Gear Trains

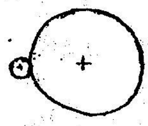
(combination of gears).

why ?

→ large centre distance

→ very high / very low requirement of velocity ratios.

For eg – $\frac{\omega_1}{\omega_2} = \frac{10}{1} = \frac{R_2}{R_1}$



→ Multiple velocity ratios are demanded.

Any gear train is a combination of –

(i) Main Driver

(ii) Main Driven

(iii) Intermediate shaft.

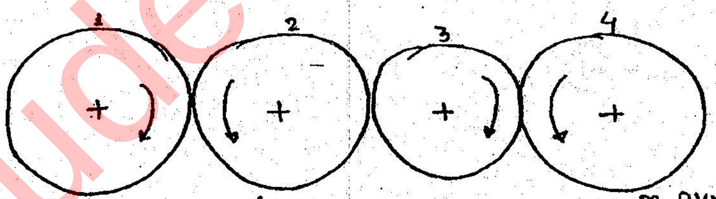
(iv) Arm

$\frac{\omega_{main\ driver}}{\omega_{main\ driven}}$ = Speed ratio of gear train.

$\frac{\omega_{main\ driven}}{\omega_{main\ driver}} = \frac{1}{S.R} =$ Train value

Simple Gear Train

Every gear is having only one gear in use.



$\boxed{m_{all} = same}$

(1, 2) : $\frac{\omega_1}{\omega_2} = \frac{T_2}{T_1}$ (i)

(2, 3) : $\frac{\omega_2}{\omega_3} = \frac{T_3}{T_2}$ (ii)

(3, 4) : $\frac{\omega_3}{\omega_4} = \frac{T_4}{T_3}$ (iii)

eq (i) x eq (ii) x eq (iii)

$\boxed{\frac{\omega_1}{\omega_4} = SR = \frac{T_4}{T_1}}$

If no. of idlers –

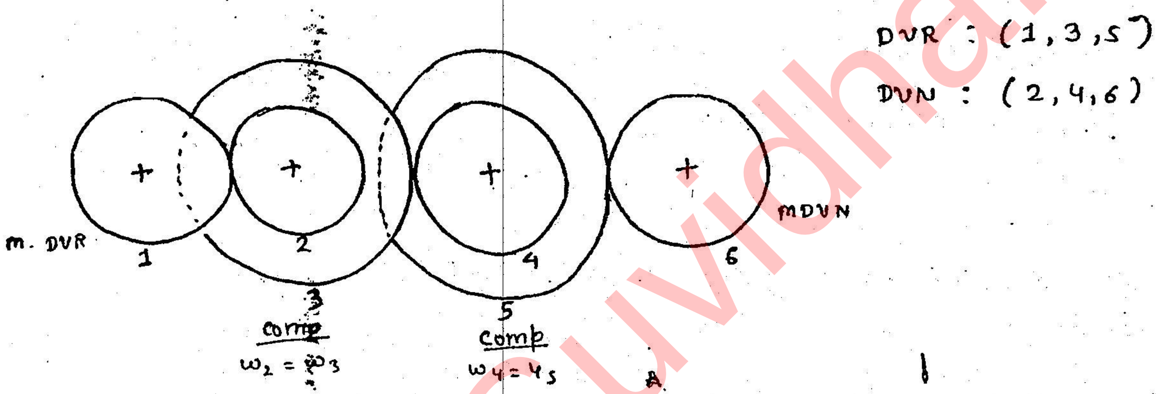
odd : Direction same

~~diff~~ even : Dir^n different.

Compound Gear Trains :-

At least one of the intermediate shaft must have more than one gears in use.

$$m_1 = m_2$$
$$m_3 = m_4$$
$$m_5 = m_6$$



$(1,2) : \dfrac{\omega_1}{\omega_2} = \dfrac{T_2}{T_1}$ — (i)

$(3,4) : \dfrac{\omega_3}{\omega_4} = \dfrac{T_4}{T_3}$ — (ii)

$(5,6) : \dfrac{\omega_5}{\omega_6} = \dfrac{T_6}{T_5}$ — (iii)

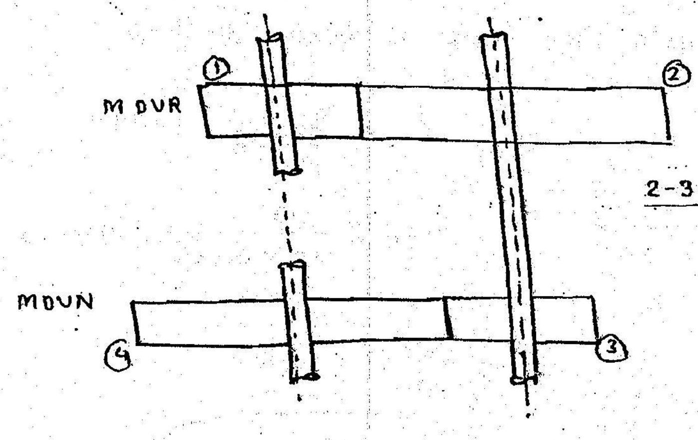
eq (i) × (ii) × (iii)

$$\frac{\omega_1}{\omega_6} = \frac{T_2 \times T_4 \times T_6}{T_1 \times T_3 \times T_5}$$

$$\boxed{S.R = \frac{\text{Product of No. of teeth on DVN}}{\text{Product of No. of teeth on DVR}}}$$

Reverted Gear Train :

That compound gear train which is used to connect coaxial shaft.



2-3 compound.

$m_1 = m_2 = m$

$m_3 = m_4 = m'$

DVR : (1, 3) ; DVN : (2, 4)

$$SR = \frac{\omega_1}{\omega_4} = \left(\frac{T_2}{T_1}\right) \times \left(\frac{T_4}{T_3}\right)$$

Note : If in the problem of reverted gear trains, speed reduction is given same.

Then take : $\boxed{\frac{T_2}{T_1} = \frac{T_4}{T_3}}$

A general concept :-

$$(r_1 + r_2) = (r_3 + r_4)$$

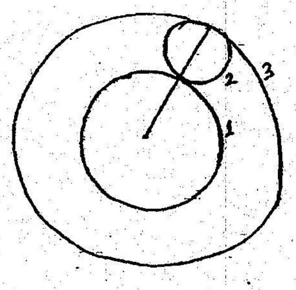
$$\frac{mT_1}{2} + \frac{mT_2}{2} = \frac{m'T_3}{2} + \frac{m'T_4}{2}$$

$$\boxed{m(T_1 + T_2) = m'(T_3 + T_4)}$$

If all gears have same modulus.

$$T_1 + T_2 = T_3 + T_4$$

for ex :-

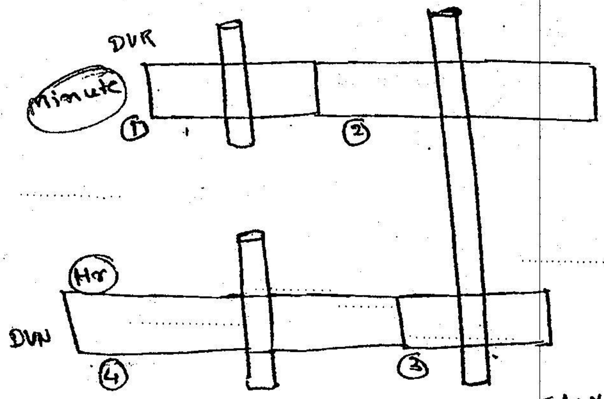


mod → same

$r_1 + 2r_2 = r_3$

$$\boxed{T_1 + 2T_2 = T_3}$$

Q - A reverted gear train is designed to rotate ~~m~~ hour hand ~~of the~~ clock with the help of minute hand. Assuming the module of all the wheels same. Determine the appropriate no. of teeth on the wheels. Any wheel should not have 11 teeth or less.



$\omega_1 = \frac{2\pi}{60 \times 60}$ rad/s

$\omega_4 = \frac{2\pi}{12 \times 60 \times 60}$ rad/s

$\frac{\omega_1}{\omega_4} = 12$

$\frac{T_2}{T_1} \times \frac{T_4}{T_3} = 12$ — (1)

$T_1 + T_2 = T_3 + T_4$ — (2)

Any teeth > 11

4 × 3
3 × 4
6 × 2
12 × 1

Assume —

$T_1 = 12$

Assume —

$\frac{T_2}{T_1} = 4$; $\frac{T_4}{T_3} = 3$

$T_2 = 4T_1 = 48$

$12 + 48 = T_3 + 3T_3$

$T_3 = 15$

$T_4 = 45$

$T_1 = 12$
$T_2 = 48$
$T_3 = 15$
$T_4 = 45$

Assume →

$\frac{T_2}{T_1} = 3$; $\frac{T_4}{T_3} = 4$

$T_2 = 3T_1 = 36$

$12 + 36 = T_3 + 4T_3$

$T_3 = 9.6$ ✗

Assume —

$\frac{T_2}{T_1} = 6$; $\frac{T_4}{T_3} = 2$

$T_2 = 6T_1 = 72$

$12 + 72 = T_3 + 2T_3$

$T_3 = 28$

$T_4 = 56$

Assume —

$\frac{T_2}{T_1} = 2$; $\frac{T_4}{T_3} = 6$

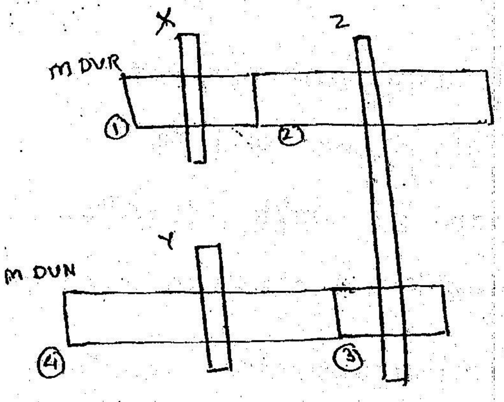
$T_2 = 2T_1 = 24$

$12 + 24 = T_3 + 6T_3$

$7T_3 = 36$

$T_3 = 5.1$ ✗

Q48 →



$m = 2$ mm

$m' = 3$ mm

$\omega_4 < \frac{\omega_1}{12}$

$\frac{\omega_1}{\omega_4} > 12 \Rightarrow \frac{T_2 T_4}{T_1 T_3} > 12$

$T_1 = T_3 = 24$

$\therefore \frac{T_2 T_4}{24 \times 24} > 12$

$T_2 T_4 > 6912$ — (1)

$m(T_1 + T_2) = m'(T_3 + T_4)$

$2(24 + T_2) = 3(24 + T_4)$

$48 + 2T_2 = 72 + 3T_4$

$3T_4 = 2T_2 - 24$

$T_4 = \frac{2}{3}(T_2 - 12)$ — (2)

$T_2 \times \frac{2}{3}(T_2 - 12) > 6912$

$T_2^2 - 12T_2 > \frac{6912 \times 3}{2}$

$(T_2^2 - 12T_2 - 10368) > 0$

$T_2 > 108$

Assume —

$T_2 = 109 \Rightarrow$

$\Rightarrow T_4 = \frac{2}{3}(109 - 12)$

$T_4 = 64.66$

$= 65$

$T_3 = \cancel{26}\ 24 - 1 = 23$

$2(24 + T_2) = 3(23 + 65)$

$T_2 = 108$

$T_1 = 24$, $T_2 = 109$

$T_3 = 23$; $T_4 = 65$

$\frac{\omega_1}{\omega_4} = \frac{108 \times 65}{24 \times 23} = 12.71$

$\boxed{\text{if } T_4 = 63}$ ×

centre dist^n

$= r_1 + r_2$

$= \frac{mT_1}{2} + \frac{mT_2}{2}$

$= 13\ \ 132$

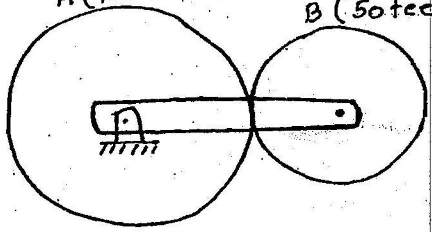
Axis Rotation

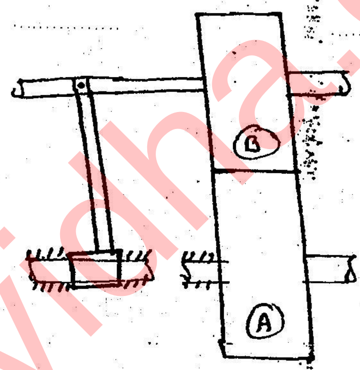
Epicyclic Gear Trains :-

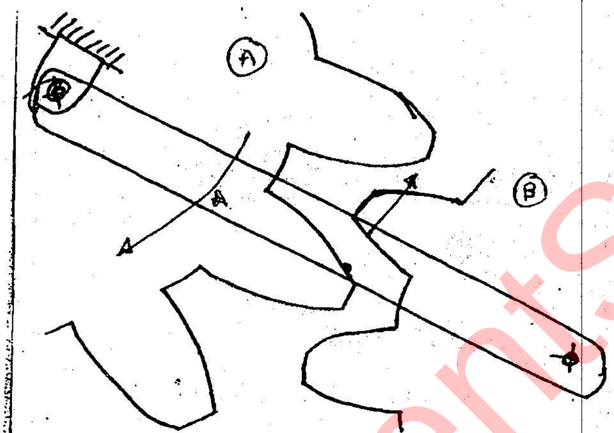
" Apart from the rotation of gear if any gear axis is rotating w.r.t some other axis, then the train will be known as epi-cyclic gear train. It can be simple epicyclic, compound epicyclic, Reverted epicyclic ~~and~~ bevel epicyclic & so on."

To rotate the axis of gear a very important link is used which is known as arm or carrier and it is not a gear.

A (100 Teeth) B (50 teeth)







let us take gear A rotated by 100 rpm (cw).

Gear B → By gear A → 200 (AC)
Gear B → Arm → 100 (AC)
↓
300 rpm (AC)

l = 4
j = 3
h = 1

$F = 3(4-1) - 2 \times 3 - 1$
$= 9 - 6 - 1$
$F = \underline{\underline{2}}$

Arm → 200 rpm (cw)
Gear A → 200 rpm (AC) > zero

Gear A → 200 rpm (AC)
Arm → 300 rpm (cw) > 100 rpm (cw)

Q → A|B – compound.

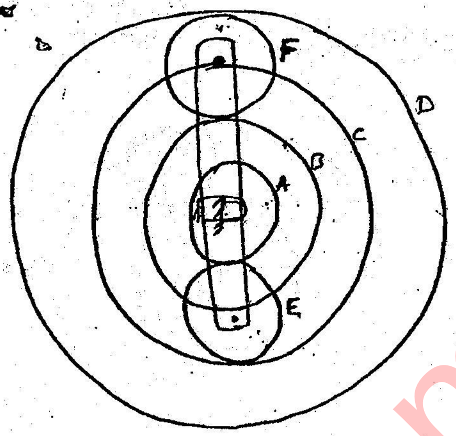
All gears have same module

$T_A = 20$; $T_B = 30$; $T_E = T_F = 10$

Gear D → fixed.

arm → 100 rpm (Ac)

find – $N_{five\ gears} = ?$



$$T_A + 2T_E = T_C$$

$$T_C = 40$$

$$T_B + 2T_F = T_D$$

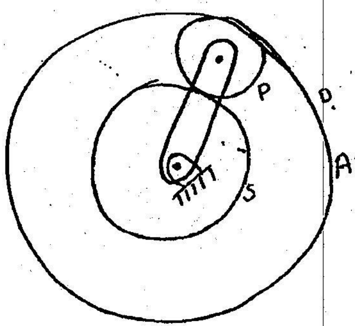
$$T_D = 50$$

Motion	Arm	A/B 20/30	E 10	C 40	F 10	D 50
1) Arm Fixed let gear A rotates by +x rpm (cw)	0	+x	$-x\frac{20}{10}$	$-x\frac{20}{10}\times\frac{10}{40}$	$-x\cdot\frac{30}{10}$	$-x\frac{30}{10}\cdot\frac{10}{50}$
2) Arm Free	y	y+x	y−2x	$y-\frac{x}{2}$	y−3x	$y-\frac{3}{5}x$

Given → $y - \frac{3x}{5} = 0$ — (I) $x = $ ✓

$y = -100$ — (II) $y = $ ✓

Planetary Gear Train :- Epi-Cyclic



m_{all} = same

$r_s + 2 r_p = r_D$

$T_s + 2 T_p = T_D$

I - Input

Sun	Ring
Fixed	Input
Input	Fixed

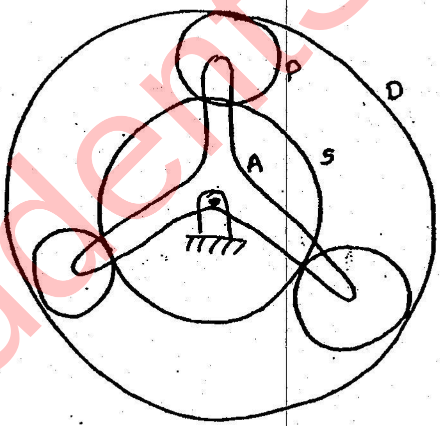
II - Input

Arm ;

Generally in planetary gear trains No. of planet gears are more than one why ?

1) For the balancing of Gear Trains.
2) for the load distribution among the no. of planets in high power transmission.

Q 49 →



$T_D = \frac{252}{3.5} = 72$

$T_s + 2 T_p = 72$

$N_B = 0$

$N_s = 5 N_A$

Arm	S T_s	P T_p	D (72)
0	$+x$	$-x\frac{T_s}{T_p}$	$-x\frac{T_s}{T_p}\cdot\frac{T_p}{72}$
y	$y+x$	$y-x\frac{T_s}{T_p}$	$y-x\frac{T_s}{72}$

$y + x = 5y$

$4y = x$

$y - x\frac{T_s}{72} = 0$

$y\left(1 - \frac{T_s}{18}\right) = 0$ $\quad y \neq 0$

$\underline{\underline{T_s = 18}}$

Fixing torque / Holding torque on epicyclic gear train -

$\Sigma T = (T_{input} + T_{output} + T_{fixing}) = 0$ — ①

~~Given~~ : Power :

$\eta_{GT} \rightarrow$ Efficiency of gear train

$\eta_{GT} \times P_{inp} + P_{output} = 0$

$\eta_{GT} \times T_{inp} \times \omega_{input} + T_{output} \times \omega_{output} = 0$ — ②

Note :- If η_{GT} is not given in the problem then take $\boxed{\eta_{GT} = 1}$

Q34 →

$N_{input} = +100$

$N_{output} = +250$

$T_{input} = +50$

$T_{Fixed} = ?$

$\eta_{GT} = 1$ (Assume)

$50(100) + T_{output} \times (250) = 0$

$T_{output} = -20$

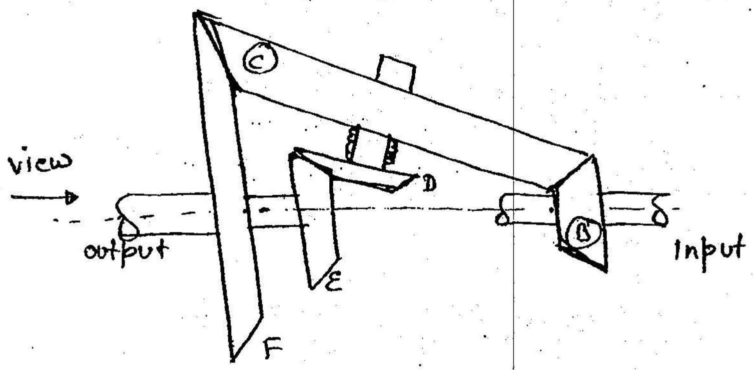
$(+50) + (-20) + T_{fixing} = 0$

$\underline{\underline{T_{fixing} = -30}}$

(ACW)

Direction consideration in Bevel Epicyclic Gear Train :-

Q49. Humpage Reduction gear of Lathe m/c -



$T_B = 19$

$T_C = 57$

$T_D = 20$

$T_E = 40$

$T_F = 76$

$N_B = +300$

$N_F = -500$

$N_E = ??$

direction of bevel gear is opposite.

Arm	B	C/D	F	E
0	$+x$	$\pm x \frac{19}{57}$	$-x \frac{19}{57} \times \frac{57}{76}$	$-x \frac{19}{57} \times \frac{20}{40}$
y	$y+x$	$y \pm \frac{x}{3}$	$y - \frac{3}{4}$	$y - \frac{3}{5}$

$$y + x = 300$$

$$y - \frac{x}{4} = -500$$

$$\frac{5x}{4} = 800$$

$$x = \frac{3200}{5} = 640$$

$$y = -340$$

$$N_E = \left(-340 - \frac{640}{4}\right) = -446.66$$

$$= 447 \text{ AC}$$

Terminology of Helical or Spiral Gear :-

1 - Driver

2 - Driven

$\phi \rightarrow$ pressure angle.

$\mu \rightarrow$ coeff. of friction , $\phi = \tan^{-1}(\mu)$

$\psi \rightarrow$ spiral angle (Helix angle)

$\theta \rightarrow$ Angle b/w the shafts.

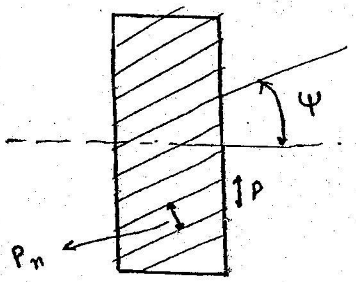
$P = P_c \rightarrow$ circular pitch

$P_N \rightarrow$ Normal pitch

$m \rightarrow$ module

$m_n \rightarrow$ normal module

$L \rightarrow$ Lead
$\lambda \rightarrow$ Lead angle] worm gear.



$$P_n = P\cos\psi$$

$$\boxed{m_n = m\cos\psi}$$

For two mating gear :-

$$m_{n_1} = m_{n_2}$$

$$\boxed{m_1\cos\psi_1 = m_2\cos\psi_2}$$

$\theta = \psi_1 + \psi_2$

= If same hand gears are in contact

$\theta = (\psi_1 - \psi_2)$

= If opposite hand gears are in contact.

Note :- If in the problem it is not given that which hand gears are in contact then take same hand

$$\theta = \psi_1 + \psi_2$$

i) Centre distance :-

$$= (R_1 + R_2)$$

$$= \frac{m_1 T_1}{2} + \frac{m_2 T_2}{2}$$

$$= \frac{m_n}{\cos\psi_1} \cdot \frac{T_1}{2} + \frac{m_n T_2}{\cos\psi_2 2}$$

$$= \frac{m_n}{2}\left[\frac{T_1}{\cos\psi_1} + \frac{T_2}{\cos\psi_2}\right]$$

ii) <u>velocity Ratio</u> -

$$VR = \frac{\omega_2}{\omega_1}$$

$$V_1 \cos\psi_1 = V_2 \cos\psi_2$$

$$\boxed{\frac{V_2}{V_1} = \frac{\cos\psi_1}{\cos\psi_2}}$$

$$VR = \frac{\omega_2}{\omega_1} = \frac{V_2/r_2}{V_1/r_1} = \frac{V_2}{V_1} \times \frac{r_1}{r_2}$$

$$= \frac{\cos\psi_1}{\cos\psi_2} \times \frac{\frac{m T_1}{2}}{\frac{m T_2}{2}}$$

$$= \frac{\cos\psi_1}{\cos\psi_2} \cdot \frac{T_1}{T_2} \cdot \frac{\frac{m_n}{\cos\psi_1}}{\frac{m_n}{\cos\psi_2}}$$

$$VR = \boxed{\frac{\omega_2}{\omega_1} = \frac{T_1}{T_2}}$$

(iii) <u>efficiency</u> :-

$$\eta = \frac{P\,o/p}{P\,i/p} = \frac{P_2}{P_1} = \frac{F_2 V_2}{F_1 V_1}$$

$$\boxed{\eta = \frac{\cos(\psi_2 + \phi)\cos\psi_1}{\cos(\psi_1 - \phi)\cos\psi_2}}$$

max efficiency.

$$\boxed{\eta_{max} = \frac{1 + \cos(\theta + \phi)}{1 + \cos(\theta - \phi)}}$$

when

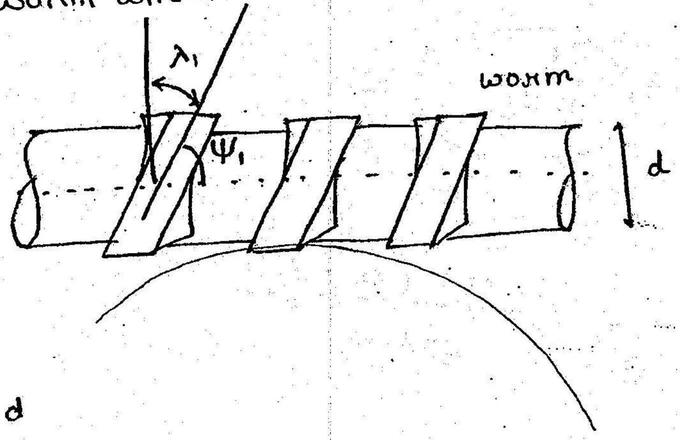
$$\theta = \psi_1 + \psi_2$$

$$\psi = \frac{\theta + \phi}{2}$$

Terminology of worm and worm wheel :-

1 - worm

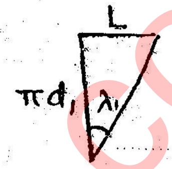
2 - worm wheel.



L = Lead

= Pitch (single start

= 2p — double start



$$\tan\lambda_1 = \frac{L}{\pi d_1}$$

$$\psi_1 + \lambda_1 = 90^\circ$$

$$\psi_1 = (90 - \lambda_1)$$

$$\theta = 90^\circ = \psi_1 + \psi_2$$

$$90^\circ = 90^\circ - \lambda_1 + \psi_2$$

$$\boxed{\psi_2 = \lambda_1}$$

Spiral angle of wheel = Lead angle of worm

i) centre distance -

$$= (P_1 + P_2)$$

$$= \frac{m_1 T_1}{2} + \frac{m_2 T_2}{2}$$

$$= \frac{m_2}{2}\left[\frac{m_1}{m_2} T_1 + T_2\right]$$

$$= \frac{m_2}{2}\left[\frac{\frac{m_n}{\cos\psi_1}}{\frac{m_n}{\cos\psi_2}} T_1 + T_2\right]$$

$$= \frac{m_2}{2}\left[\frac{\cos\lambda_1}{\cos(90-\lambda_1)} T_1 + T_2\right]$$

$$= \frac{m_2}{2}\left[T_1 \cot\lambda_1 + T_2\right]$$

velocity Ratio (VR) –

$$VR = \frac{\omega_2}{\omega_2} = \frac{L/r_2}{2\pi}$$

$$VR = \frac{L}{2\pi r_2} = \frac{L}{\pi d_2}$$

Efficiency :–

$$\eta = \frac{\cos(\lambda_1 + \phi)}{\cos(90^\circ - (\lambda_1 + \phi))} \times \frac{\cos(90 - \lambda_1)}{\cos \lambda_1}$$

$$\eta = \frac{\tan \lambda_1}{\tan(\lambda_1 + \phi)}$$

$$\boxed{\eta_{max} = \frac{1 - \sin\phi}{1 + \sin\phi}}$$