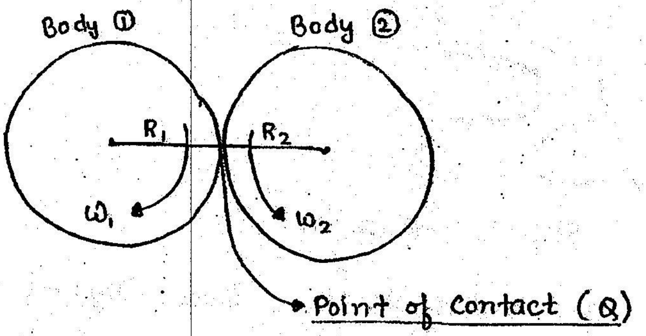
Gears

⇒ Drive used in power transmission.



By Body ① By Body ②

$$\boxed{R_1\omega_1 = R_2\omega_2}$$ # Cond. of slipping

for pure rolling (No slipping) -

$$R_1\omega_1 = R_2\omega_2$$

$$\frac{\omega_1}{\omega_2} = \frac{R_2}{R_1} = \text{const}$$

- Friction is static (f_s) → variable friction.

$$\boxed{0 \le f_s \le \mu N}$$

- All those drive in which slip is possible → Negative Drives.
  for eg :- Belt, Rope, chain drives . . . .

- In case of slip -

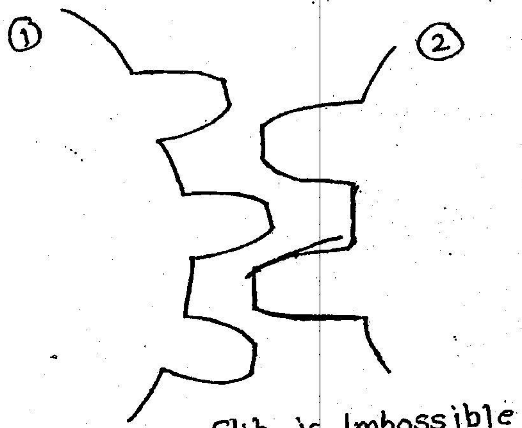
$$R_1\omega_1 \neq R_2\omega_2$$

$$\frac{\omega_1}{\omega_2} \neq \frac{R_2}{R_1} \neq \text{const}.$$

- Kinetic friction loss - (Some energy loss)

- In some system, a very high level of accuracy is demanded for the velocity ratio to be const.

$$\boxed{\frac{\omega_1}{\omega_2} = \text{const}}$$



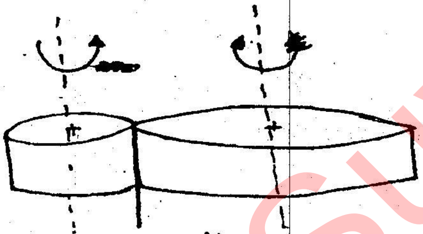
Slip is Impossible

⇒ Positive Drive (Gear Drive).

Classification of Gear :-

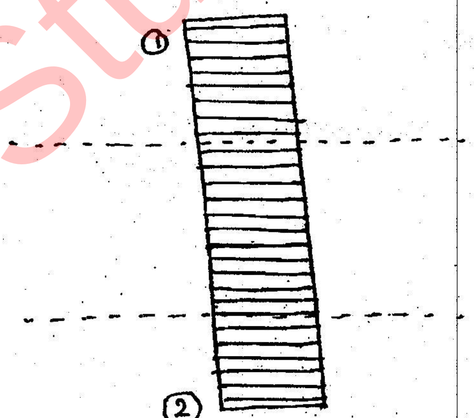
A] A/c to the axis of shafts connected :-

(i) Both Axes are parallel :-



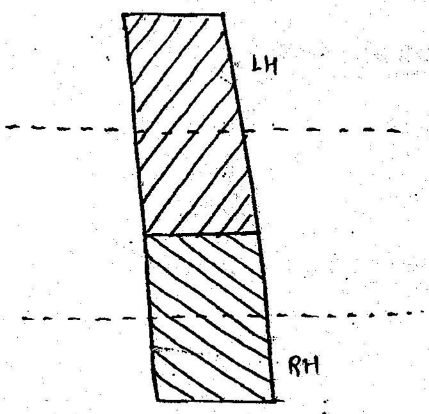
- Pure rolling motion can be transmitted b/w two cylindrical surfaces in contact.
- Teeth are straight and parallel to the axis of rotation.

Spur Gear :-



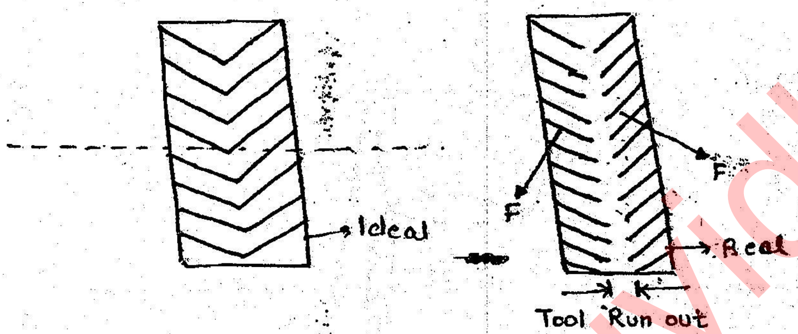
- 99 % failed, 1 % used
- used in low power transmission of very low speed
- No axial thrust thus our bearing is safe
- Instantaneous engagement which results in Impact stresses on profile.

Helical gear :-



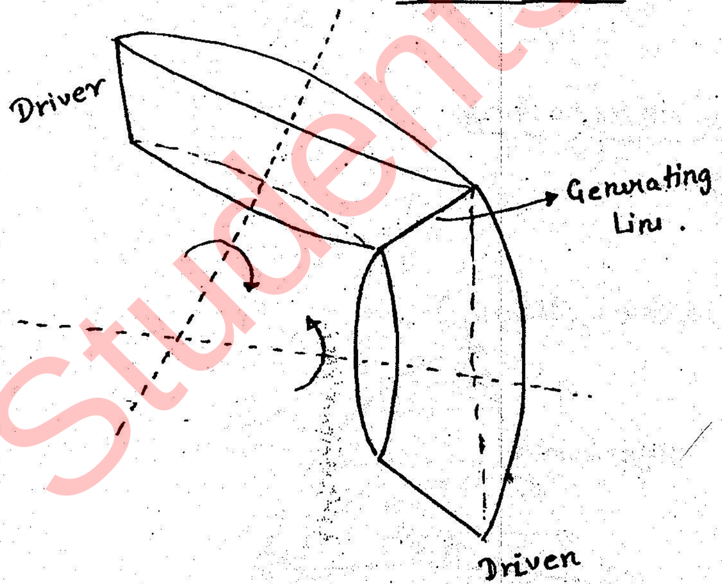
- Teeths are straight but inclined to the axis of rotation.
- 98% used.
- can be Right hand / Left hand gears.
- Always opposite hand helical gear must come in contact.
- Axial thrust
- Gradual Engagement ⇒ No Impact stresses.

Double Helical Gear :- (Herringbone Gear)



ii) Both are Intersecting :-

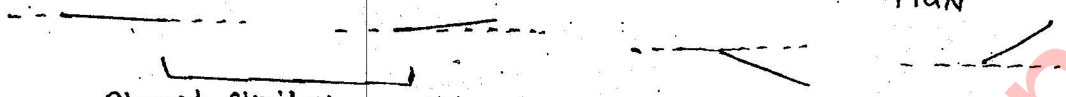
Bevel Gear



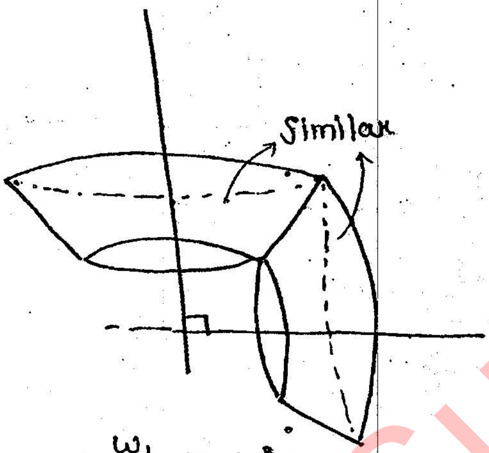
Pure rolling motion can be transmitted b/w two conical surfaces in contact.

Bevel gear

- St. bevel gear — Plan ↓
- Zero Bevel — Plan ↓
- Helical bevel.
  - RH → Plan
  - LH → Plan

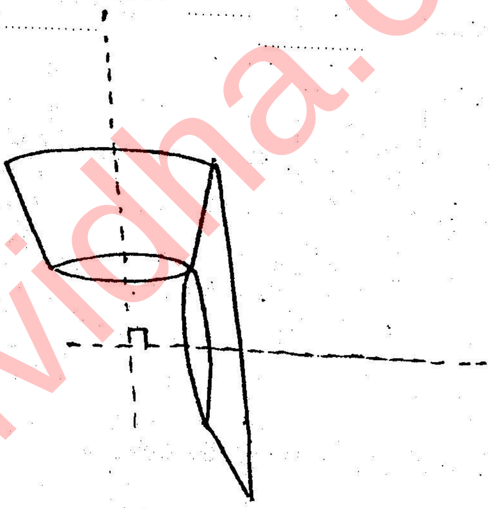


Almost similar (Impact stresses) | 1% used



$$\frac{\omega_1}{\omega_2} = 1$$

$\boxed{\text{Mitre Gear}}$



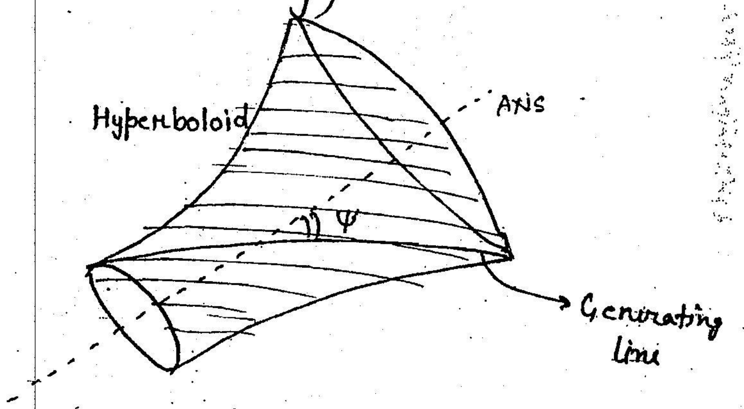
$$\frac{\omega_1}{\omega_2} = \text{const} \neq 1$$

iii) Axes are Neither ‖ nor Intersecting –

- Pure rolling is impossible.
- Rolling is possible
  ↳ (Rotation + Partial Sliding)

Spiral Gear :-
(Skew - Bevel Gear)

ψ → spiral angle / Helix angle.



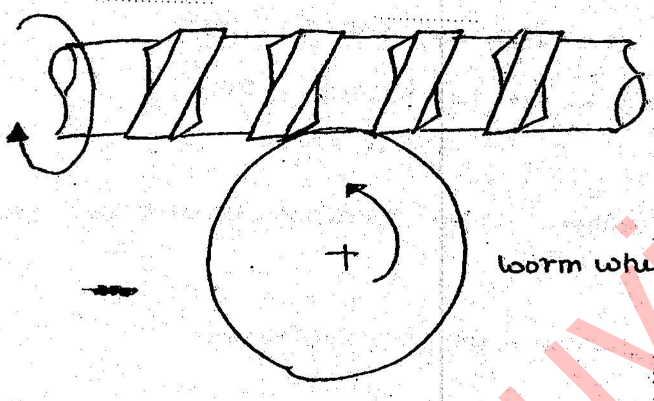
Sometime when the space b/w the shaft (offset) is very less, then some portion of Hyperboloid is used to form spiral Gears known as Hypoid Gears.

worm and worm wheel :-

spiral

- very high spiral angle
- very less dia

- very less spiral angle
- very high dia.

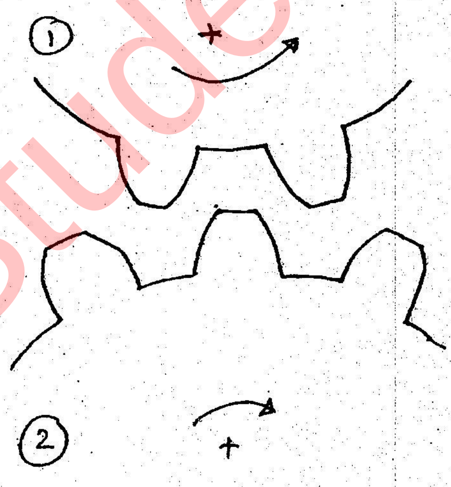


- Driver = worm
- Famous for high speed Reduction ratios

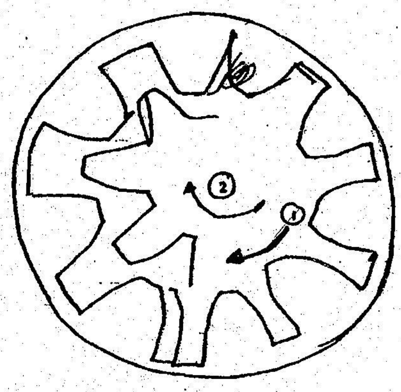
W	:	WW
10	:	1
30	:	1
⋮		
presently used → 1250	:	1

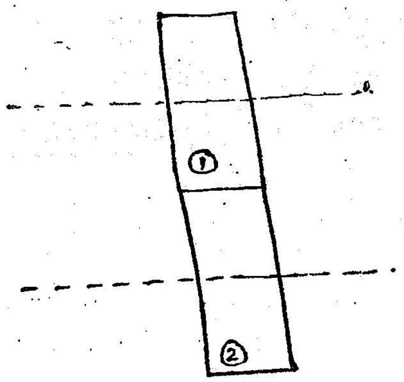
B) Acc to the type of Gearing :-

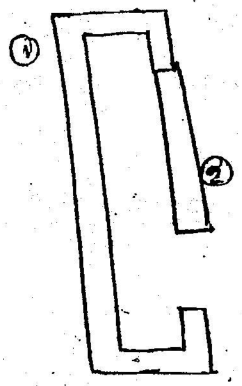
External Gearing
(>95% used)



Internal Gearing
(<5% used)







Bigger : GEAR
Smaller : PINION

Bigger : ANNULAR (RING)
Smaller : PINION.

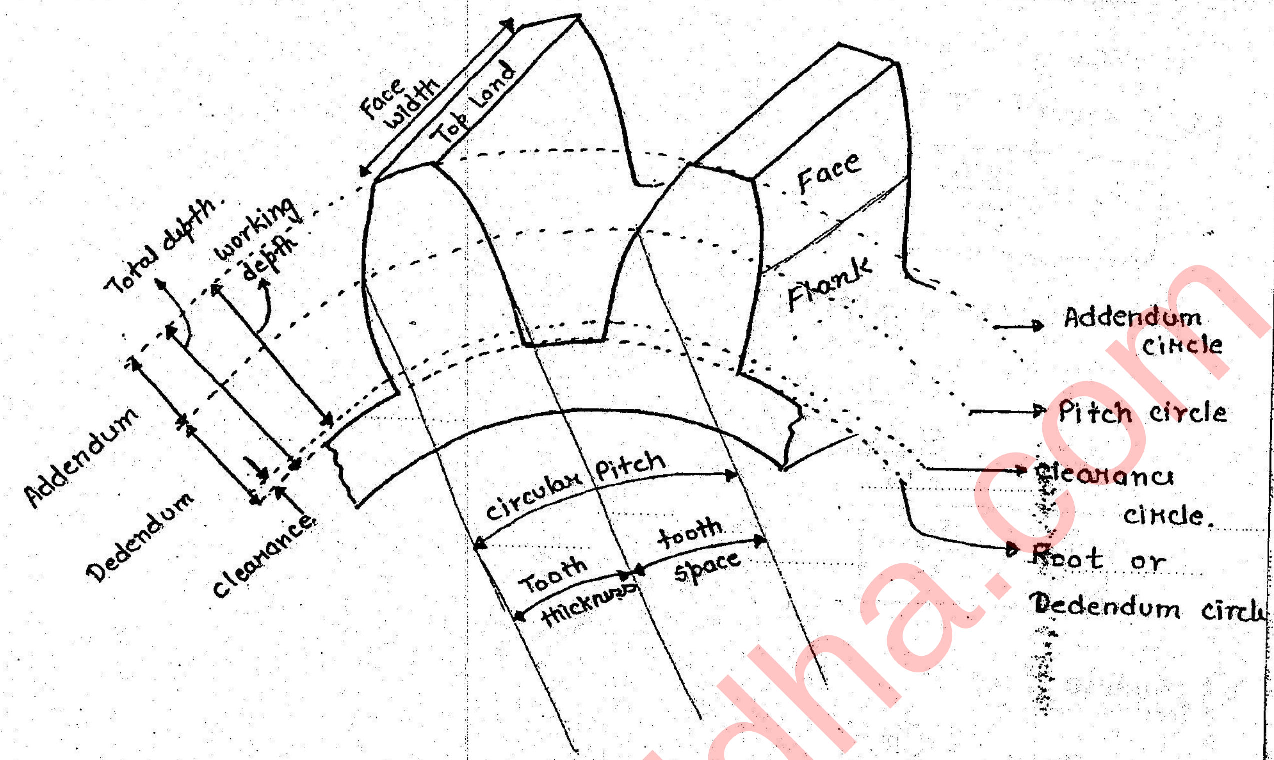
Note :-

1) If more than one gear are mounted on same shaft
   → compound gears
   = speed same

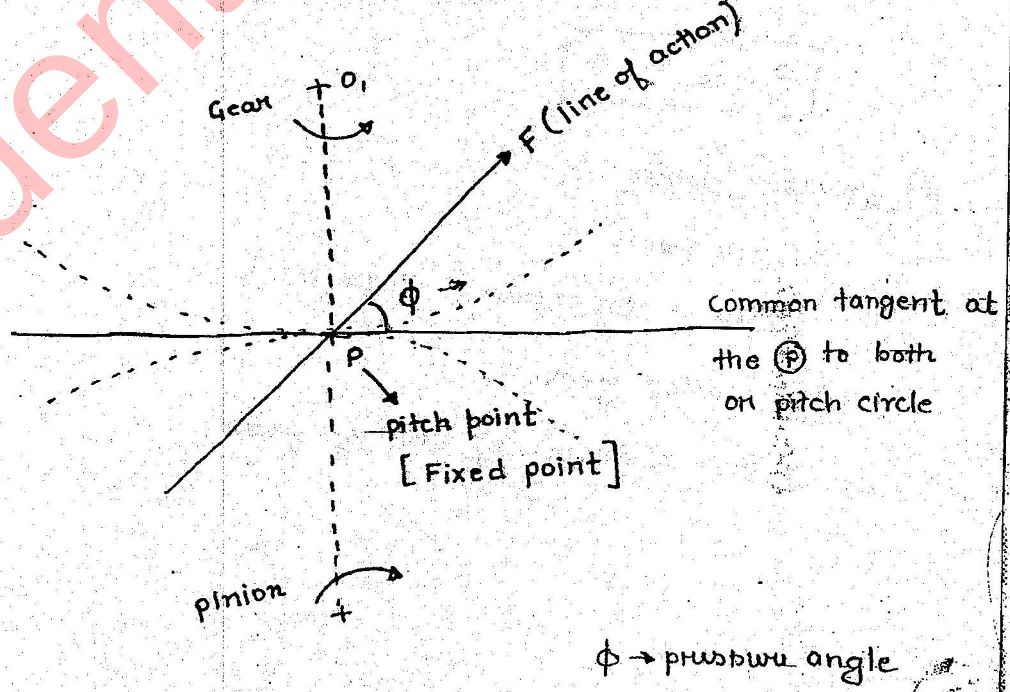
2) Generally in power transmissions, the smaller bodies are made as drivers.

   Power : $P = T \times \omega$ → High for small bodies.
   → less torque is req.

Gear Terminology –



1) **Pitch circle** : " It is an imaginary circle in the gear where the pure rolling is observed when the mating gears are transmitting power. Being an imaginary circle it can not be the physical characteristics of gear but being the most important circle it is one of the biggest specification of gear. The size of the gear is specified by the diameter of pitch circle.



2) Circular Pitch (P_c) :-

If PCD = D

No. of teeth = T

$$\boxed{P_c = \frac{\pi D}{T}}$$

for two mating gears -

$$P_{c_1} = P_{c_2}$$

$$\frac{\pi D_1}{T_1} = \frac{\pi D_2}{T_2}$$

$$\boxed{\frac{D_1}{T_1} = \frac{D_2}{T_2}}$$

3) Module (m) :-

$$m = \frac{D\ (mm)}{T}$$

for two mating gears → $m_1 = m_2$

4) Diametral Pitch (P_d) :- $\left(\frac{1}{m}\right)$

$$P_d = \frac{T}{D}$$

$$P_c \cdot P_d = \frac{\pi D}{T} \times \frac{T}{D}$$

$$\boxed{P_c \cdot P_d = \pi}$$

5) working depth :-

By one gear -

(Add. + dedendum - clearance)

By mating gears -

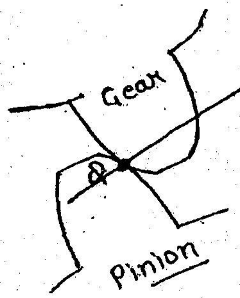
⇒ Sum of addn. of both of gears.

~~Toth~~

~~Tooth thickness — Tooth~~ tt

Tooth space − Tooth thickness of mating gears ⇒ Backlash.

To avoid the jamming of teeth due to thermal expansion.



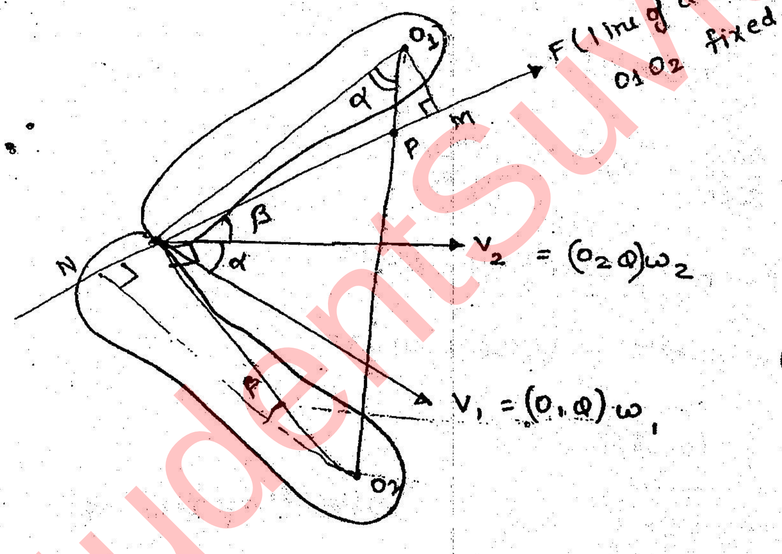
common normal at Q
(line of action).

pressure angle = angle b/w the line of action and the common tangent at the pitch point.

$\phi = 20^\circ$ to 25° ×

less power will be transmitted : $(F \cos\phi \uparrow)$

Law of gearing :-



$V_2 = (O_2 Q)\omega_2$

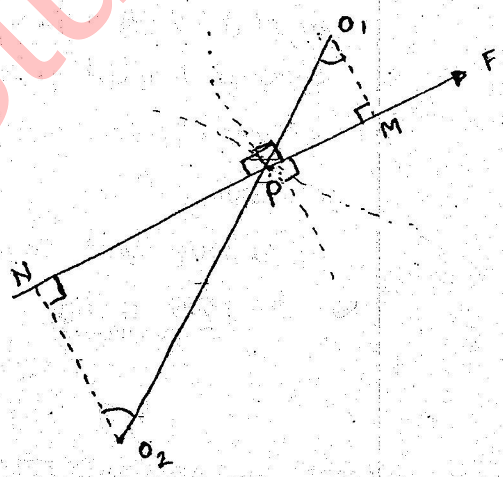
$V_1 = (O_1 Q)\omega_1$

For proper contact :-

$V_1 \cos\alpha = V_2 \cos\beta$

$(O_1 \cancel{Q})\omega_1 \cdot \frac{O_1 M}{\cancel{O_1 Q}} = (O_2 \cancel{Q})\omega_2 \frac{O_2 N}{\cancel{O_2 Q}}$

$\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M}$ — ①



In $\Delta O_1 PN$

* $\frac{\omega_1}{\omega_2} = \frac{O_2 N}{O_1 M} = \frac{O_2 P}{O_1 P} = \frac{PN}{PM}$

If these two bodies are gears –

$$\frac{\omega_1}{\omega_2} = \text{const}$$

$$\frac{O_2P}{O_1P} = \text{const}$$

($O_1 O_2$ already fixed)

$P \rightarrow$ fixed point.

"Line of action must always pass through the fixed point (pitch point) on the line joining the centre of rotation of gear"

Mating profile must be designed in such a way, such that always law of gearing is satisfied – conjugate profile → Involute profile, → Cycloidal profile.

velocity of sliding –

$$V_{sliding} = |V_1 \sin\alpha - V_2 \sin\beta|$$

$$= \cancel{O_1Q}\, \omega_1 \frac{QM}{\cancel{O_1Q}} - \cancel{O_2Q}\, \omega_2 \frac{QN}{\cancel{O_2Q}}$$

$$= \omega_1 (QP + PM) - \omega_2 (PN - QP)$$

$$= \omega_1 QP + \cancel{\omega_1 PM} - \cancel{\omega_2 PN} + \omega_2 QP$$

$$V_{sliding} = \omega_1 QP + \omega_2 QP$$

$$\boxed{V_{sliding} = (\omega_1 + \omega_2) QP}$$

~~when (P) of two math~~

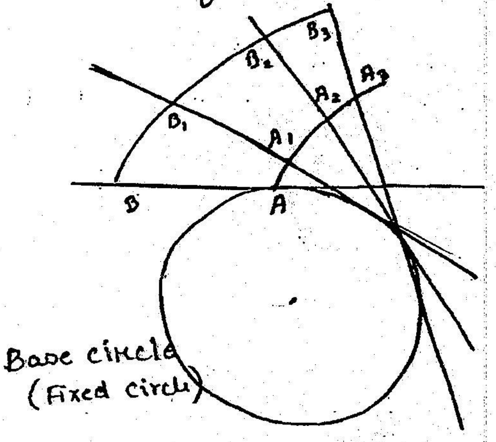
In pitch circle when two common tangent intersect at pitch point, then $QP = 0$

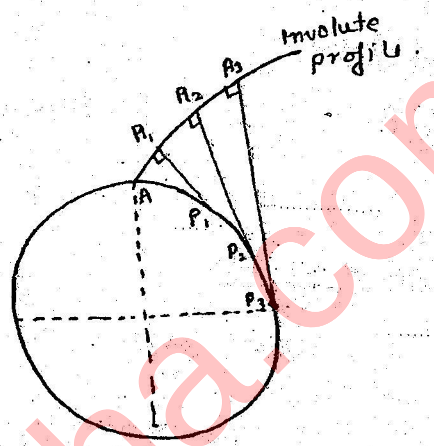
$\therefore V_{sliding} = 0$

$\therefore$ At pitch point there will be pure rolling.

Involute Profile :- (By nature Conjugate)

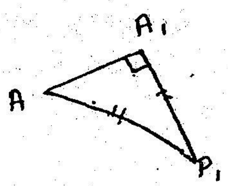
"It is defined as the locus of the point on the line which rolls without slipping on the fixed circle"
and this fixed circle is known as base circle.





In reality →

$$\left.\begin{matrix} AP_1 \\ P_1P_2 \\ P_2P_3 \\ \vdots \end{matrix}\right\} = 0 \quad \text{Differential length.}$$

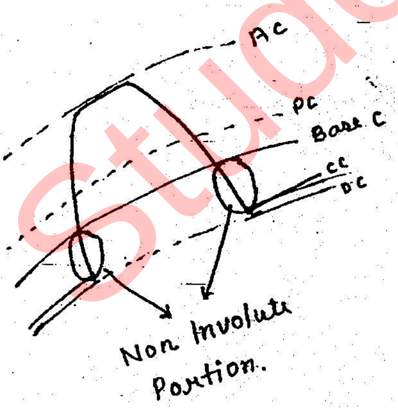


differential portion of circle having centre P_1 and rod P_1A_1

Arc $(AA_1) = P_1A_1$
Arc $(AP_2) = P_2A_2$

- Normal drawn at any point on Inv. curve will become tangent to its base circle Automatically.

Actual position of Base circle in an Involute :-



In external gears -

if $r_{base}\downarrow = \phi\uparrow$

limitation $[20^\circ, 25^\circ]$

In external gears, Non-Involute portion can never be eliminated.

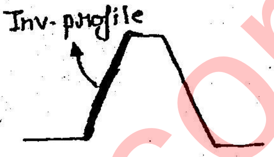
Velocity Ratio –

$$\frac{\omega_p}{\omega_g} = \frac{T}{t} > 1$$

$$\frac{\omega_g}{\omega_p} = \frac{t}{T} < 1$$

Gear Ratio –

$$g = \frac{T}{t} \quad ; \quad \text{Always} \geq 1$$

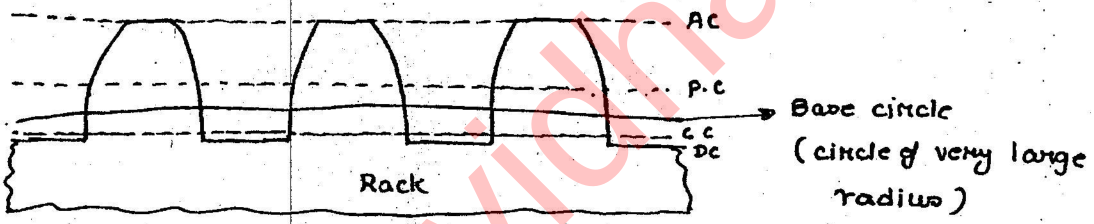


Rack :

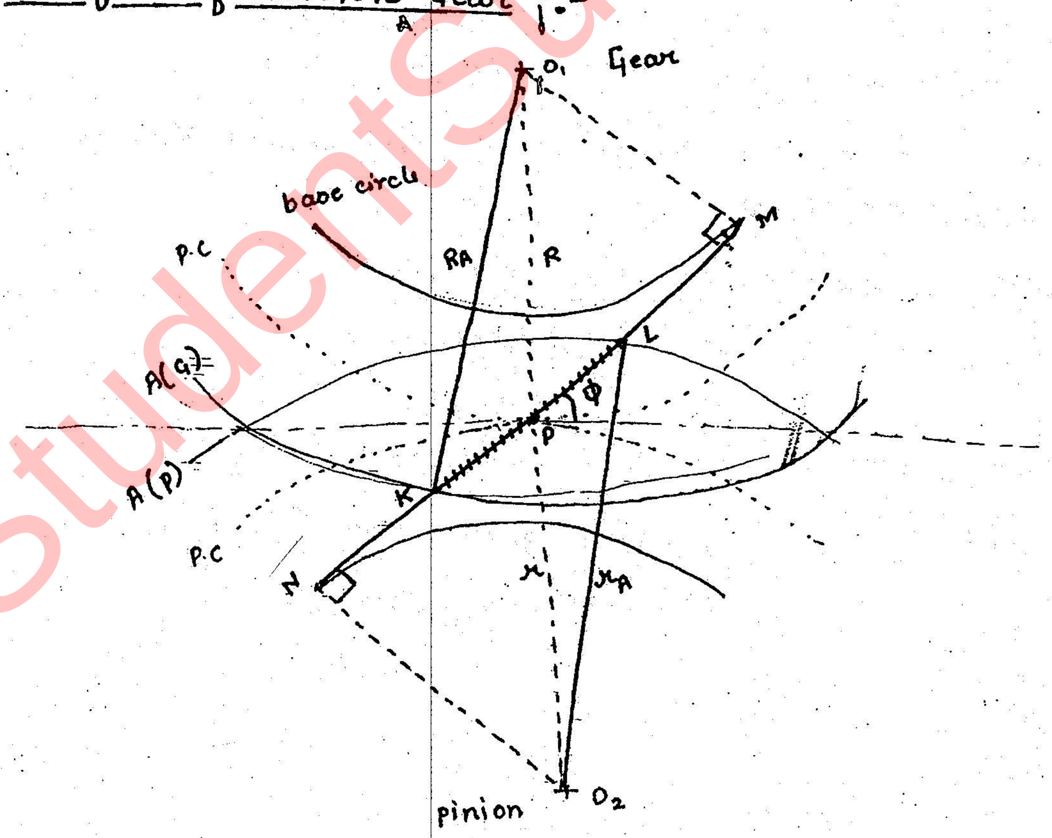
Gear of ∞ PCD –

⇓

Biggest gear



Analysis of Involute Gear :-



start of engagement : K

end of engagement : L

Line of action :

1) Pass through (P) ← Law of Gearing

2) Tangent to both of base circle ← Involute property.

- Point of contact is changing but line of action is not changing.
  ⇒ ϕ = const.

- Point of contact is travelling along the ~~ret~~ Line of action.
  ⇒ locus of Q = st. Line

- the time interval in which Q is travelling from start to end of engagement
  ⇒ one engagement period.

the distance travelled by Q in this Period –

⇓

KL → Path of contact

KP — Path of Approach

PL — Path of recess

$$O_1M = R\cos\phi$$

$$PM = R\sin\phi$$

In ΔO_1KM –

$$R_A^2 = R^2\cos^2\phi + (KP + R\sin\phi)^2$$

$$KP = \sqrt{R_A^2 - R^2\cos^2\phi} - R\sin\phi$$

Similarly,

$$PL = \sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi$$

$$KP = KP + PL$$

→ Path of contact.

** **Arc of contact ⇒**

Travel of pinion / gear along their pitch circle in one engagement period.

$$\text{Arc of App} = \frac{\text{Path of App}}{\cos \phi}$$

$$\text{Arc of Recess} = \frac{\text{Path of Recess}}{\cos \phi}$$

$$\boxed{\text{Arc of contact} = \frac{\text{Path of contact}}{\cos \phi}}$$

<u>In one engagement period</u> –

$$\text{Angle turned by pinion} = \frac{\text{Arc of contact}}{r} \quad (\text{rad})$$

$$\text{Angle turned by gear} = \frac{\text{Arc of contact}}{R} \quad (\text{rad})$$

$$\boxed{\text{Contact Ratio} = \frac{\text{Arc of contact}}{P_c}}$$

→ No. of pairs enaged in one engagement period.

min ≥ 1

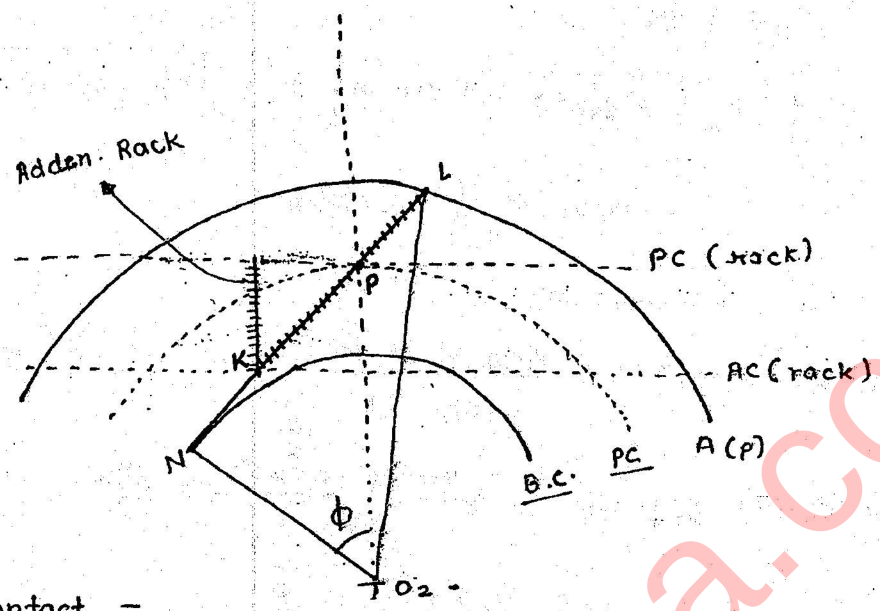
C.R , generally $\in [1.2, 1.8]$

for spur gear – $\in [2, 3]$

<u>for exm</u> :- contact ratio = 1.32

→ one pair is engaged in full engagement period. But in 32% time of engagement period, along with this pair, one more pair i.e. total two pairs are engaged. Therefore, no. of pairs engaged in one engagement period, its average value comes out to be 1.32.

Path of Contact in Rack and Pinion Arrangement :-



Path of contact -

$$KL = KP + PL$$

$$KL = \frac{\text{Adden. Rack}}{\sin\phi} + \left[\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi\right]$$

Q251

$t = 24$ Adden (each) = 7.5 mm

$T = 36$ $N_p = 450$ rpm

$m = 8$ mm

$\phi = 20°$

$$\frac{N_g}{N_p} = \frac{t}{T} = \frac{24}{36}$$

$N_s = 300$ rpm

Gear :- $R = \frac{mT}{2} = \frac{8 \times 36}{2} = 144$ mm

$R_A = (144 + 7.5) = 151.5$ mm

Pinion :- $r = \frac{mt}{2} = \frac{8 \times 24}{2} = 96$ mm

$r_A = (96 + 7.5) = 103.5$ mm

Path of contact →

$KL = KP + PL$

$= \left\{\sqrt{R_A^2 - R^2 \cos^2\phi} - R\sin\phi\right\} + \left\{\sqrt{r_A^2 - r^2\cos^2\phi} - r\sin\phi\right\}$

$= (\_\_\_)mm + (\_\_\_)mm$

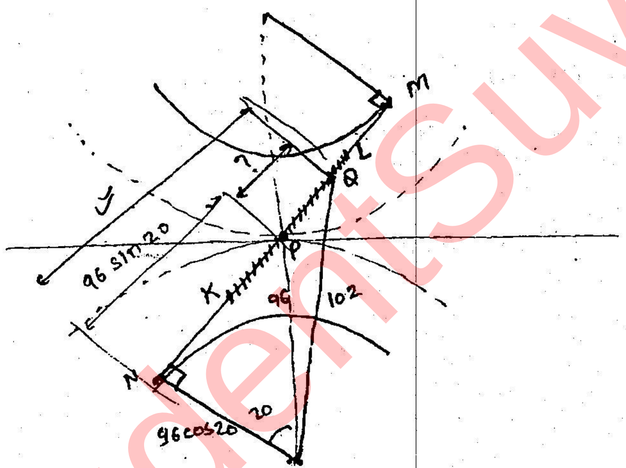
$= (\_\_\_)mm$

Arc of contact $= \dfrac{\text{Path of contact}}{\cos 20^\circ} = (\_\_\_)$ mm

i) Angle turned by pinion $= \dfrac{\text{Arc of contact}}{96} \times \dfrac{180}{\pi} = (\_\_)^\circ$

ii) $V_{sliding} = (\omega_1 + \omega_2)\,QP$

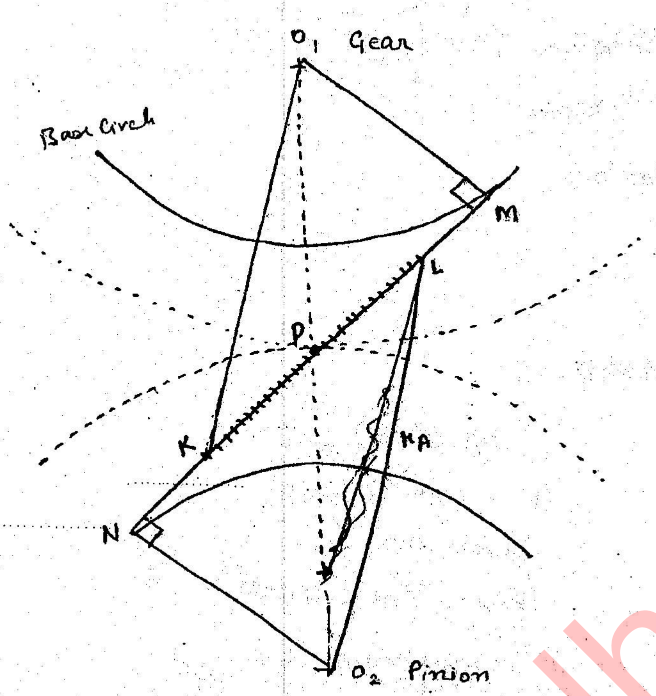
$= \dfrac{2\pi}{60}(450 + 300)\,QP$



$(102)^2 = (96\cos 20)^2 + (96\sin 20 + QP)^2$

$QP = (\_\_\_)$ mm

Interference



if $r_A > O_2M$:-

→ Involute tip of pinion will touch Non - Involute flank portion of gear.

→ Involute - Non Involute correction
= law of gearing is not be satisfied.

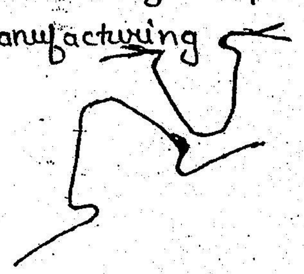
→ Involute tip of pinion will remove some material from Non-Invo flank portion of gear.
(This removal of material is a process called under cutting)

Similar will be there if $R_A > O_1N$

Interference - last safety points of K & L are N M
critical point

Methods to Prevent Interference :-

1) Under Cut-gears :-
under cutting is provided by the cutting tool at the time of manufacturing



Interference eleminated
Strength of the tooth is less at the root
Used upto low power tranission

In motion to High power transmission .-

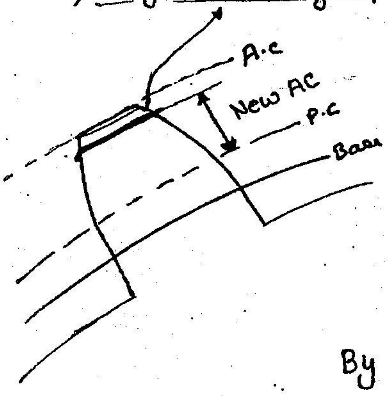
2) $\phi \uparrow$ $\longrightarrow$ limitation (20° to 25°).

if ϕ is $\uparrow$ by $\downarrow$ r_{base}

$\Rightarrow$ Non - Inv. Portion

$\Rightarrow$ Interfer $\downarrow\downarrow$

3) By stubbing the teeth :



By stubbing

ϕ = No change

Addendum $\downarrow$

Adden. circle radius $\downarrow$

$\Rightarrow$ Interference $\downarrow\downarrow$

By stubbing $\rightarrow$

= path of contact $\downarrow\downarrow$

= Arc of contact $\downarrow\downarrow$

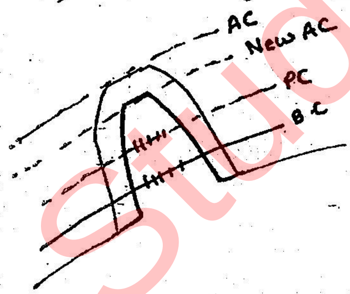
But $P_c = \frac{\pi D}{T}$ = No change

$\Rightarrow$ (contact Ratio) $\downarrow\downarrow$

limitation (min ≥ 1)

* (Best Method)

4) Increasing the No. of teeth :-



By $\uparrow$ T -

ϕ $\rightarrow$ No change

But -

Addendum $\downarrow$

Addendum circle rad $\downarrow$

Interference $\downarrow\downarrow$ (No limitations)

By $\uparrow$ T $\rightarrow$

= path of contact $\downarrow$

= Arc of contact $\downarrow$

$\longrightarrow P_c = \frac{\pi D}{T} \downarrow\downarrow$

contact Ratio $\uparrow\uparrow$ # Good Point –

A → Fractional Adden for 1 mm module in order to avoid interference.
(A_P, A_S, A_R)

Therefore, Addendum req. In order to avoid interference

$= ma$ { m → modulous }

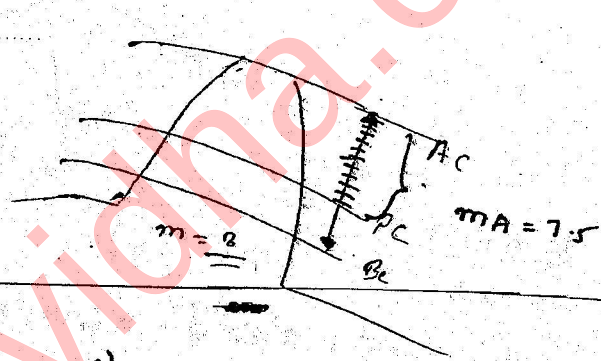
ma_P, ma_G, ma_R.

Q,

$m = 8$ mm

Add. = 7.5 m

$m \cdot A = 7.5$, $A = \frac{7.5}{m}$



In gear system –

Full Deapth Involute ($14\frac{1}{2}°$ to $20°$)

$\therefore$ Add. = std. add
= one modulus value

$mA = 1 \cdot m$

$A = 1$ $\left.\begin{matrix} A_P \\ A_S \\ A_r \end{matrix}\right\} 1$

Stub Involute ($20°$, 25)

Add : < standard Add.

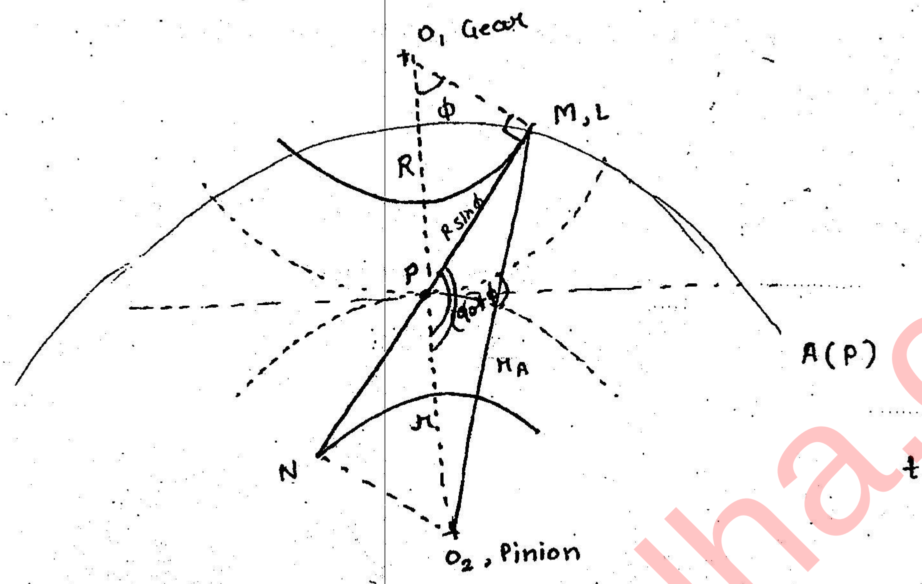
$mA < 1$ m

$A < 1$ $\left.\begin{matrix} A_P \\ A_S \\ A_r \end{matrix}\right\} < 1$

20° Stub Involute – Best gear in market.

1) lessen Interference
2) min. no. of teeth req^m is less
3) cost is less
4) stronger tooth.

⇒ Min no. of teeth requirement on the gear and pinion to avoid interference :



$t_{min} = ?$

In ΔO_2PL →

cos rule -

$$r_A^2 = r^2 + R^2\sin^2\phi - 2(r)(R\sin\phi)\cos(90+\phi)$$

$$= r^2 + (R^2 + 2rR)\sin^2\phi$$

$$= r^2\left[1 + \frac{R}{r}\left(\frac{R}{r}+2\right)\sin^2\phi\right]$$

Gear ratio $(G) = \frac{R}{r}$

$$r_A = r\sqrt{1 + G(G+2)\sin^2\phi}$$

Addendum of Pinion $= (r_A - r)$

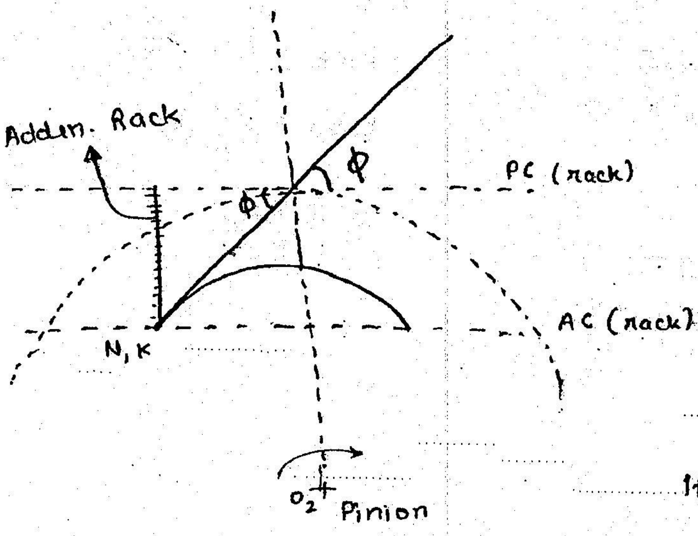
$$= r\left[\sqrt{1+G(G+2)\sin^2\phi} - 1\right] = m A_p$$

$$= \frac{\cancel{m}\, t_{min}}{2}\left[\sqrt{1+G(G+2)\sin^2\phi} - 1\right] = \cancel{m} A_p$$

$$t_{min} = \frac{2A_p}{\sqrt{1+G(G+2)\sin^2\phi} - 1}$$ *

Similarly, $$t_{min} = \frac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$ *

Min. no. of teeth required on the pinion in order to avoid interference in Involute rack and pinion arrangement -



$$\sin\phi = \frac{\text{Addn Rack}}{r\sin\phi}$$

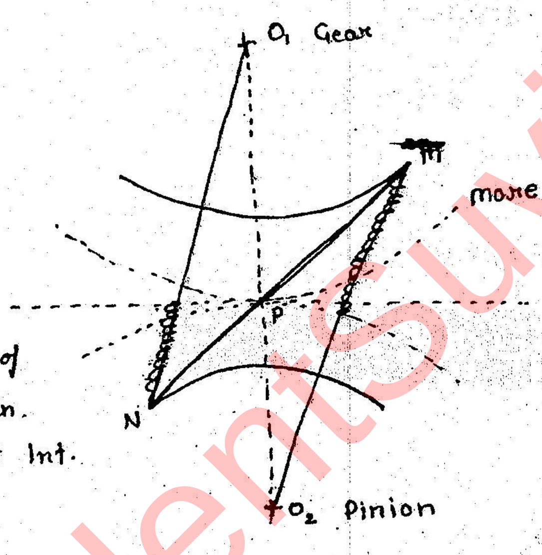
$$\text{Addn Rack} = r\sin^2\phi$$

$$\frac{\cancel{m}\, t_{min}}{2}\sin^2\phi = \cancel{m}\, A_R$$

$$\boxed{t_{min} = \frac{2A_r}{\sin^2\phi}}$$

If ~~pinion~~ rack is saved then ~~gea~~ pinion is also saved

Note :-



If Gear / Pinion
↳ Having same Addendum.

- First gear must be safe.

$$T_{min} = \frac{2A_s}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi} - 1}$$

- Pinion will be automatically safe

$$\boxed{t_{min} = \frac{T_{min}}{g}}$$

$g = 3$

$T_{min} = 28$

$t_{min} = \frac{28}{3} = 9.33$

≈ 10

$T_{min} = 30$ X

If Gear / pinion –

Having different Addendum.

- $T_{min} = \dfrac{2 A_g}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1}$

- $t_{min} = \dfrac{2 A_p}{\sqrt{1 + G(G+2)\sin^2\phi} - 1}$

- $\dfrac{T_{min}}{t_{min}} = g$

For ex :- $g = 3$

$T_{min} = 40$ 42

$t_{min} = 14$

$\dfrac{42}{14} = 3$

$g = \dfrac{T_{min}}{t_{min}}$

Note :-

~~Note~~ Normal ~~co~~ thrust b/w the teeth $= F$

Tangential component of force $= F\cos\phi$

* (power component of force)

Radial component of force $= F\sin\phi$

Torque = [redacted]

Power : $P = T \times \omega$

$= T \times \dfrac{2\pi N}{60}$

$$\boxed{P = \frac{2\pi N T}{60}} \text{ watt}$$

efficiency :-

$$\eta = \frac{P_o}{P_i} = \frac{F_o \times v_o}{F_i \times v_i} = \frac{T_o \times \omega_o}{T_i \times \omega_i}$$

Note :- The power component of force is 95 % of the normal thrust b/w the teeth ,

$\cancel{F}\cos\phi = 0.95\,\cancel{F}$

$\cos\phi = .95$

$\phi =$ ______

Q52 :

$g = 3$

$A_p = A_s = 1$

$\phi = 20^\circ$; $t_{min} = ?$

$$T_{min} = \frac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1} = 45$$

$$t_{min} = \frac{45}{3} = 15$$

if $t_{min} = 15 - 3 = 12$

$g = 3$, $T_{min} = 12 \times 3 = 36$

$$T_{min} = 36 = \frac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

$A_g = 0.8$

$= 20\%$ stubbing

how much pressure angle be changed

$$T_{min} = 36 = \frac{2A_g \rightarrow 1}{\sqrt{1+\underset{3}{\frac{1}{G}}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

Q50 :

$\phi = 20^\circ$

$m = 10$ mm

$A_p = A_g = 1$

$T = 50$, $t = 13$ $\Big]$ $g = \frac{50}{13}$

i) $$T_{min} = \frac{2A_g}{\sqrt{1+\frac{1}{G}\left(\frac{1}{G}+2\right)\sin^2\phi}-1} = 60$$

ii) $$T_{min} = 50 = \frac{2A_g \rightarrow 1}{\sqrt{1+\underset{50/13}{\frac{1}{G}}\left(\frac{1}{G}+2\right)\sin^2\phi}-1}$$

Amit Kakkar

$g = 4$

$A_p = A_s = 1$

$\phi = 20^\circ$

find $T_{min} = ?$

$16 \times 4 = 64$

$$T_{min} = \frac{2 A_g}{\sqrt{1 + \frac{1}{G}\left(\frac{1}{G} + 2\right)\sin^2\phi} - 1} = 62$$

$$t_{min} = \frac{62}{4} = 15.5 = 16$$

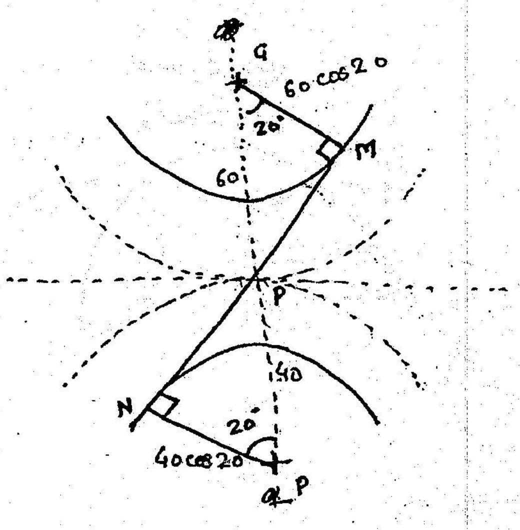
Effect of variation of centre distance due to vibration on the performance of involute gear :-

centre distance = 100 mm

$R = 60$ mm

$r = 40$ mm

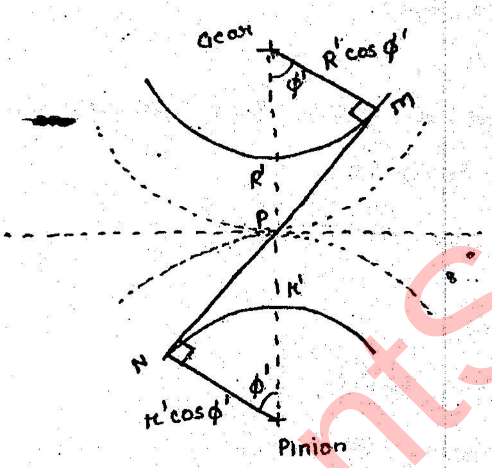
$\phi = 20^\circ$



At the moment

→ let the centre distance is ↑ by 2%

100 mm ⟶ 102 mm



Base circle will not change, else everything will change

∴ same gear is studied.

$$R' \cos \phi' = 60 \cos 20^\circ$$

$$r' \cos \phi' = 40 \cos 20^\circ$$

$$\underbrace{(R' + r')}_{102} \cos \phi' = 100 \cos 20^\circ$$

$$\cos \phi' = \frac{100 \cos 20}{102}$$

$$\phi' = \text{?}$$

Due to vibration

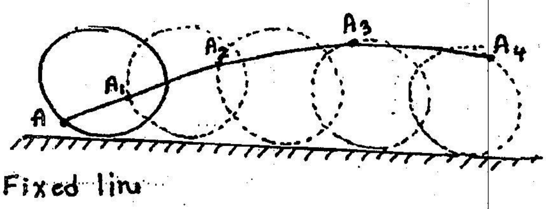
→ close to zero but ≠ 0 practically.

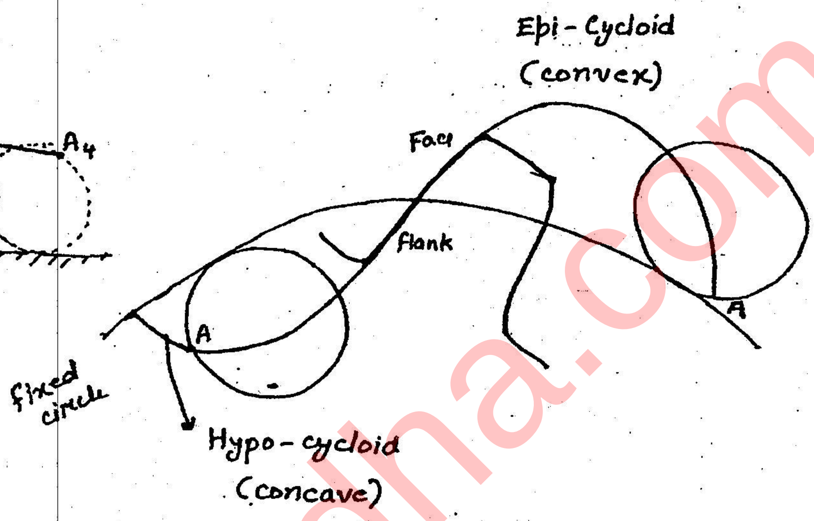
- centre distance is changing
- PC is changing.
- P is changing.
- ϕ is changing
- r_{base} → No change

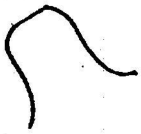
$$\boxed{\frac{\omega_1}{\omega_2} = \frac{O_2N}{O_1M}}$$ ⟶ base circle radius const. → velocity ratio

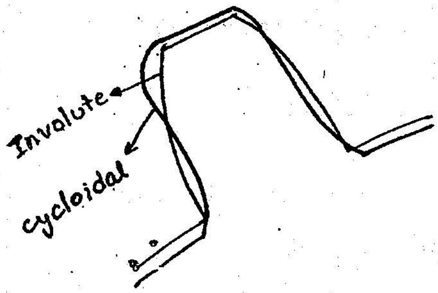
Cycloidal Profile

"It is defined as the locus of the point on the circumference of circle which rolls without slipping on the fixed straight line"









- Per tooth cost is more but overall cost of the gear is less.
- Interference is absent.
- Flanks → wide
  (stronger tooth)
- concave convex connection.
  (less wear
  life is more)

ϕ → changing

* ϕ_{max} → At start of engagement

* 0 → At pitch point

* ϕ_{max} → At the end of engagement

(In reverse direction)

Gradual change.