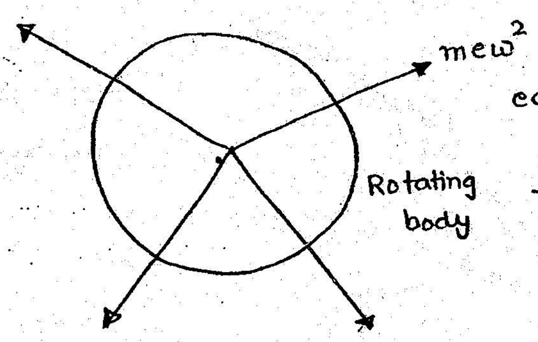
Balancing



eccentricity (e)

Rotating body → const. magnitude but direction continuously changing.

Reciprocating body
↓
unbalance force of changing magnitude having same direction

$$F_I = mr\omega^2\left(\cos\theta + \frac{\cos 2\theta}{n}\right)$$

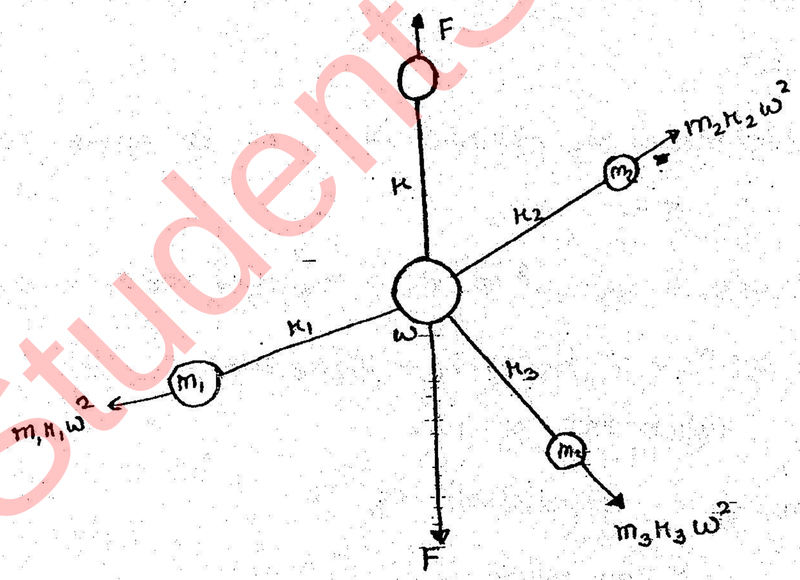
Balancing

→ $\Sigma F = 0$ static Balancing $\Sigma F = 0$ ex :- (if all masses are in same plane)

moments automatically balanced

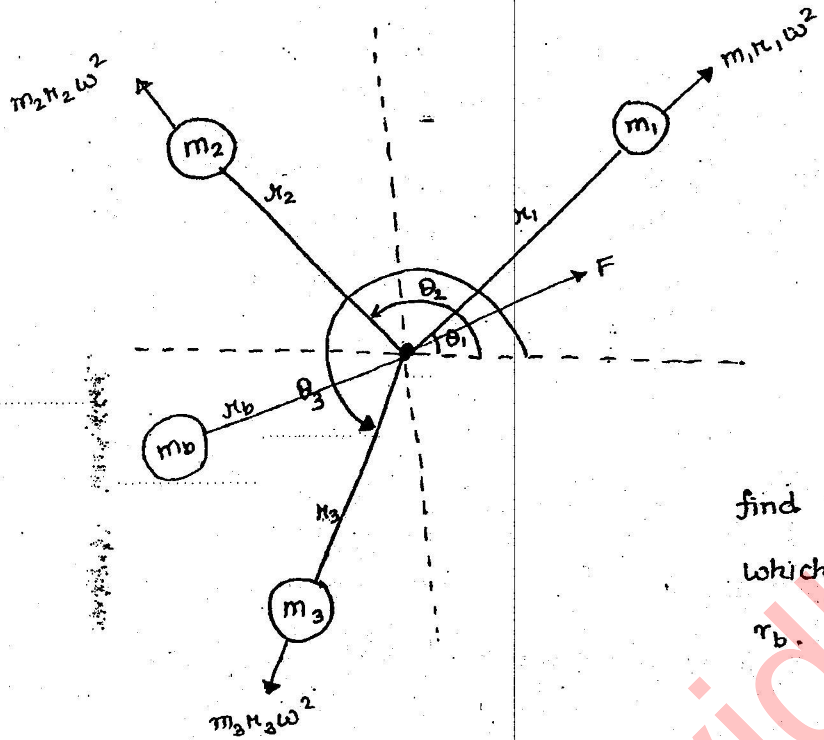
→ $\Sigma M = 0$ Dynamic Balancing $\Sigma F = 0$ & $\Sigma M = 0$

(ex :- min. 2 masses in two diff plane).



Static balancing of rotating masses :-

(All the masses are in same plane).



weight of the masses are neglected.

find the balancing mass m_b which is applied at a radius r_b.

$$F_x = m_1r_1\omega^2\cos\theta_1 + m_2r_2\omega^2\cos\theta_2 + m_3r_3\omega^2\cos\theta_3$$

$$F_y = m_1r_1\omega^2\sin\theta_1 + m_2r_2\omega^2\sin\theta_2 + m_3r_3\omega^3\sin\theta_3$$

total unbalanced force -

$$F = \sqrt{F_x^2 + F_y^2}$$

After applying balancing mass m_b at radius r_b at an angle θ_B

$$\Sigma F_x = 0 \quad ; \quad \Sigma F_y = 0$$

$$m_1r_1\omega^2\cos\theta_1 + m_2r_2\omega^2\cos\theta_2 + m_3r_3\omega^2\cos\theta_3 + m_Br_B\omega^2\cos\theta_B = 0$$

$$m_1r_1\cos\theta_1 + m_2r_2\cos\theta_2 + m_3r_3\cos\theta_3 + m_Br_B\cos\theta_B = 0$$

$\therefore$ Balancing is independent of ω.

Need of balancing is dependent of ω.

$$m_Br_B\cos\theta_B = -\Sigma mr\cos\theta \quad \text{— (I)}$$

$$m_Br_B\sin\theta_B = -\Sigma mr\sin\theta \quad \text{— (II)}$$

$eq\ (10)^2 + eq\ (11)^2$

$= m_B^2 r_B^2 = (-\Sigma mr \cos\theta)^2 + (-\Sigma mr \sin\theta)^2$

$$\boxed{m_B r_B = \sqrt{(-\Sigma mr \cos\theta)^2 + (-\Sigma mr \sin\theta)^2}}$$

$eq\ (11) \div eq\ (10)$

$$\boxed{\tan\theta_B = \frac{-\Sigma mr \sin\theta}{-\Sigma mr \cos\theta}}$$

The final sign of numerator and denominator will decide the quadrant of balancing mass

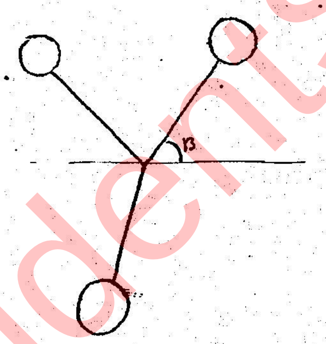
eg :- $\Sigma mr \sin\theta = -3$

$\Sigma mr \cos\theta = 4$

$\tan\theta_B = \frac{-(-3)}{-(4)} = \frac{+3}{-4}$ $\begin{matrix} \rightarrow \sin\theta_B \\ \rightarrow \cos\theta_B \end{matrix}$

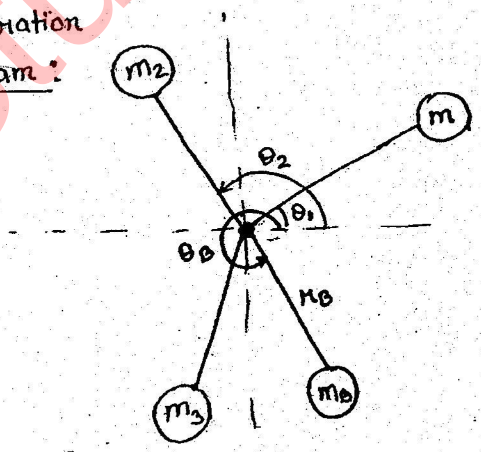
θ_B is in 2^{nd} Quadrant.

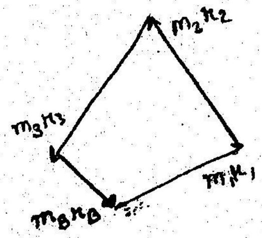
Graphical :-



m	r	
m_1	r_1	$m_1 r_1$
m_2	r_2	$m_2 r_2$
m_3	r_3	$m_3 r_3$

configuration Diagram :





force polygon

1 mm = ____

$m_B r_B$ measure & descale.

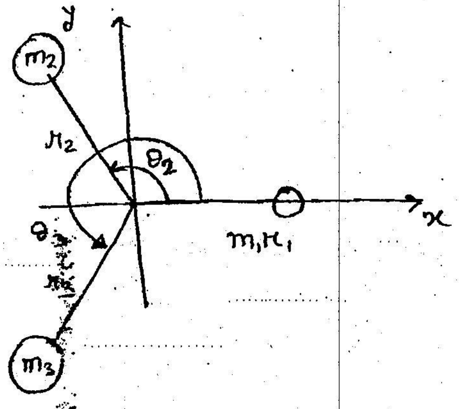
Q> 3 masses are rotating in a same plane

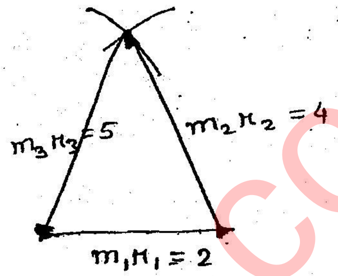
$m_1r_1 = 2$

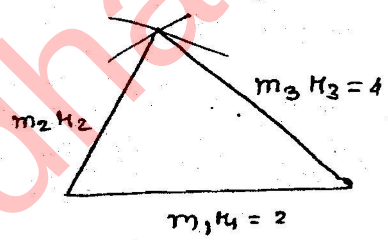
$m_2r_2 = 4$

$m_3r_3 = 5$

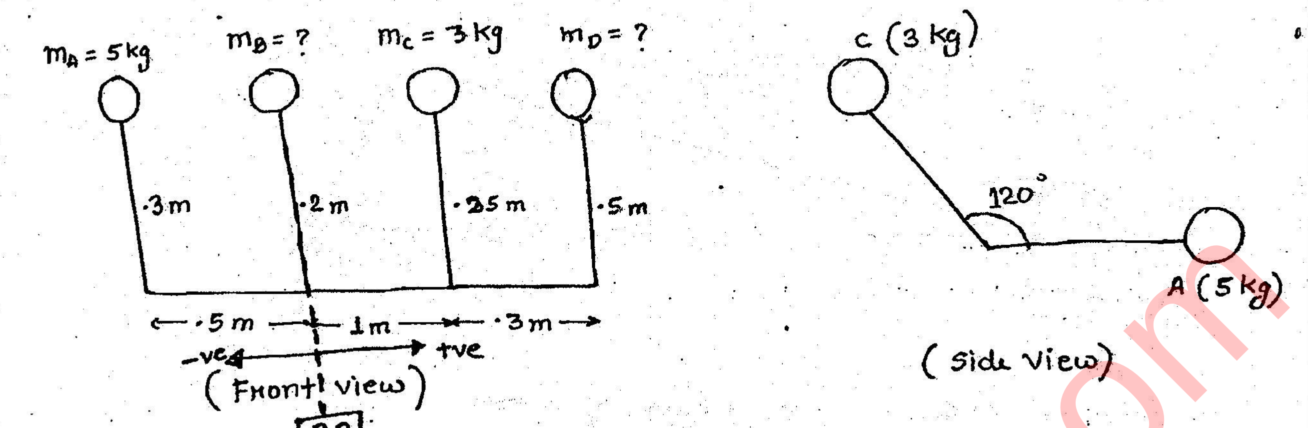
what should be the orientation of mass 2 and 3 with respect to mass 1 in order to have complete static balance.







Dynamic Balancing :- All mass are not rotating in same plane.



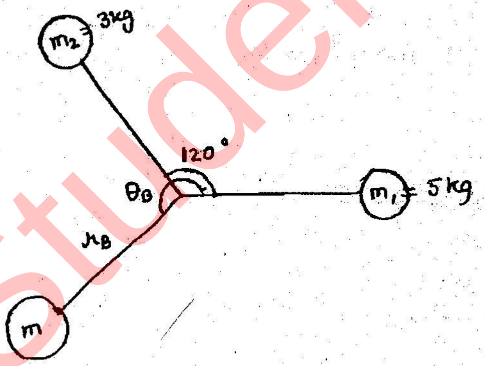
$m_B = ?$, $m_D = ?$, $\theta_B = ?$, $\theta_D = ?$

Find the masses in the plane B and D along with their position in order to have complete balancing.

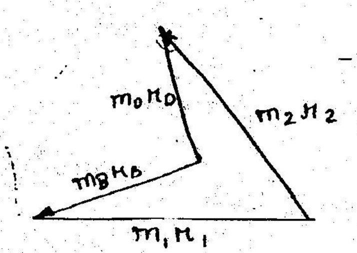
→ mostly ref. plane is considered as the unknown mass location

Graphical →

Plane	m	r	mr	Distance of plane from R.P	mrl ($kg-m^2$)
A	5	.3	1.5	-0.5	-.75
B	m_B	.2	.2 m_B	0	0
C	3	.25	.75	1	0.75
D	m_D	.5	.5 m_D	1.3	0.65 m_D



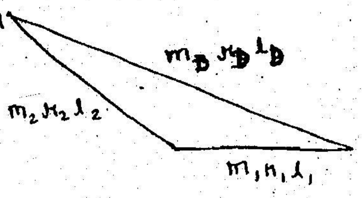
Force polygon -



$m_B r_B$ known (measure & descale)

m_B = known

Momentum

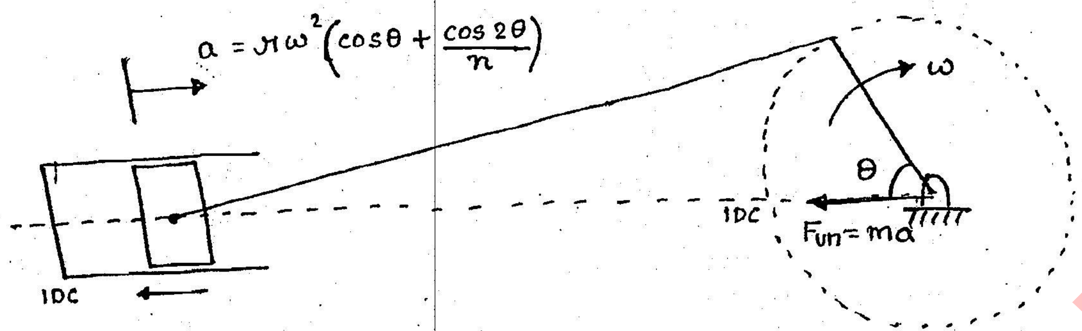


0.65 m_D = known (measure & descale)

m_D = known

θ_D = known

Balancing of Reciprocating masses :-



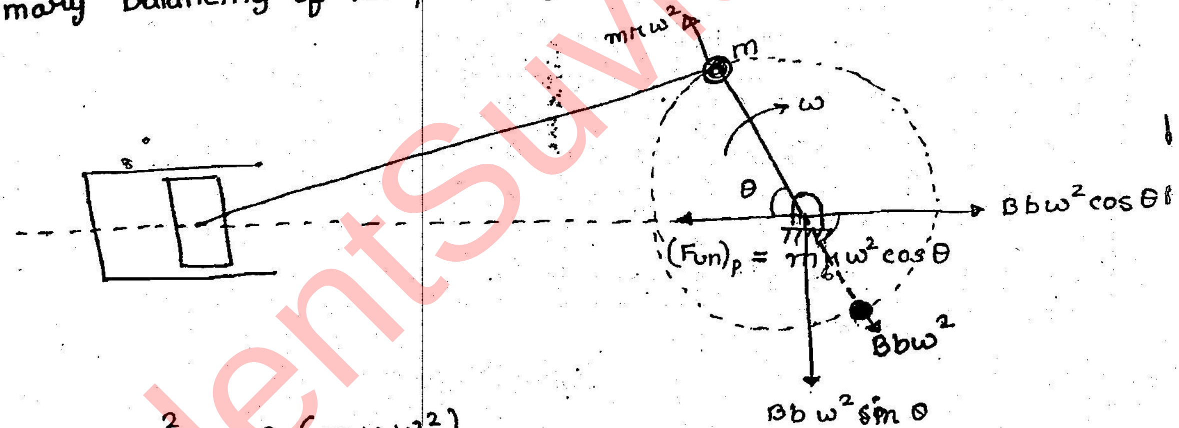
$F_{un} = m a$ $\qquad$ $m \rightarrow$ mass of piston

$F_{un} = m r \omega^2 \left(\cos\theta + \frac{\cos 2\theta}{n}\right)$

$$\boxed{F_{un} = \underbrace{m r \omega^2 \cos\theta}_{\text{Primary Unbalanced Force}} + \underbrace{\frac{m r \omega^2 \cos 2\theta}{n}}_{\text{Secondary Unbalanced Force}}}$$

Primary Unbalanced Force >>> Secondary Unbalanced Force

- Primary balancing of Reciprocating mass :-



$B b \omega^2 = c\,(m r \omega^2)$

Along line of stroke : $0 < c < 1$

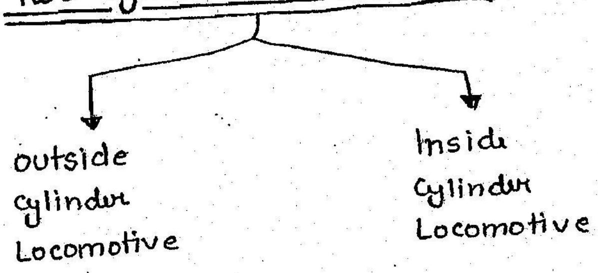
$F_{un\,p\,(\text{along LOS})} = m r \omega^2 \cos\theta - c\, m r \omega^2 \cos\theta$

$= (1 - c)\, m r \omega^2 \cos\theta$

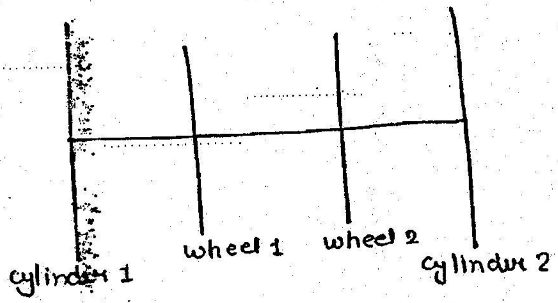
$\perp$ to L.O.S $\rightarrow$

$F_{un\,p\,(\perp \text{ to LOS})} = c\, m r \omega^2 \sin\theta$

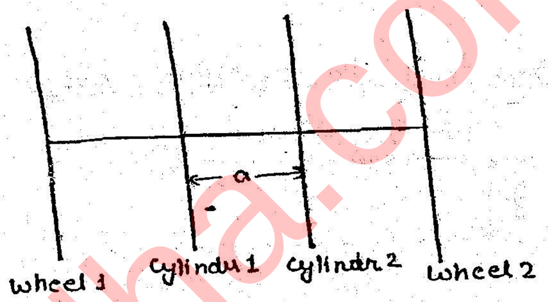
Two Cylinder Locomotive :-

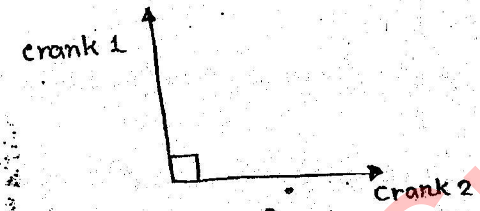


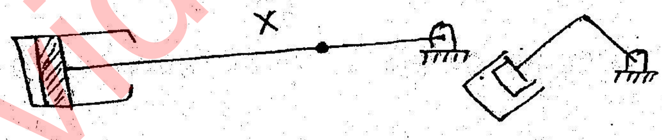
Outside Cylinder Locomotive :-



Inside Cylinder Locomotive :-

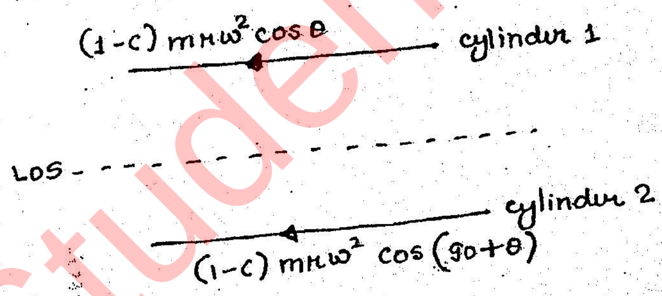






Effects of partial balancing in two cylinder locomotive :-

i) Variation in tractive force :-



$$\text{Tractive force} = (1-c)\,mr\omega^2 \cos\theta + (1-c)\,mr\omega^2 \cos(90+\theta)$$

$$= (1-c)\,mr\omega^2 \cos\theta + (1-c)\,mr\omega^2(-\sin\theta)$$

$$\boxed{T.F = (1-c)\,mr\omega^2\left[\cos\theta - \sin\theta\right]}$$

for tractive force to be maximum –

$$\frac{d}{d\theta}(\cos\theta - \sin\theta) = 0$$

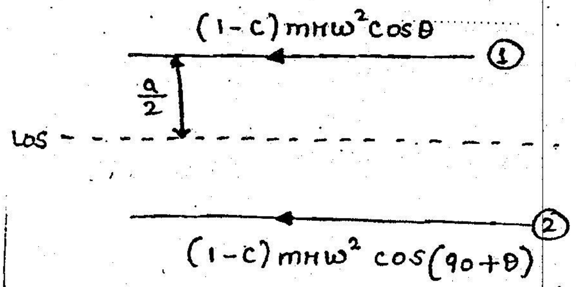
$$-\sin\theta - \cos\theta = 0$$

$$\tan\theta = -1$$

$$\theta = 135^\circ \;;\; \underline{315^\circ}$$

Variation in tractive force $= \underline{\underline{\pm\sqrt{2}\,(1-c)\,mr\omega^2}}$

2) <u>Variation in Swaying couple</u> :-



$$\text{Swaying couple} = (1-c)mr\omega^2\cos\theta \times \frac{a}{2} - (1-c)mr\omega^2\cos(90+\theta)\times\frac{a}{2}$$

$$= (1-c)mr\omega^2\cos\theta\times\frac{a}{2} - (1-c)mr\omega^2(-\sin\theta)\times\frac{a}{2}$$

$$= \frac{a}{2}(1-c)\,mr\omega^2\,[\cos\theta + \sin\theta]$$

for swaying couple to be maximum –

$$\frac{d}{d\theta}(\cos\theta + \sin\theta) = 0$$

$$-\sin\theta + \cos\theta = 0$$

$$\tan\theta = 1$$

$$\theta = 45^\circ \;;\; \underline{225^\circ}$$

$\theta = 45^\circ$; swaying couple $= \frac{a}{2}(1-c)\,mr\omega^2\left[\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right]$

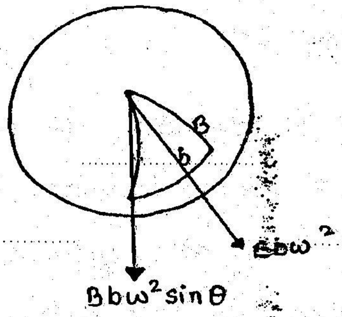
$$= \frac{a}{\sqrt{2}}(1-c)\,mr\omega^2$$

$\theta = 225^\circ$; Swaying couple $= -\frac{a}{\sqrt{2}}(1-c)\,mr\omega^2$

Variation in swaying couple $= \pm \frac{a}{\sqrt{2}}(1-c)\,m r \omega^2$

3) Hammer Blow :-

Balanced mass can not be attached reverse to crank because of design ~~constraint~~ constant. Therefore they are attached to the wheel.



$Bb\omega^2 \sin\theta$

Hammer blow $= Bb\omega^2$

$\boxed{Bb\omega^2 \le \text{static load per wheel}}$

→ A limit on speed is imposed.

Q → In 2 cylinder Locomotive rotating mass per cylinder is 200 kg. Reciprocating mass / cylinder = 400 kg. Stroke = 0.8 m. Cylinder planes gap = 1 m; wheel's plane gap = 1.5 m; N = 290 rpm. Both of the cranks are 90° to each other. Balance masses are attached on the wheels at a distance of 0.6 m from wheel's centre in order to balence rotation & $\frac{3}{4}$ reciprocation

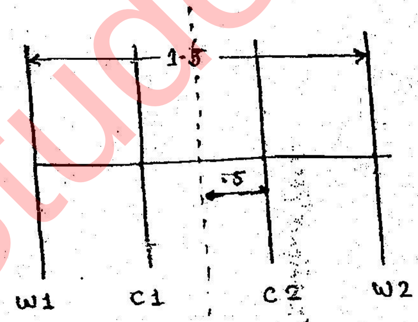
Find -

i) the balance masses along with their position in order to have balancing.

ii) hammer blow.

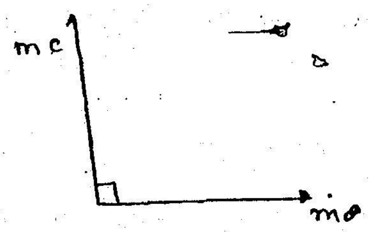
Mass of crank pin $= 200 + \frac{3}{4} \times 400$

$= 500$ kg.



Plane	m	r	mr	l	mrl
A	–	–	–	0	0
B	–	–	–		
C	–	–	–	–	
D	–	–	–	–	

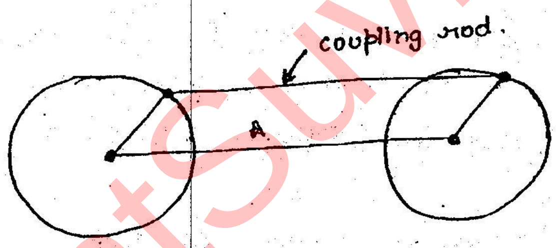
moment of polygon
↓
$m_A \rightarrow$ known
$\theta_A \rightarrow$ known



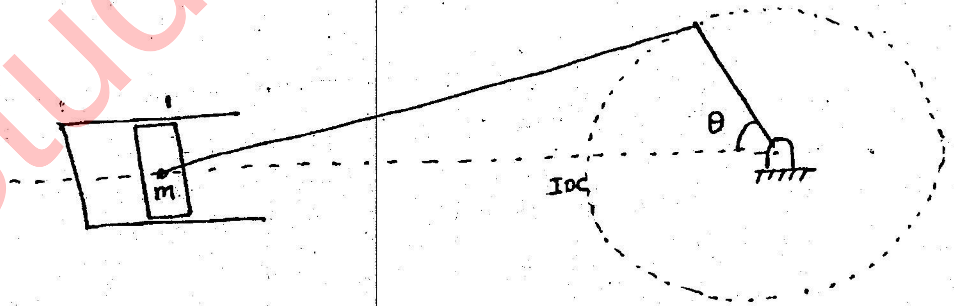
Stroke = 0.8 m
r = 0.4 m

Introduction of Coupling rod in Locomotive :-

Coupling rod ~~is~~ was basically introduced to decrease the amount of Hammer blow by decreasing the balanced mass requirement in order to balance the reciprocating masses of the cylinder through splitting the reciprocating balancing b/w the driving and trailing wheel. This was the development from the passenger locomotive to express locomotive in which two - 2 wheel were coupled and to develop superfast locomotive three - three wheels were coupled.



Secondary Balancing :-

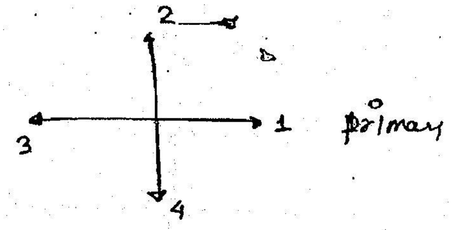


$$F_{un} = m r \omega^2 \cos\theta + \frac{m r \omega^2 \cos 2\theta}{n}$$

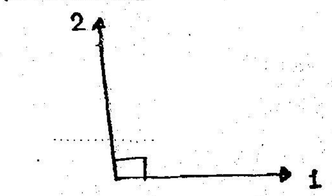
$$F_{sec} = \frac{m r \omega^2}{n} \cos 2\theta$$

$$= \frac{m r (2\omega)^2}{4n} \cos 2\theta$$

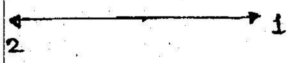
$$= \frac{m r}{4n} (2\omega)^2 \cos 2\theta$$



Primary

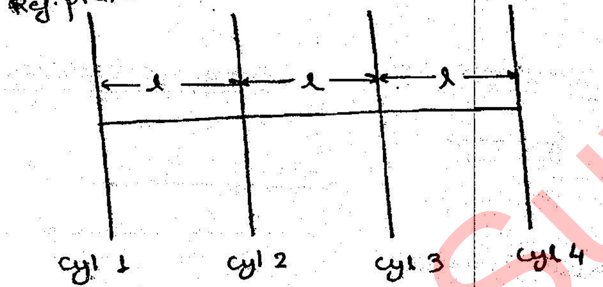


Secondary



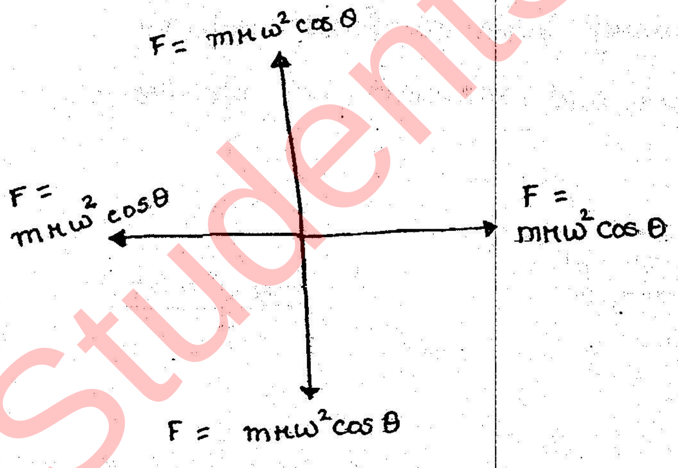
Roll of respective crank positions of Multicylinder engine in balancing

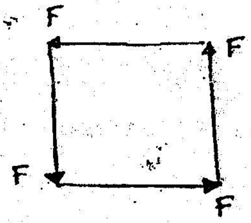
4-cylinder Inline Engine :



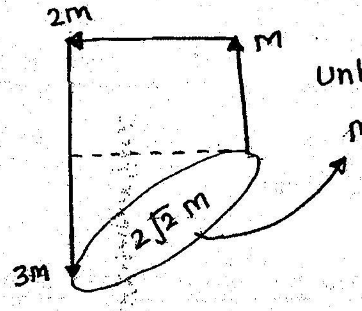
$\rightarrow$ $m r \omega^2 \cos\theta$

$m r \omega^2 \cos(90+\theta)$



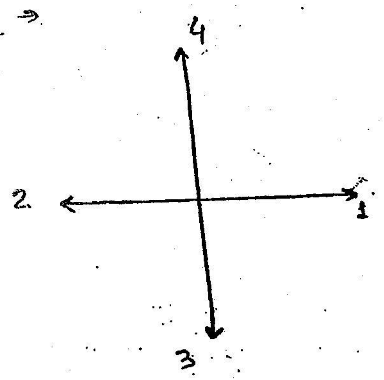


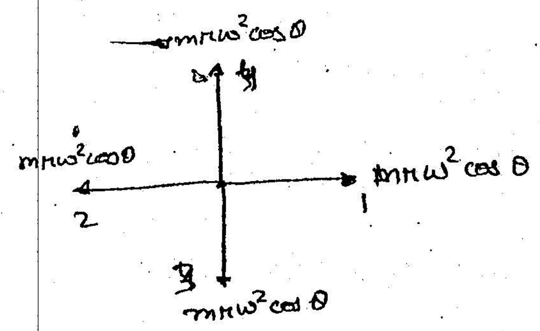
Primary Unbalanced force = 0



Unbalanced Moment

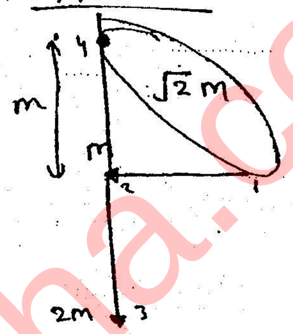
Q →



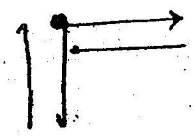


Forced Polygon

Moment



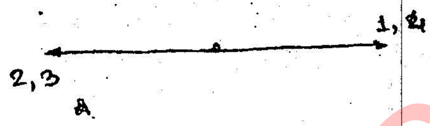
Balanced

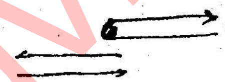


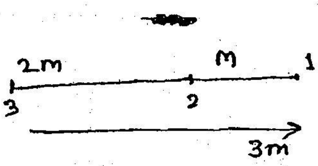
Q →

Primary Force Balanced

Moment Balanced

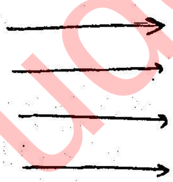




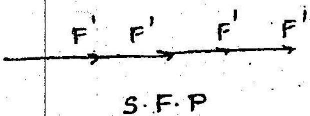


* Best crank position

4 cylinder Inline engine is primary balanced (Not completely balanced) because secondary force and moment are unbalanced.

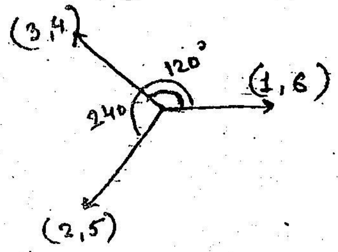


Secondary comp



$$F' = \frac{mr\omega^2\cos 2\theta}{n}$$

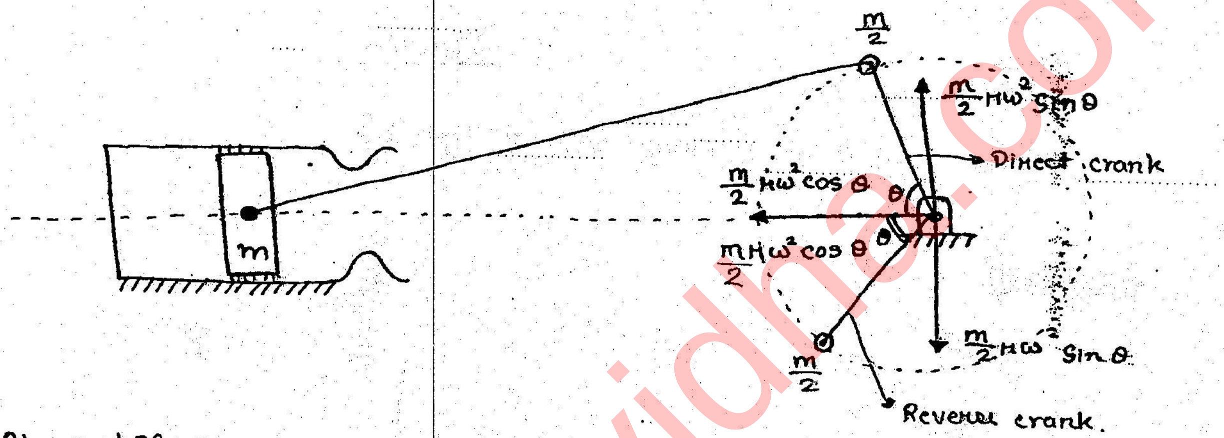
(*) 6 cylinder Inline engine is completely balanced. (both primary & secondary balanced)



Direct and Reverse crank Method :- (mostly applied in radial engine)

• Single Cylinder engine

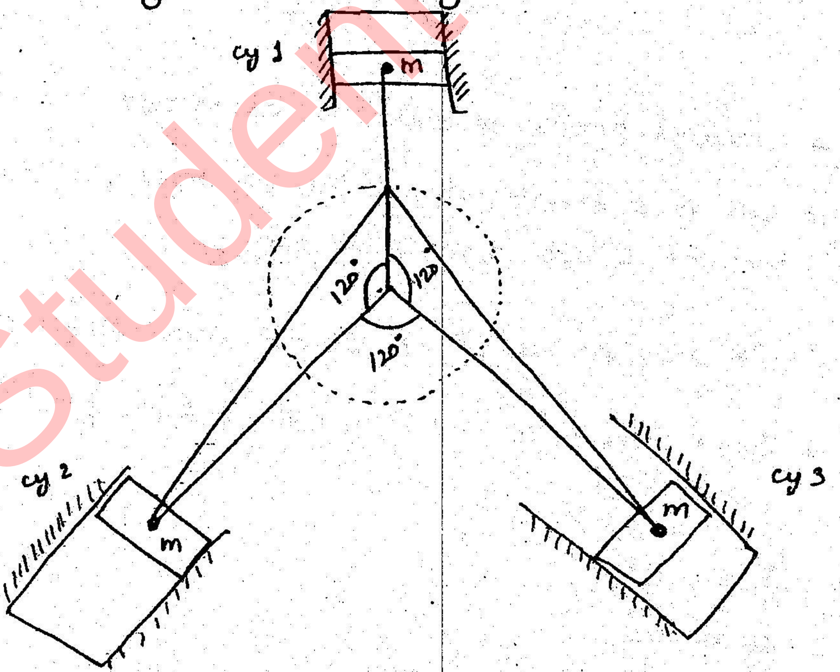
This method is basically applied in the multi cylinder engines to get primary and secondary unbalanced force information along with their magnitude and direction.



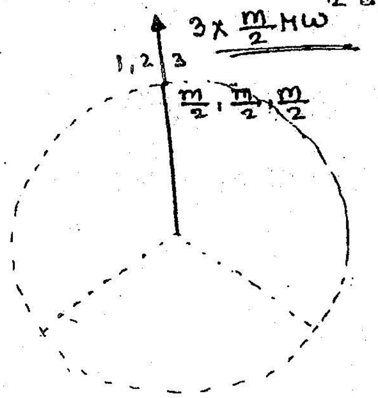
Along LOS –

$$\left(\frac{m}{2}\right)\left(\frac{r}{4h}\right)(2\omega)^2 \cos 2\theta + \frac{m}{2}\left(\frac{r}{4h}\right)(2\omega)^2 \cos 2\theta = \frac{m r\omega^2 \cos 2\theta}{2n}$$

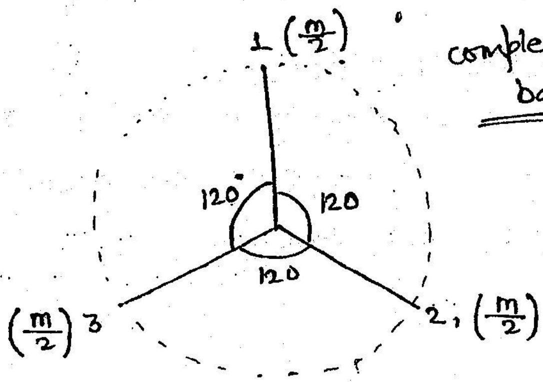
• Three cylinder Radial Engine :-



Primary →



Direct

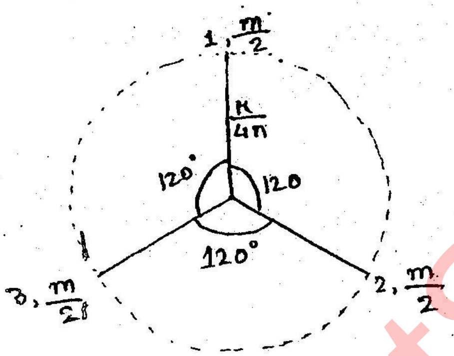


completely balanced

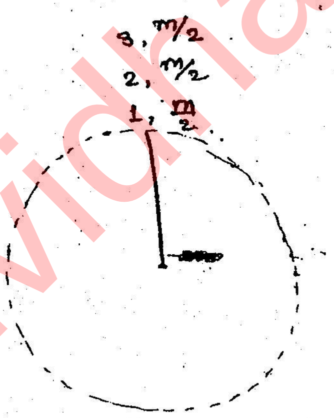
Reverse

$$F_P = \frac{3}{2} m r \omega^2 \quad (\text{Along LOS of cyl. 1})$$

Secondary



Direct



Reverse

1° Direct → actual position of crank → along LOS.

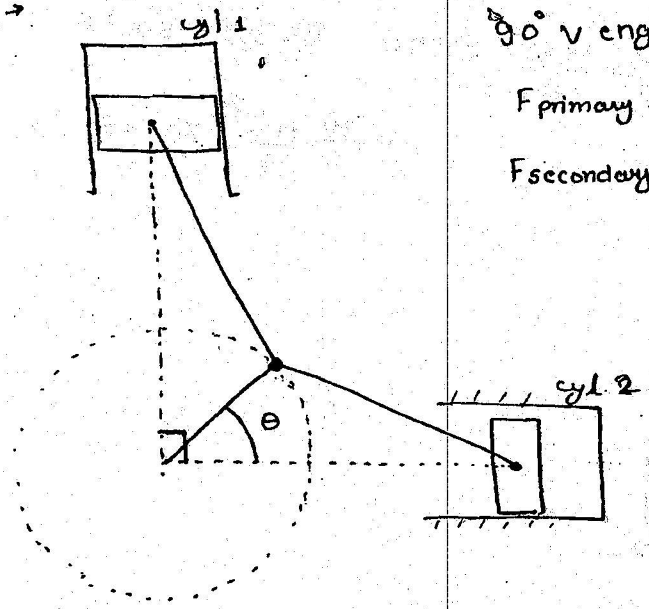
Reverse → LOS se 1° direct jitna ∠ bna ra hoga, 1° reverse utna hi ∠ opp. directⁿ me banayega.

2° → Direct → 1° Direct jitne angle pe hoga, 2° Direct uske double ∠ pe.

Reverse → LOS se 2° direct jitne ∠ pe h, 2° rev. utne hi ∠ pe opp. ∠ pe hoga.

$$F_{secondary} = \frac{3}{2} m \frac{r\omega^2}{n}$$

Q →

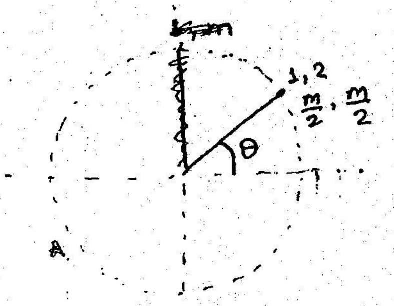


90° v engine.

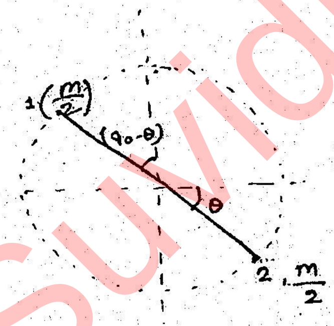
$F_{primary}$ = ?

$F_{secondary}$ = ?

Primary -



Direct



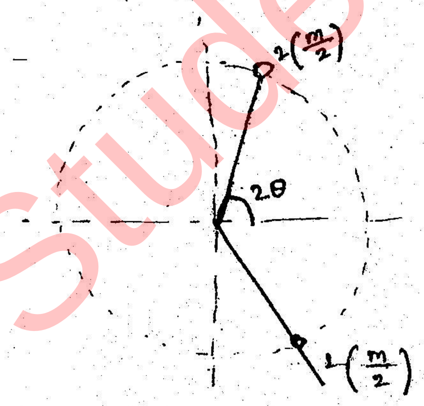
Reverse

(balanced)

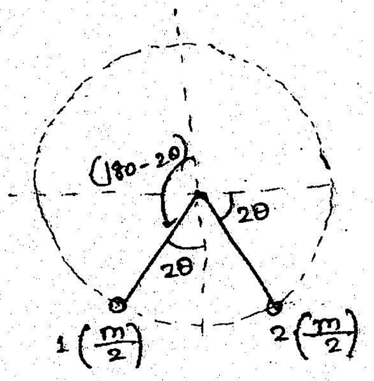
$$F_p = mr\omega^2 \simeq$$

(along crank)

Secondary -



Direct



~~dore~~ Reverse

$$F_x = \left(\frac{m}{2}\right)\frac{r\omega^2}{n}\cos 2\theta + \cancel{\frac{m}{2}\,\frac{r\omega^2}{n}\sin 2\theta} + \frac{m}{2}\,\frac{r\omega^2}{n}\cos 2\theta$$

$$- \cancel{\frac{m}{2}\,\frac{r\omega^2}{n}\sin 2\theta}$$

$$F_x = \frac{mHw^2}{r} \cos 2\theta$$

$$F_y = \frac{m}{2} \times \cancel{\frac{2w^2}{r} \sin 2\theta} - \frac{m}{2} \frac{Hw^2}{r} \cos 2\theta - \frac{m}{2} \cancel{\frac{Hw^2}{r} \sin 2\theta}$$

$$- \frac{m}{2} \frac{Hw^2}{r} \cos 2\theta$$

$$F_y = - \frac{mHw^2}{r} \cos 2\theta$$

$$F = \sqrt{F_x^2 + F_y^2}$$

$$F = \sqrt{2} \left[\frac{mHw^2}{r} \cos 2\theta \right]$$