vertical Engine :-



$$F' = F + mg$$

(default)

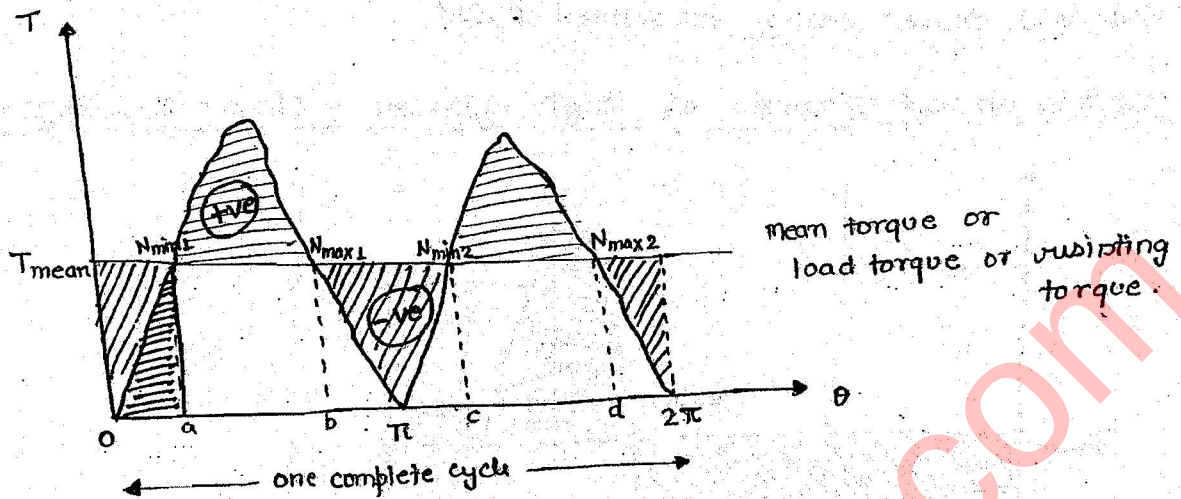


$$F' = F - mg$$

$$\text{Piston effort} = F \pm mg$$

$$F' = F_P - F_I \pm f_k \pm mg$$

Turning Moment Diagram of Single Cylinder double acting Steam Engine



$$W_{\text{cycle}} = \int_0^{2\pi} T \cdot d\theta$$

$$T_{\text{mean}} = \frac{W_{\text{cycle}}}{\text{cycle angle}}$$

$$\text{coefficient of fluctuation of speed } (C_s) = \frac{N_{\text{max}} - N_{\text{min}}}{N_{\text{mean}}} \quad \left[\begin{array}{l} 0.03 \text{ if } 3\% \\ 0.06 \text{ } \pm 3\% \end{array} \right]$$

$$\begin{cases} N_{\text{max}} \rightarrow \{ N_{\text{max}1}, N_{\text{max}2}, \dots \} \\ N_{\text{min}} \rightarrow \{ N_{\text{min}1}, N_{\text{min}2}, \dots \} \end{cases}$$

$N_{\text{max}} - N_{\text{min}} \rightarrow$ fluctuation still left.

$$\Delta E = \frac{I \omega_2^2}{2} - \frac{I \omega_1^2}{2}$$

coefficient of fluctuation of energy (C_E) =

$$C_E = \frac{E_{\text{max}} - E_{\text{min}}}{W_{\text{cycle}}}$$

$$N_{\text{max}} \rightarrow \omega_{\text{max}}$$

$$E_{\text{max}} \rightarrow \frac{1}{2} I \omega_{\text{max}}^2$$

$$N_{\text{min}} \rightarrow \omega_{\text{min}}$$

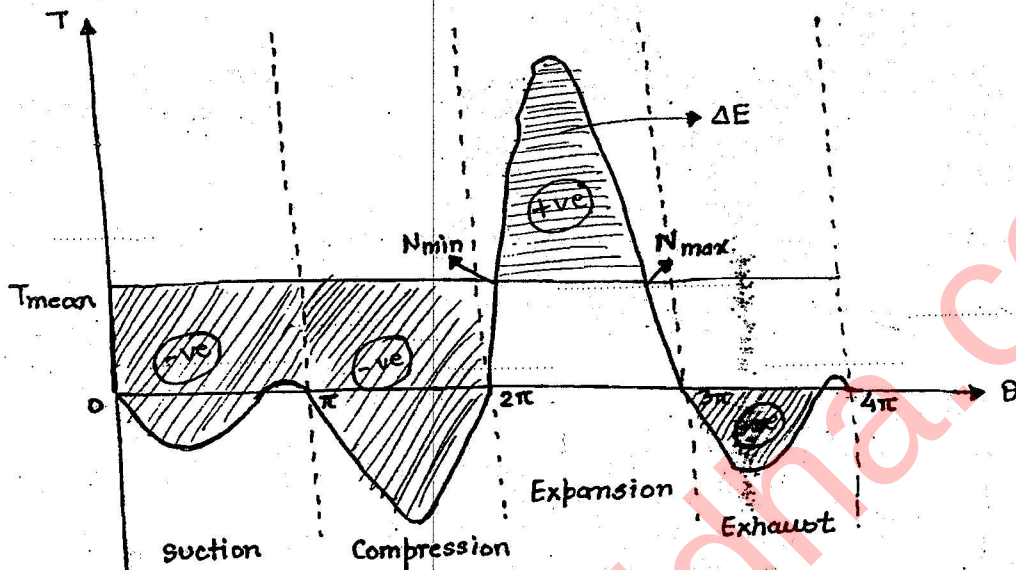
$$E_{\text{min}} = \frac{1}{2} I \omega_{\text{min}}^2$$

Sum of all +ve area above T_{mean} line = Sum of all -ve area below T_{mean} line.

Function of flywheel -

- store and deliver energy at desired speed .

Turning Moment Diagram of single cylinder 4 stroke I.C Engine :



Energy absorbed by flywheel = Energy supplied.

during suction, due to inertia force some portion is above θ -line.

during exhaust, it is due to pressure force of gases in opp. direction i.e. comp. gases is giving some work.

Energy absorbed during expansion is greater than suction + comp + Exhaust.

Maximum Fluctuation in energy (ΔE) = $E_{max} - E_{min}$

$$\Delta E = \frac{1}{2} I \omega_{max}^2 - \frac{1}{2} I \omega_{min}^2$$

$$= \frac{1}{2} I (\omega_{max}^2 - \omega_{min}^2)$$

$$= \frac{1}{2} I (\omega_{max} + \omega_{min}) (\omega_{max} - \omega_{min})$$

$$= \frac{I (\omega_{max} - \omega_{min})}{\omega} \cdot \omega \times \omega$$

$$** \boxed{\Delta E = I \omega^2 C_s}$$

$I \rightarrow$ mass MOI of flywheel

if m = mass of the flywheel

R = radius.

if disc $\rightarrow I = \frac{mR^2}{2}$

Ring $\rightarrow I = mR^2$

31, Q-26 $\rightarrow T = 10,000 + 2000 \sin 2\theta - 1800 \cos 2\theta$

$C_s = \pm 0.25\%$

$N = 250 \text{ rpm}$

$L = 80 \text{ mm}$

$r = 20 \text{ mm}$

$m_f = 120 \text{ kg}$

Load = const.

cycle time -

$\sin n\theta \rightarrow \frac{2\pi}{n}$

$\sin 2\theta = \pi$; $\cos 2\theta = [0 \text{ to } \pi]$

$W_{\text{cycl}} = \int_0^\pi T \cdot d\theta$

$\int_0^\pi (10,000 + 2000 \sin 2\theta - 1800 \cos 2\theta) d\theta$

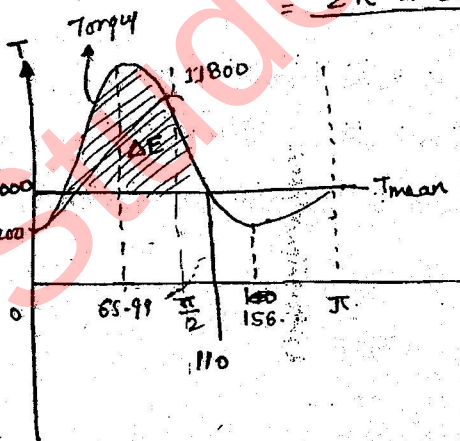
$= \frac{10000 \pi}{\pi}$

$T_{\text{mean}} = \frac{W_{\text{cycl}}}{\pi} = 10,000 \text{ N-m}$

Power = $\frac{2\pi NT}{60}$

$= \frac{2\pi \times 250 \times 10,000}{60}$

$\Rightarrow 261799.38 \text{ watt} \rightarrow 261.8 \text{ kW}$



$T =$

$\frac{dT}{d\theta} = 0$

$(2000) 2 \cos 2\theta + 3600 \sin 2\theta = 0$

$\tan 2\theta = \frac{4000}{3600}$

$2\theta = 131.98^\circ$

$\theta = 65.99^\circ$

$$T_{engine} = T_{mean}$$

$$\tan 2\theta = 0.9$$

$$2\theta = 41.987^\circ ; 221.987^\circ$$

$$\theta = 20.77^\circ ; 110.97^\circ$$

$$\Delta E = \int_{20.9736}^{110.9736} T \cdot d\theta - \int_{20.9736}^{110.9736} T_{mean} d\theta$$

$$= \int_{20.9736}^{110.9736} (T - T_{mean}) d\theta$$

$$= 2690.774 \text{ N-m}$$

$$\int \left((2000 \sin(2x)) - (1000 \cos(2x)) \right) d\theta$$

$$= 154170 \times \frac{\pi}{180}$$

$$= 2690.77$$

$$\rightarrow \Delta E = I \omega^2 C_s$$

$$2690.77 = I (\omega)^2 \times 0.005$$

$$I = 785.17 \text{ kg-m}^2$$

$$\rightarrow \theta = 45^\circ$$

$$T_{engine} = 12000 \text{ N-m}$$

$$T_{mean} = 10000 \text{ N-m}$$

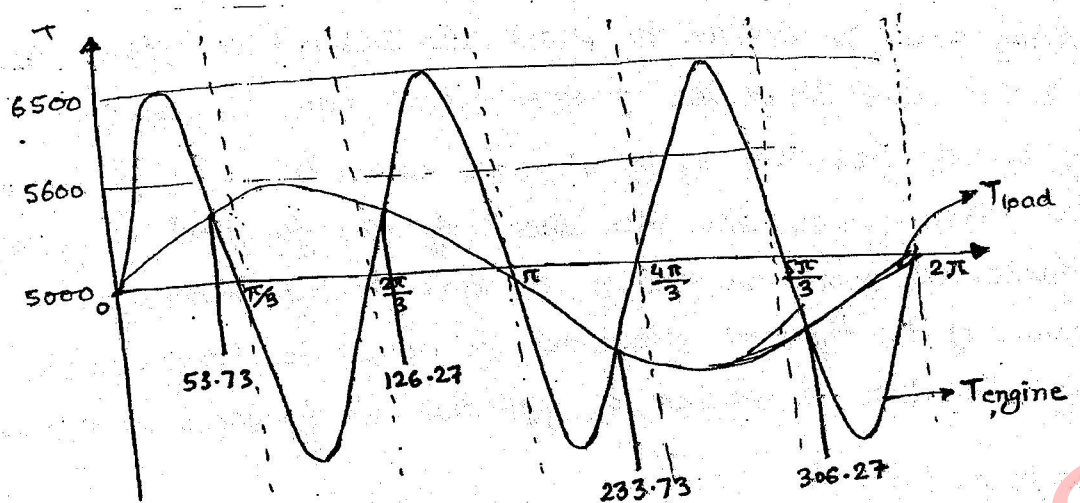
$$(T - T_{mean}) = I \alpha$$

$$\alpha = 2.547 \text{ rad/s}^2$$

Q → The eqⁿ of T_m for an engine is $5000 + 1500 \sin 3\theta$ N-m where θ is the crank angle measured from IDC. The resisting torque exerted is given by $T_{res} = 5000 + 600 \sin \theta$ N-m. Calculate the max fluctuation of energy.

$$\rightarrow T = 5000 + 1500 \sin 3\theta \rightarrow \left[0, \frac{2\pi}{3}\right]$$

$$T_{res} = 5000 + 600 \sin \theta \rightarrow [0, 2\pi]$$



$$T_{load} = T_{engine}$$

$$1500 \sin 3\theta = 600 \sin \theta$$

$$1500 (3 \sin \theta - 4 \sin^3 \theta) = 600 \sin \theta = 0$$

$$\sin \theta (13 - 20 \sin^2 \theta) = 0$$

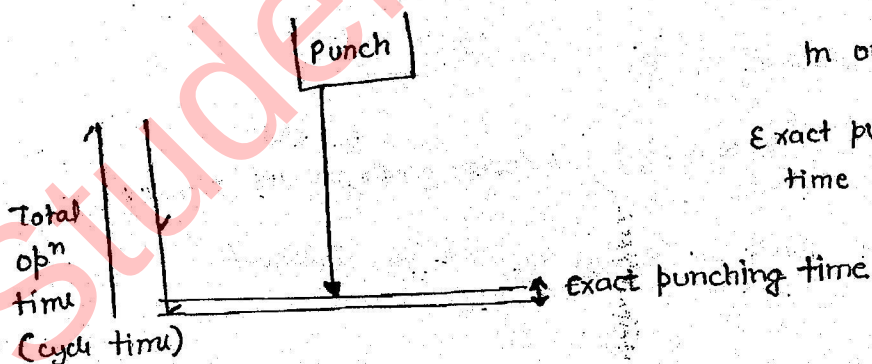
$$\sin \theta = 0 \quad ; \quad \sin^2 \theta = \frac{13}{20}$$

$$\theta = (0, \pi, 2\pi) \quad ; \quad (53.73, 126.27, 233.73, 306.27)$$

$$\Delta E = \int_{233.73}^{306.27} (T - T_{mean}) d\theta = 1656.6$$

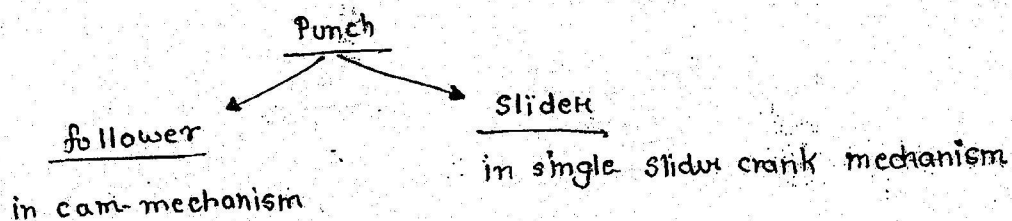
Requirement of flywheel in power press :

power press - used for metal forming operations.



In one cycle $\rightarrow$ two stroke covers

Exact punching time $\ll$ cycle time



Q - A punching press is desired to punch 720 holes/hr. Each hole dia is 3 mm and thickness of sheet is 2 mm. It requires 20 J of energy per mm sq. of sheared area if each punching operation takes $\frac{1}{3}$ sec and the speed of the flywheel is going to be fluctuated from 120 rpm to 80 rpm during punching. Find mass of the flywheel required in order to have desired punching operation if radius of gyration of flywheel is 0.3 m.

→ 720 holes / hr

1 hr → 720 hole

3600 sec → 720 hole, 1 hole → $\frac{3600}{720} = 5$ sec
cycle time

Exact punching time = $\frac{1}{3}$ sec

sheared area = $\pi D t$

$$= 6\pi \text{ mm}^2$$

Total energy (E) = $6\pi \times 20 = 120\pi$ Joule → 5 sec

$$1 \text{ sec} \rightarrow \frac{120\pi}{5}$$

$$= 24\pi \text{ watt}$$

OR, power = energy req. for cutting one hole × No. of holes per second

$$= 120\pi \times \frac{1}{5} \Rightarrow 24\pi \text{ watt}$$

$$\Delta E = 120\pi - \left(24\pi \times \frac{1}{3}\right) \rightarrow \text{Energy given by motor during punching}$$

$$= 112\pi \text{ Joule} \rightarrow \text{Energy supplied by flywheel.}$$

$$\Delta E = I \omega^2 C_s$$

$$C_s = \frac{N_{\max} - N_{\min}}{N_{\text{mean}}} = \frac{40}{100} = \frac{2}{5}$$

variation in problem -

- i) Total cycle time = 5 sec
Stroke length = 200 mm
Sheet thickness = 5 mm

$$5 \text{ sec} \rightarrow 400 \text{ mm}$$

$$400 \text{ mm} \rightarrow 5 \text{ sec}$$

$$5 \text{ mm} = \frac{5}{400} \times 5 = \text{ (Exact punching time)}$$

- ii) total cycle time = 5 sec

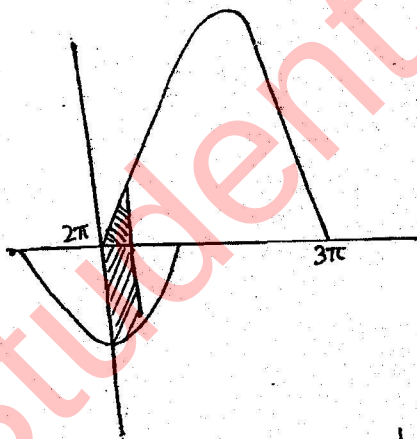
$$\text{exact punching} = 20^\circ$$

$$360^\circ \rightarrow 5 \text{ sec}$$

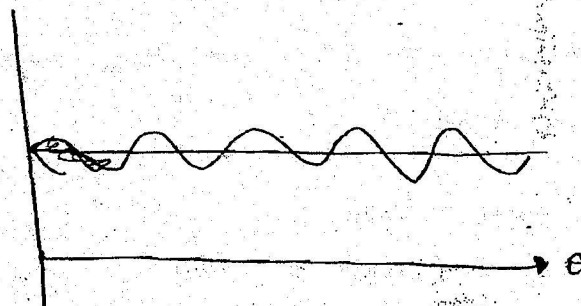
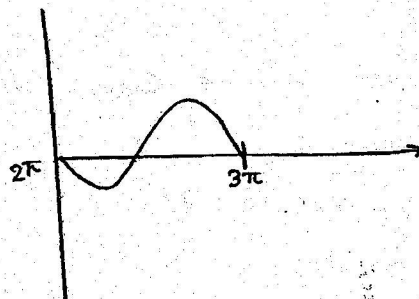
$$20^\circ \rightarrow \frac{5}{360} \times 20 = \text{exact punching time}$$

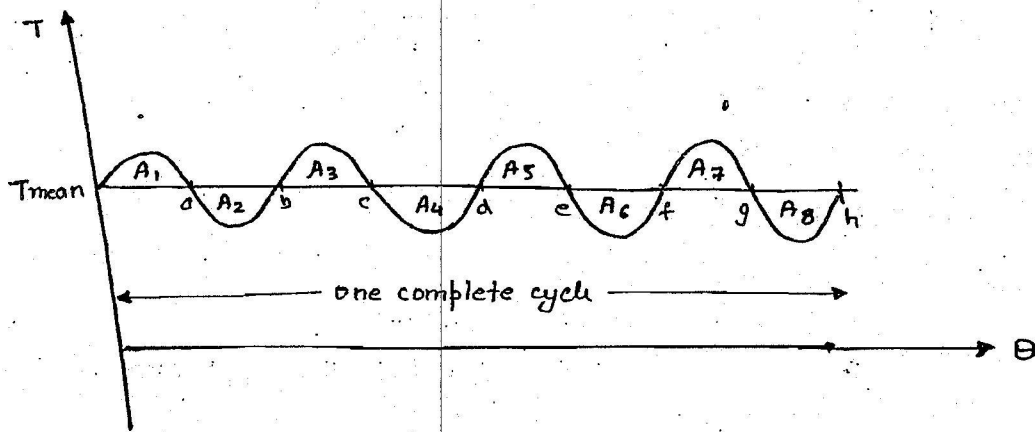
Requirement of flywheel in multicylinder :-

Multicylinder concept came into picture to increase the power as well as to increase the uniformity of power so that fluctuation in the engine can be reduced upto certain extent and the requirement of the flywheel can be suppressed upto certain limits.



6 cylinder engine $\rightarrow$





$$E_0 = E$$

$$E_a = E + A_1$$

$$E_b = E + A_1 - A_2$$

$$E_c = E + A_1 - A_2 + A_3$$

$$E_d = E + A_1 - A_2 + A_3 - A_4$$

$$E_e = E + A_1 - A_2 + A_3 - A_4 + A_5$$

$$E_f = E + A_1 - A_2 + A_3 - A_4 + A_5 - A_6$$

$$E_g = E + A_1 - A_2 + A_3 - A_4 + A_5 - A_6 + A_7$$

$$E_h = E + A_1 - A_2 + A_3 - A_4 + A_5 - A_6 + A_7 - A_8 = E$$

E_{max}

E_{min}

$$\Delta E = I \omega^2 C_s$$

Q11 →

$$E_0 = 0$$

$$E_{180} = 100 \rightarrow E_{max}$$

$$E_B = 100 - 75$$

$$E_c = 25 + 49 = 74$$

$$E_d = 74 - 77 = -3$$

$$E_e = -3 + 43 = 40$$

$$E_f = 40 - 73 = -33 \rightarrow E_{min} \rightarrow N_{min}$$

$$E_g = -33 + 33 = 0$$

$$\Delta E = E_{max} - E_{min}$$

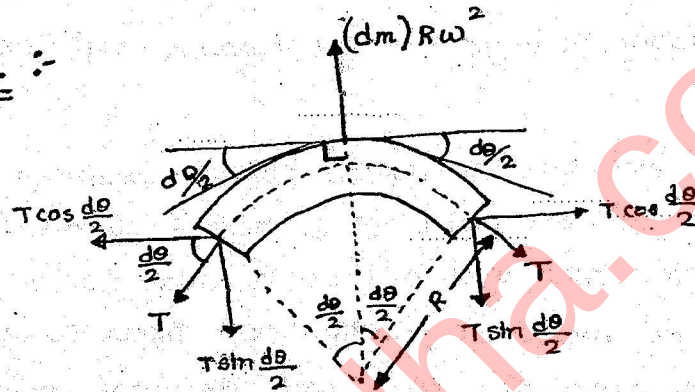
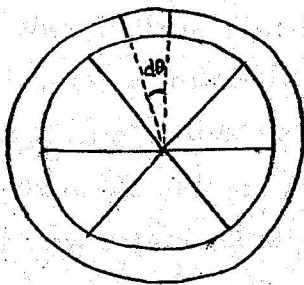
$$= 100 - (-33) = 133$$

Variation -

① Scale - $1 \text{ mm}^2 = 20 \text{ J}$
 $100 \text{ mm}^2 = 20 \times 100 = 2 \text{ KJ}$

② TM (scale) $1 \text{ mm} = 5 \text{ N-m}$
 $\theta \text{ scale } 1 \text{ mm} = 5 \text{ degree}$
 $= 1 \text{ mm}^2 = 5 \times \left(\frac{5 \times \pi}{180} \right)$

Design of flywheel rim :-



cross section area of the rim = A

radius of flywheel = R

vertical equilibrium -

$$dm R \omega^2 = 2T \sin \frac{d\theta}{2}$$

Volume of mass element $(dv) = (R d\theta) A$

Mass of small mass element $(dm) = \rho \times dv$
 $= \rho (R d\theta) A$

$$dm = \rho A R d\theta$$

$$\therefore (\rho A R) d\theta R \omega^2 = 2T \times \frac{d\theta}{2} \quad (\because d\theta \text{ is very small})$$

$$\rho A R^2 \omega^2 d\theta = T \times d\theta$$

$$T = \rho A R^2 \omega^2$$

$$T = \rho A V^2 \quad (V = R\omega)$$

$$\frac{T}{A} = \rho V^2$$

$$\tau = \rho V^2 \quad (\text{tangential or hoop stress})$$

$$\tau = \rho v^2$$

$$\tau \leq \tau_b \text{ (bearing limit)}$$

$$\tau_b = \rho v_{\max}^2$$

$$v_{\max} = \sqrt{\frac{\tau_b}{\rho}}$$

Rim type flywheel -

This is suitable for low speed because bearing limit is less but during energy storage the fluctuation will be less for this flywheel.

As we increase the no. of spokes bearing limit will increase. It means it can run at a higher speed, but mass is also increasing which will act as a load, and increase the fuel cost. As mass is distributed near to the axis of rotation of the flywheel the MOI will decrease this will increase the speed fluctuation.

If flywheel is disc type, MOI is minimum. It means for a given energy storage speed fluctuation will be more. But the advantage of disc flywheel is that it can be run at a higher speed (because τ_b increases).

Best flywheel design.

