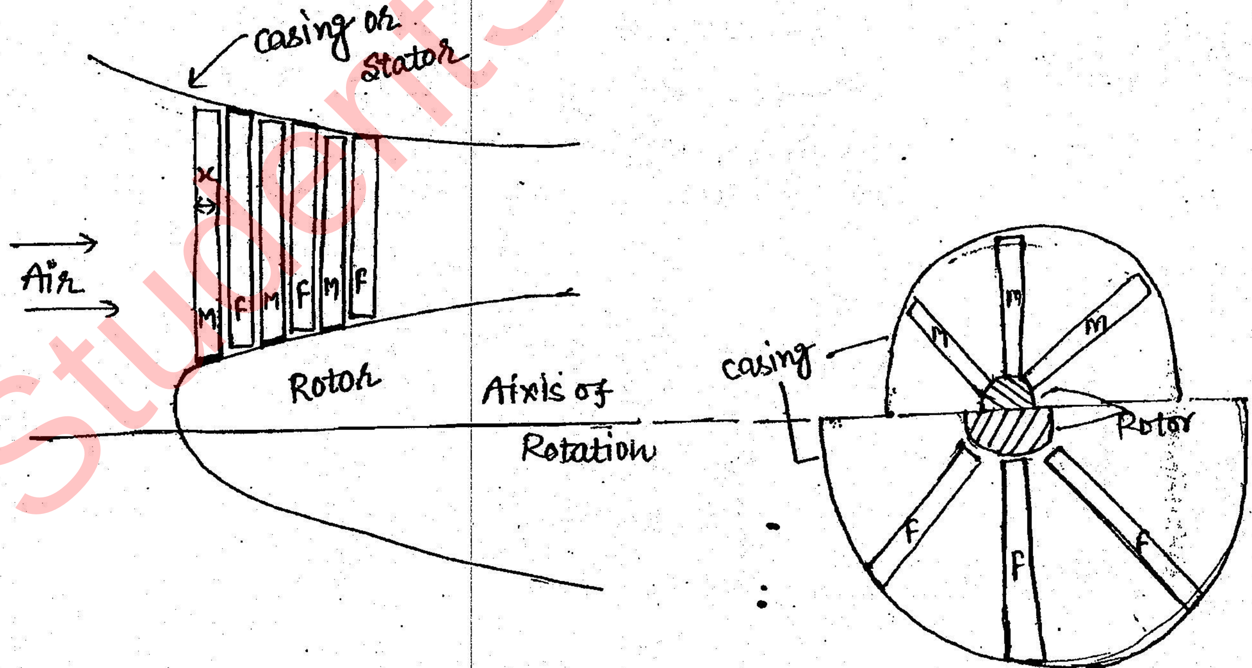
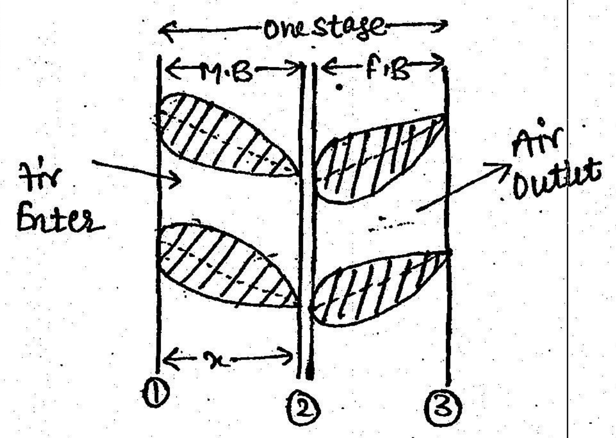
AXIAL FLOW COMPRSSOR



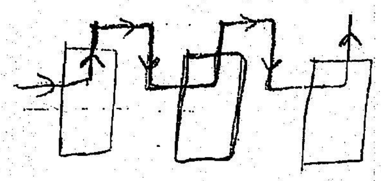


$P_1 < P_2 < P_3$

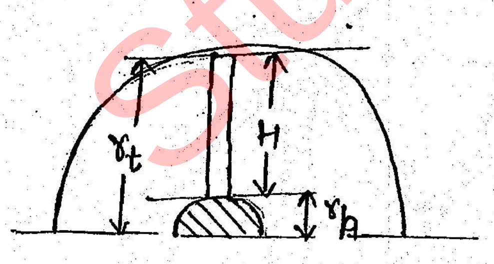
$V_1 < V_2 > V_3 \simeq V_1$

$V_{r_1} > V_{r_2}$

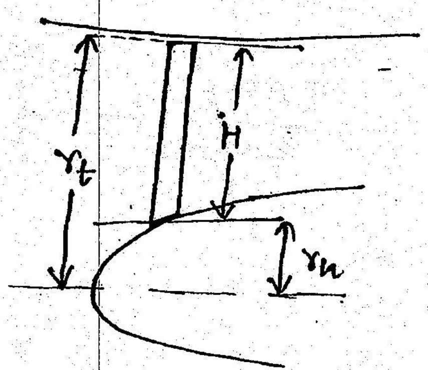
⊛ In axial flow compressor air flow in a direction parallel to the axis of rotation at it consist of no. of stages. Each stage consist of a set of moving and fixed blade. The blade crossectional passage is made diverging from inlet to outlet for both fixed blade & moving blade. There is increase in static pressure due to diffusion in the moving blade along with increase in kinetic energy, which is converted into pressure in the fixed blade due to impulse effect. The pressure ratio per stage is very less (1.22 to 1.4) as compare to centrifugal compressor (4–6). The hight of blade is decreased from inlet to outlet as pressure increases in order to get a constant axial flow velocity (to neutralise axial thrust on the rotor).



· In multistage centrifugal can't be used because losses is very high. (Because of Bending)
But in the case of Axial compressor air is moving in the parallel direction and exit in parallel dirⁿ.



$r_h \rightarrow$ Hub, $r_t \rightarrow$ tip



H = Hight of blade

LECTURE-7
19/09/2016

$$\boxed{H = r_t - r_h}$$

$$A = \pi (r_t^2 - r_h^2)$$

$$\boxed{A = \frac{\pi}{4}(D_t^2 - D_h^2)}$$

$$A = \pi \left(\frac{D_t + D_h}{2}\right)\left(\frac{D_t - D_h}{2}\right).$$

$$\boxed{A = \pi D_m \cdot H}$$

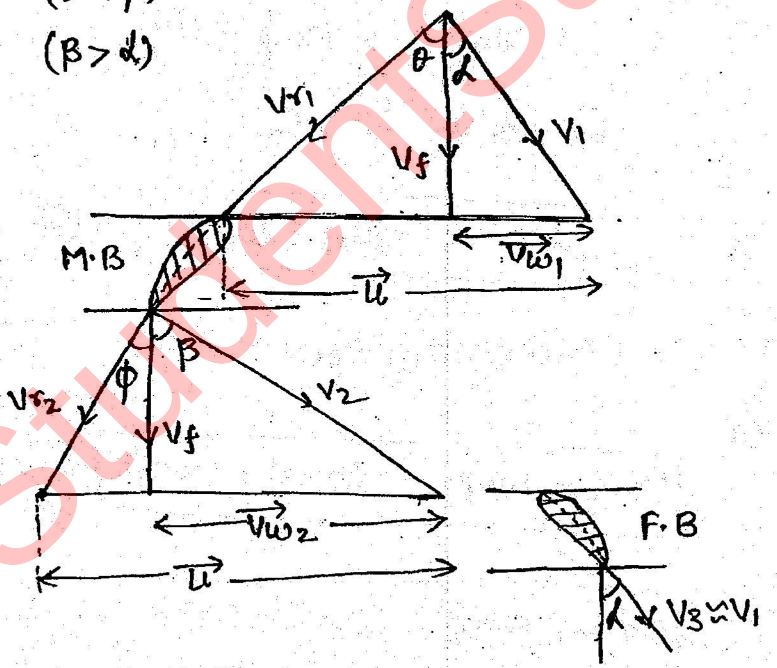
$$\boxed{D_m = \left(\frac{D_t + D_h}{2}\right) = \text{Mean Dia}}$$

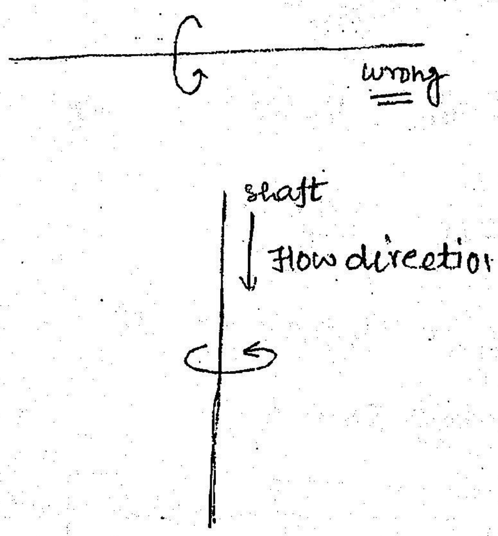
$$\boxed{\dot{m} = \rho A V_f}$$

Velocity Diagram for axial flow compressor :-

$(\theta > \phi)$

$(\beta > \alpha)$

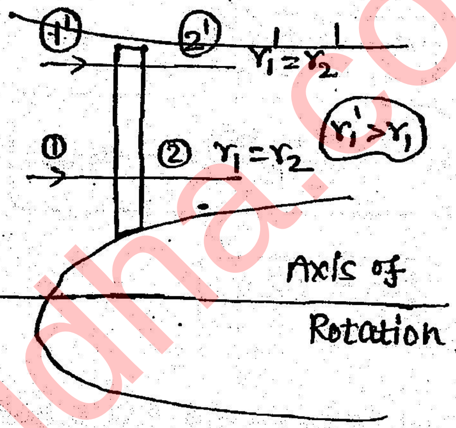




where α and β are the flow angle measured from the axial direction.

θ and ϕ are the blade angle at inlet and outlet measured from axial direction.

As fluid enters and exit at the same radius but in actual the radius increases along the blade height towards the outer tip. Blade speed is taken as constant corresponding to mean diameter for simple easy calculations.



$$\boxed{u_1 = u_2 = u = \frac{\pi D_m N}{60}}$$

$$\boxed{D_m = \frac{D_t + D_h}{2}}$$

From inlet Δ

$$\tan\theta = \frac{u - V_{w_1}}{V_f}$$

$$u - V_{w_1} = V_f \tan\theta \quad \text{— (a)}$$

$$\tan\alpha = \frac{V_{w_1}}{V_f}$$

$$V_{w_1} = V_f \tan\alpha \quad \text{— (b)}$$

adding eq (a) & (b)

$$\boxed{u = V_f (\tan\theta + \tan\alpha)} \quad \text{— (1)}$$

From outlet Δ

$$\tan\beta = \frac{V_{w_2}}{V_f}$$

$$V_{w_2} = V_f \tan\beta \quad \text{— (c)}$$

$$\tan\phi = \frac{u - V_{w_2}}{V_f}$$

$$u - V_{w_2} = V_f \tan\phi \quad \text{— (d)}$$

On adding (c) & (d)

$$\boxed{u = V_f (\tan\beta + \tan\phi)} \quad \text{— (2)}$$

On equating eq (1) & (2)

$$\boxed{\tan\alpha + \tan\theta = \tan\beta + \tan\phi} \quad \text{— (3)}$$

$$\boxed{\tan\theta - \tan\phi = \tan\beta - \tan\alpha} \quad \text{— (4)}$$

$$\text{work}/\text{stage} = \dot{m}(V_{w_2}-V_{w_1})u$$

from eq(c) & (b)

$$\boxed{w/st = \dot{m}\,V_f u(\tan\beta - \tan\alpha)}$$

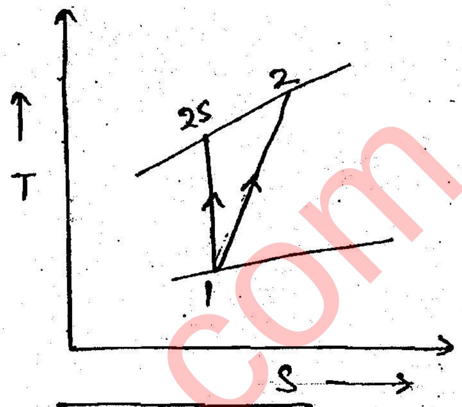
$$\boxed{w/st = \dot{m}\,u V_f(\tan\theta - \tan\phi)}$$

If N stages of equal work input

$$\boxed{W_{total} = N \times w/st}$$

by SFEE

$$\boxed{W_{total} = \dot{m}C_p(T_2-T_1)}$$



$$\boxed{\eta_c = \frac{T_{2s}-T_1}{T_2-T_1}}$$

DEGREE OF REACTION :-

It gives an indication about the contribution of moving part compare to complete stage.

In case of pump and compressor it may be defined as enthalpy or pressure rise in the moving blade compare to the total enthalpy or pressure rise in the stage.

$$\boxed{R = \frac{(\Delta h)_m}{(\Delta h)_m + (\Delta h)_f}}$$

$$V_{w_2}u_2 - V_{w_1}u_1 = \left(\frac{V_2^2-V_1^2}{2}\right) + \left(\frac{u_2^2-u_1^2}{2}\right) + \left(\frac{V_{r_1}^2-V_{r_2}^2}{2}\right)$$

$$\boxed{R_{\substack{\text{centrifugal}\\ \text{compressor (CC)}}} = \frac{(u_2^2-u_1^2)+(V_{r_1}^2-V_{r_2}^2)}{2(V_{w_2}u_2 - V_{w_1}u_1)}}$$

for centrifugal compressor $\alpha = 90^\circ$, $V_{w_1} = 0$ [most times]

$V_1 = V_{f_1}$

$$R = \frac{(u_2^2-u_1^2)+(V_{r_1}^2-V_{r}^2)}{2V_{w_2}u_2}$$

$$R = 1 - \frac{(V_2^2 - V_1^2)}{2V_{w_2}u_2}$$

$\boxed{V_1 = V_{f_1}}$ & $V_2^2 = V_{w_2}^2 + V_{f_2}^2$

$$R = 1 - \frac{V_{w_2}^2 + V_{f_2}^2 - V_{f_1}^2}{2V_{w_2}u_2}$$

Now if $\boxed{V_{f_1} = V_{f_2}}$

$$\boxed{R = 1 - \frac{V_{w_2}}{2u_2}}$$

For Radial Blade, $\phi = 90°$ & $V_{w_2} = u_2$

$$\boxed{R = 50\%}$$

FOR AXIAL COMPRESSOR

$$(V_{w_2} - V_{w_1})u = \underbrace{\left(\frac{V_2^2 - V_1^2}{2}\right)}_{F.B} + \underbrace{\left(\frac{V_{r_1}^2 - V_{r_2}^2}{2}\right)}_{M.B}$$

$$\boxed{R_{A.C} = \frac{V_{r_1}^2 - V_{r_2}^2}{2(V_{w_2} - V_{w_1})u}}$$

$\cos\theta = \frac{V_f}{V_{r_1}} \Rightarrow \boxed{V_{r_1} = V_f \sec\theta}$

$\cos\phi = \frac{V_f}{V_{r_2}} \Rightarrow \boxed{V_{r_2} = V_f \sec\phi}$

$$(V_{w_2} - V_{w_1})u = uV_f(\tan\theta - \tan\phi)$$

$$R = \frac{V_f^2(\sec^2\theta - \sec^2\phi)}{2uV_f(\tan\theta - \tan\phi)}$$

$$R = \frac{V_f}{2u}\frac{(1 + \tan^2\theta - 1 - \tan^2\phi)}{(\tan\theta - \tan\phi)}$$

$$\boxed{R = \frac{V_f}{2u}(\tan\theta + \tan\phi)}$$

50% REACTION STAGE :-

In a 50% Reaction stage pressure rise is equally divided b/w the fixed and moving blade.

$$R = \frac{1}{2} = \frac{\left(\frac{V_{r_1}^2 - V_{r_2}^2}{2}\right)}{\left(\frac{V_2^2 - V_1^2}{2}\right) + \left(\frac{V_{r_1}^2 - V_{r_2}^2}{2}\right)}$$

$$(V_2^2 - V_1^2) + (V_{r_1}^2 - V_{r_2}^2) = 2(V_{r_1}^2 - V_{r_2}^2)$$

$$\boxed{(V_2^2 - V_1^2) = (V_{r_1}^2 - V_{r_2}^2)}$$

$$R = \frac{V_f}{2u}(\tan\theta + \tan\phi) = \frac{1}{2}$$

$\boxed{u = V_f(\tan\theta + \tan\phi)}$ ← only for 50% stage.

$\boxed{u = V_f(\tan\theta + \tan\alpha)}$ ← General eqn

On Equating

$$\cancel{\tan\theta} + \tan\phi = \cancel{\tan\theta} + \tan\alpha$$

$$\boxed{\alpha = \phi}$$

$\tan\alpha + \tan\theta = \tan\beta + \tan\phi$

$\boxed{\beta = \theta}$

$\cos\beta = \cos\theta$

$\frac{V_f}{V_2} = \frac{V_f}{V_{r_1}}$

$\boxed{V_2 = V_{r_1}}$

as $\alpha = \phi$, $\cos\alpha = \cos\phi$

$\frac{V_f}{V_1} = \frac{V_f}{V_{r_2}} \Rightarrow \boxed{V_1 = V_{r_2}}$

PB:-

Speed of an axial flow compressor is 15000 rpm and the mean Dia. is 0.6 m. The actual flow velocity is constant and is 225 m/s. The velocity of whirl at inlet is 85 m/s and the workdone is 45 KJ/kg of air in a stage. The inlet condition are 1 bar and 300 K. Assuming the stage η as 89%. Calculate -

(1) The fluid deflection angle.

(2) pressure ratio

(3) Degree of Reaction.

$N = 15000$ rpm

$D_m = 0.6$ m

$V_{f_1} = V_{f_2} = 225$ m/s

$V_{w_1} = 85$ m/s

① $u = \frac{\pi D_m N}{60}$

$u = 471.23$ m/s

$w/st = (V_{w_2} - V_{w_1})u$

$45\times10^3 = (V_{w_2} - 85)\times 471.23$

$V_{w_2} = 180.5$ m/s

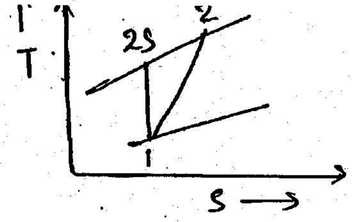
$\tan\theta = \frac{u - V_{w_1}}{V_f}$ $\theta = 59.76^\circ$

$\tan\phi = \frac{u - V_{w_2}}{V_f}$ $\phi = 52.26^\circ$

Fluid deflection $= (\theta - \phi) = 7.5^\circ$

② $w/st = C_p(T_2 - T_1)$

$T_2 = 344.77$ K



$\eta_c = 0.89 = \frac{T_{2s}-T_1}{T_2-T_1}$

$T_{2s} = 339.85\,K$

$\frac{T_{2s}}{T_1} = (r_p)^{\frac{\gamma-1}{\gamma}}$

$\boxed{r_p = 1.54}$

(8) $V_{r_1} = V_f \sec\theta = 446.76\ m/s$

$V_{r_2} = V_f \sec\phi = 367.5\ m/s.$

$R = \frac{V_{r_1}^2 - V_{r_2}^2}{2(V_{w_2}-V_{w_1})u} = 71.8\%$ [Right way for conventional]

OR $R = \frac{V_f}{2u}(\tan\theta + \tan\phi)$

$\boxed{R = 71.8\%}$

B: A ten stage Axial flow compressor provides an overall pressure ratio of 5 with an overall isentropic η of 87%. The temp. of the air at Inlet 15°c and work is equally divided b/w all the stages with 50% Reaction stage. The blade speed is 210 m/s and the axial velocity of flow 170 m/s. Find the blade angle at Inlet and outlet.

N = 10 stages

$r_p = 5$

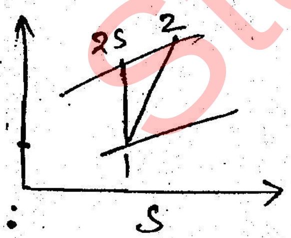
$\eta_c = 0.87$

$T_1 = 288K$

$R = \frac{1}{2}$

$u = 210\ m/s$

$V_f = 170\ m/s.$



$\left(\frac{T_{2s}}{T_1}\right) = (r_p)^{\frac{\gamma-1}{\gamma}}$

$T_{2s} = 456.14\,K$

$\eta_c = \frac{T_{2s}-T_1}{T_2-T_1} = 0.87$

$T_2 = 481.26\,K$

$w_{total} = C_p(T_2 - T_1)$

$w_{total} = 194.23\,KJ/kg$

$w/st = 19.423\,KJ/kg$

$w/st = uV_f(\tan\theta - \tan\phi)$

$\tan\theta - \tan\phi = 0.544$ — (1)

$u = V_f(\tan\theta + \tan\alpha)$

50% stage $\boxed{\alpha = \phi}$

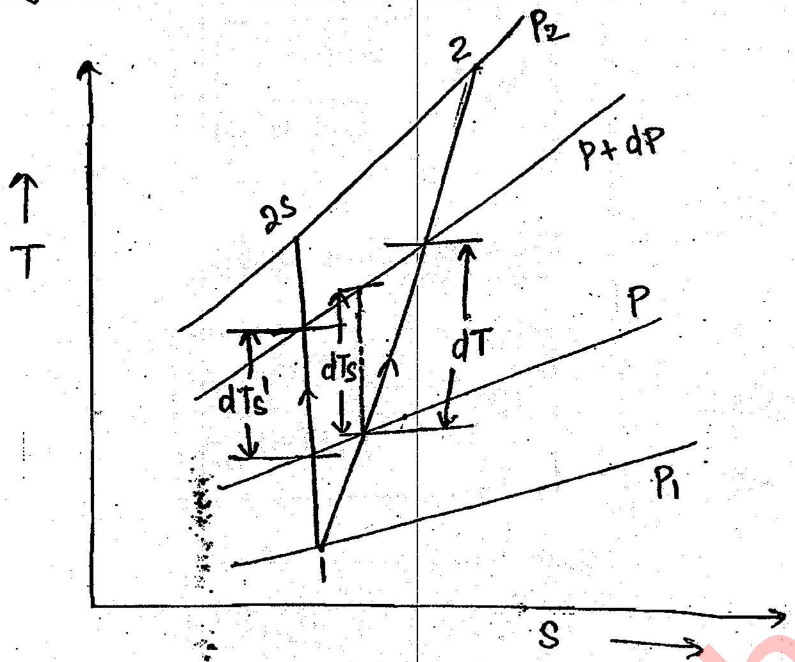
$u = V_f(\tan\theta + \tan\phi)$

$\tan\theta + \tan\phi = 1.235$ — (2)

On eq^n (1) & (2)

$\boxed{\theta = 41.65^\circ \quad \phi = 19.05^\circ}$

Polytropic efficiency (OR) small stage efficiency :-



$$\boxed{\eta_o = \frac{dT_s'}{dT}}$$

$$\boxed{\eta_{PC} = \frac{dT_s}{dT}}$$

$$dT_s > dT_s'$$

$$\boxed{\eta_{PC} > \eta_o}$$

It is isentropic efficiency of a very small stage of compression which can be assumed as constant throughout. It is defined as the ratio of isentropic temp. rise across a small stage to the actual temp. rise across the small stage.

Now Polytropic compression

$$Pv^n = c$$

& $PV = RT$

$$v = \frac{RT}{P}$$

$$P\left(\frac{RT}{P}\right)^n = c$$

$$P^{1-n} \cdot T^n = C_1$$

Taking log both side

$$(1-n)\log P + n\ln T = \ln C_1$$

On differentiation

$$(1-n)\frac{dP}{P} + n\frac{dT}{T} = 0$$

$$\frac{dT}{T} = \frac{-(1-n)}{n}\frac{dP}{P}$$

$$\boxed{dT = \left(\frac{n-1}{n}\right)\frac{dP}{P}T}$$

For isentropic compression

$$\boxed{dT_s = \left(\frac{\gamma-1}{\gamma}\right)\frac{dP}{P}T}$$

Polytropic η :-

$$\eta_{PC} = \frac{dT_s}{dT}$$

$$\boxed{\eta_{PC} = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{n}{n-1}\right)}$$

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{n}$$

$$\boxed{\left(\frac{n-1}{n}\right) = \left(\frac{\gamma-1}{\gamma}\right)\frac{1}{\eta_{pc}}}$$

OR) $\eta_{pc} = \frac{dT_s}{dT} = \left(\frac{\gamma-1}{\gamma}\right)\frac{dP}{P}\,\frac{T}{dT}$

$$\eta_{pc}\frac{dT}{T} = \left(\frac{\gamma-1}{\gamma}\right)\frac{dP}{P}$$

s η_{pc} remains constant throughout.

n integrating from initial to final ondition.

$$\eta_{pc}\ln\left(\frac{T_2}{T_1}\right) = \left(\frac{\gamma-1}{\gamma}\right)\ln\left(\frac{P_2}{P_1}\right)$$

$$\eta_{pc} = \frac{\ln(P_2/P_1)^{\left(\frac{\gamma-1}{\gamma}\right)}}{\ln(T_2/T_1)}$$

$$\left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{T_{2s}}{T_1}\right)$$

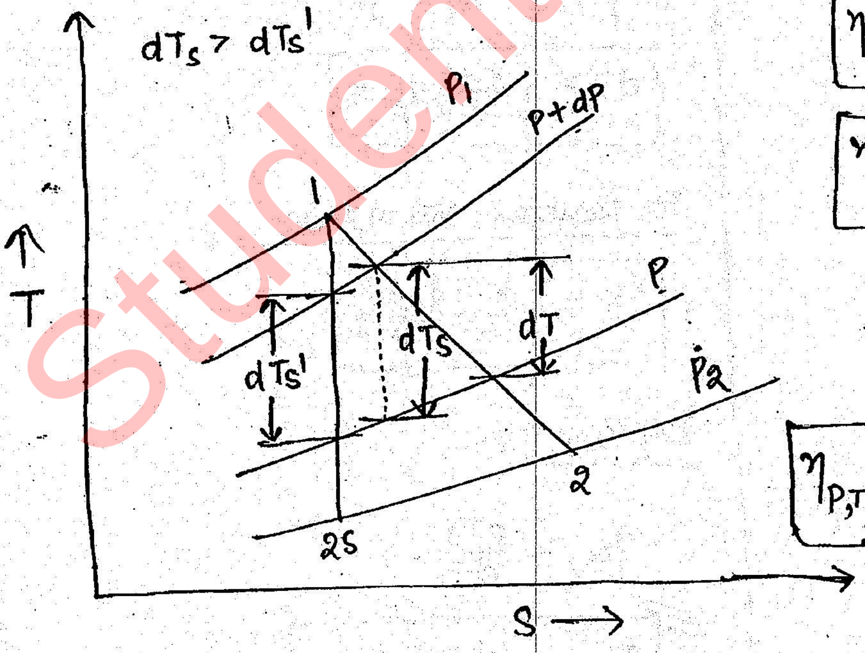
$$\boxed{\eta_{pc} = \frac{\ln(T_{2s}/T_1)}{\ln(T_2/T_1)}}$$

For stagnation state

$$\boxed{\eta_{pc} = \frac{\ln(T_{02s}/T_{01})}{\ln(T_{02}/T_{01})}}$$

$$\boxed{\eta_{pc} = \left(\frac{\gamma-1}{\gamma}\right)\left(\frac{n}{n-1}\right)}$$

IN CASE OF TURBINE (Expansion)



$$\boxed{\eta_0 = \frac{dT}{dT_s'}}$$

$$\boxed{\eta_{PT} = \frac{dT}{dT_s}}$$

$$\boxed{\eta_0 > \eta_{PT}}$$

$$\boxed{\eta_{P,T} = \left(\frac{\gamma}{\gamma-1}\right)\left(\frac{n-1}{n}\right) = \frac{dT}{dT_s}}$$