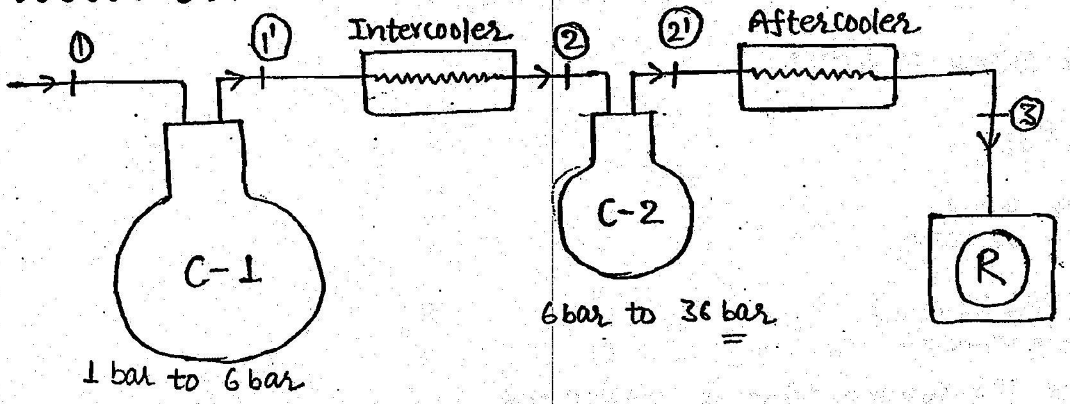
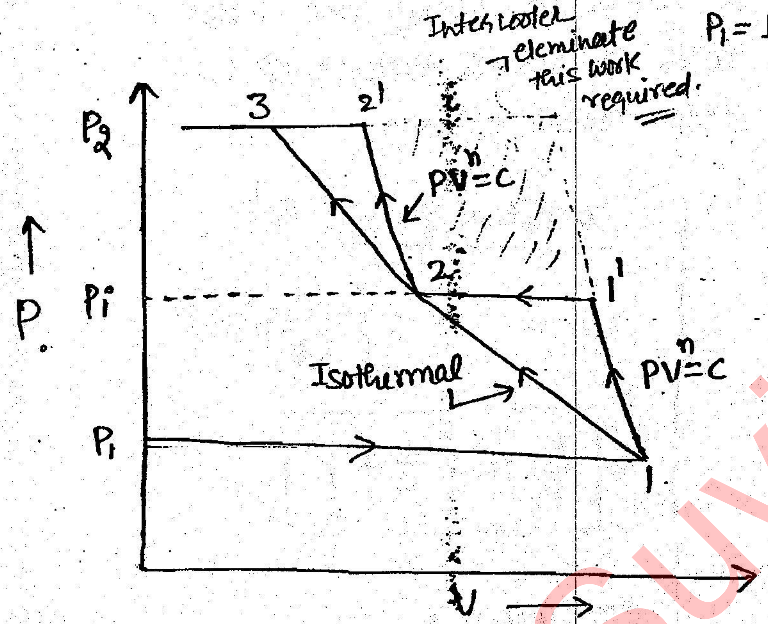
MULTI STAGE COMPRESSION :-





$P_1 = 1$ bar want to convert $P_2 = 36$ bar

$c = 5\%$ $n = 1.3$

$$\eta_v = 1 - \frac{5}{100}\left[(36)^{\frac{1}{1.3}} - 1\right]$$

$\eta_v = 26\%$

in multistage compression.

6 bar 6 bar to 36 bar

$\boxed{\eta_v = 85\%}$

$P_1 = 1$ bar

Single → 5.6 bar

Double → 5.6 to 35 bar

Triple → 35 bar to 180 bar.

(*) In a single stage compression η_v decreases with increase in pressure ratio therefore beyond certain pressure ratio multistaging become compulsary. It also save the construction cost of material and leakage across the piston. If intercooling is used in b/w the stages work input required by the compressor can also be minimized.

Condition for minimum work input :-

$$W_c = W_{c_1} + W_{c_2} = \left(\frac{n}{n-1}\right) mRT_1\left[\left(\frac{P_i}{P_1}\right)^{\frac{n-1}{n}} - 1\right] + \left(\frac{n}{n-1}\right) mRT_2\left[\left(\frac{P_2}{P_i}\right)^{\frac{n-1}{n}} - 1\right]$$

let $\boxed{\left(\frac{n-1}{n}\right) = x}$

$$w_c = \frac{mRT_1}{x}\left[\frac{P_i^x}{P_1^x} - 1\right] + \frac{mRT_2}{x}\left[\frac{P_2^x}{P_i^x} - 1\right]$$

For w_c to be minimum $\frac{dw_c}{dP_i} = 0$

$$\frac{mRT_1}{x} \times \frac{x P_i^{x-1}}{P_1^x} + \frac{mRT_2}{x} \times \frac{P_2^x(-x)}{P_i^{x+1}} = 0$$

$$T_1 \frac{P_i^{x-1}}{P_1^x} + \left[-\frac{T_2 P_2^x}{P_i^{x+1}}\right] = 0 \Rightarrow \frac{T_1 P_i^{x-1}}{P_1^x} = \frac{T_2 P_2^x}{P_i^{x+1}}$$

$$P_i^{2x} = \frac{T_2}{T_1} P_2^x \cdot P_1^x$$

$$\boxed{P_i = \sqrt{P_2 P_1}\left[\frac{T_2}{T_1}\right]^{\frac{n}{2(n-1)}}}$$

For perfect intercooling $T_2 = T_1$

$\boxed{P_i = \sqrt{P_2 P_1}}$ (OR) $\boxed{\frac{P_i}{P_1} = \frac{P_2}{P_i}}$

* If there are n stages in multistage compression then the pressure ratio across each stage will be equal.

$$\boxed{\frac{P_2}{P_1} = \frac{P_3}{P_2} = \frac{P_4}{P_3} = \text{-------------} = \frac{P_{N+1}}{P_N} = K}$$

$$\frac{P_{N+1}}{P_1} = P_{o,r} = \text{overall pressure ratio}$$

$P_2 = P_1 K$

$P_3 = KP_2 = K^2 \cdot P_1$

$P_4 = KP_3 = K^3 P_1$

$\vdots$

$\boxed{P_{N+1} = K^N \cdot P_1}$

$K = \frac{P_2}{P_1}$ = Pressure ratio across a stage $= P_{r,s}$

$\boxed{P_{o,r} = (P_{r,s})^N}$ ← No. of stages

$$W_{C_1} = \left(\frac{n}{n-1}\right) mRT_1 \left[\left(\frac{P_i'}{P_i}\right)^{\frac{n-1}{n}} - 1 \right]$$

$$W_{C_2} = \left(\frac{n}{n-2}\right) mRT_2 \left[\left(\frac{P_2}{P_i'}\right)^{\frac{n-1}{n}} - 1 \right]$$

as perfect intercooling

$$T_2 = T_1 \text{ and } \frac{P_i'}{P_1} = \frac{P_2}{P_1} \Rightarrow \boxed{W_{C_1} = W_{C_2}}$$

For perfect intercooling with minimum work input the pressure ratio with each stage will be equal & each stage compressor required equal amount of work.

: Find the minimum no. of stages required when air enters at 1 bar and 27°c and delievers at 350 bar. The max^m discharge temp. is limited to 145°C. Take the perfect intercooling in between stages and $n = 1.3$.

$T_1 = 300K$, $T_1' = 418K$

$$k = \frac{P_i'}{P_1} = \left(\frac{T_1'}{T_1}\right)^{\frac{n}{n-1}}$$

$k = 4.209$

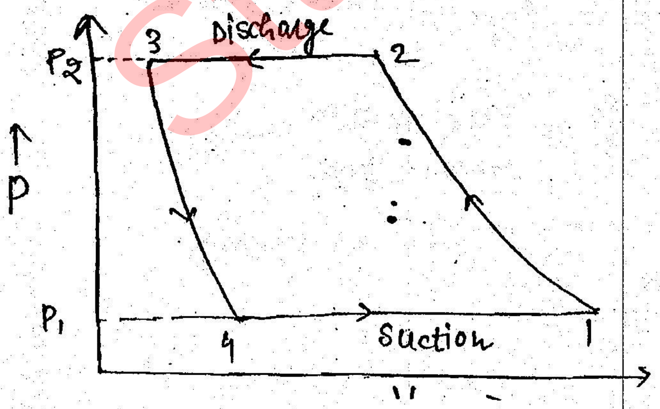
$$P_{N+1} = k^N \cdot P_1$$

$$350 = (4.209)^N \times 1$$

$$\boxed{N = 4.075}$$

FREE AIR DELIEVERED (FAD) :-

It works as a reference point to compare the volume handling capacity of different compressors. Volume of air when converted to 1 bar pressure and 15°c temp. is known as free air delievered.



C_1	C_2
$P_1 = 35$ bar	$P_2 = 84$ bar
$T_1 = 470K$	$T_2 = 505K$
$V_1 = 1.23\ m^3/min$	$V_2 = 0.79\ m^3/min$

mass at Suction :-

$$m_s = \frac{P_1(V_1-V_4)}{RT_1}$$

mass at Discharge

$$m_d = \frac{P_2(V_2-V_3)}{RT_2}$$

Reference

$P_t = 1$ bar

$T_t = 15°C$

$V_t =$ FAD

$$m_t = \frac{P_t V_t}{RT_t}$$

$$\boxed{m_s = m_d = m_t}$$

$$\boxed{\frac{P_1(V_1-V_4)}{T_1} = \frac{P_2(V_2-V_3)}{T_2} = \frac{P_t \cdot V_t}{T_t}}$$

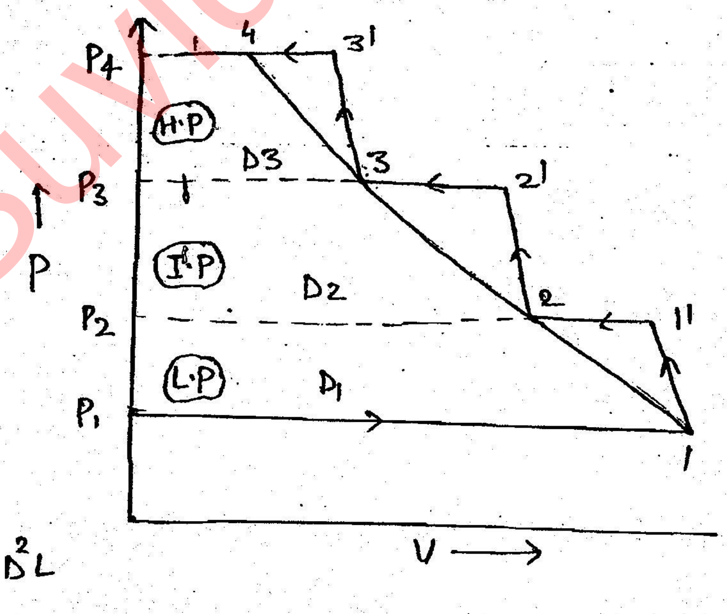
FAD

Cylinder Dimensions in Multistage compression :-

The mass of air passing through each cylinder will be equal in multi stage compression under idead condition.

$$m_{a_1} = m_{a_2} = m_{a_3}$$

$$\frac{P_1 V_{a_1}}{RT_1} = \frac{P_2 V_{a_2}}{RT_2} = \frac{P_3 V_{a_3}}{RT_3}$$



For perfect intercooling

$$\boxed{T_1 = T_2 = T_3}$$

$$P_1 V_{a_1} = P_2 V_{a_2} = P_3 V_{a_3}$$

$$\boxed{\eta_v = \frac{V_a}{V_s}} \Rightarrow V_a = \eta_v \cdot \frac{\pi}{4} D^2 L$$

$$P_1 \cdot \eta_{v_1} \cdot D_1^2 L_1 = P_2 \eta_{v_2} D_2^2 L_2 = P_3 \eta_{v_3} D_3^2 L_3$$

In multicylinder stroke length is kept equal to get uniform torque and to minimize construction cost.

$$\boxed{L_1 = L_2 = L_3}$$

$$P_1 \eta_{v_1} D_1^{2} = P_2 \eta_{v_2} D_2^{2} = P_3 \eta_{v_3} D_3^{2}$$

$$\boxed{\eta_v = 1 - C\left[(K)^{\frac{1}{n}} - 1\right]}$$

If clearance ratio is kept equal for all stages then

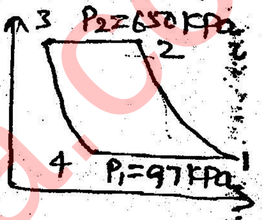
$$\boxed{\eta_{v_1} = \eta_{v_2} = \eta_{v_3}} \qquad \boxed{P_1 D_1^2 = P_2 D_2^2 = P_3 D_3^2}$$

Ques A Reciprocating compressor has a bore of 300 mm and stroke of 400 mm and it runs at a speed of 400 rpm. $C = 5\%$, $n = 1.3$, $P_1 = 97$ KPa, $T_1 = 20°C$, $P_2 = 650$ KPa and if free air condition are 105 KPa and 15°C. Determine –

(i) η_v

(2) FAD in m^3/min

3) Power required in kw.



1) $\eta_v = 1 - C\left[\left(\frac{P_2}{P_1}\right)^{\frac{1}{n}} - 1\right]$

$$\boxed{\eta_v = 83.4\%}$$

2) $\eta_v = \frac{V_a}{V_s}$

$V_{a_1} = \eta_v \times \frac{\pi}{4} d^2 L \times N$

$V_{a_1} = 9.43\ m^3/min$

$$\frac{P_1 V_{a_1}}{T_1} = \frac{P_t V_t}{T_t}$$

$$\frac{97 \times 9.43}{293} = \frac{105 \times V_t}{288}$$

$$\boxed{V_t = 8.56\ m^3/min}$$

(3) $P = \left(\frac{n}{n-1}\right) P_1 V_{a_1}\left[\left(\frac{P_2}{P_1}\right)^{\frac{n-1}{n}} - 1\right]$

$$= \frac{1.3}{0.3} \times 97 \times \frac{9.43}{60}\left[\left(\frac{650}{97}\right)^{\frac{0.3}{1.3}} - 1\right]$$

$$\boxed{P = 36.4\ kw}$$

Ques A two stage compressor with perfect intercooling delivers 5 kg/min of air at 16 bar. The inlet conditions are 1 bar and 300K. $n = 1.3$ Calculate –

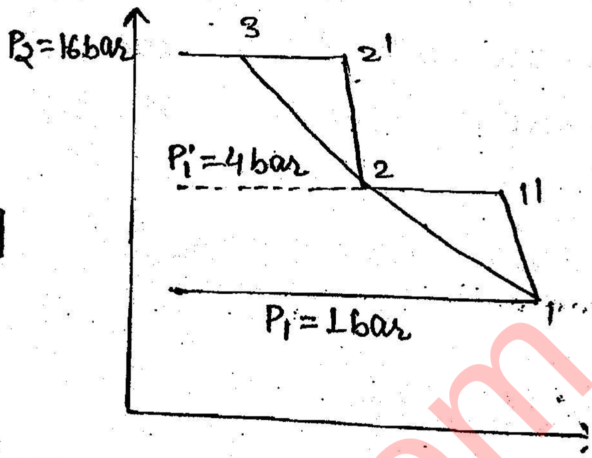
i) Power required in kw

ii) Isothermal η

iii) Heat transferred in each cylinder

iv) Heat rejected in the intercooler.

$r_i = \sqrt{P_1 P_2} = \underline{\underline{4 \text{ bar}}}$



(1) $P = 2 \times \frac{n}{n-1} \times m R T_1 \left[(4)^{\frac{n-1}{n}} - 1 \right]$

$\boxed{P = 23.4 \text{ kW}}$ $\quad [P = 2 \times W_{c_1}]$

(2) $\eta_{iso} = \frac{W_{iso}}{W_{act}}$

$W_{iso} = m R T_1 \ln(16) = \underline{\underline{19.09 \text{ kW}}}$

$\eta_{iso} = \frac{19.09}{23.4} = \underline{\underline{84.8\%}}$

(3) $\frac{T_1'}{T_1} = (4)^{\frac{n-1}{n}}$

$T_1' = \underline{\underline{413.1 \text{ K}}}$

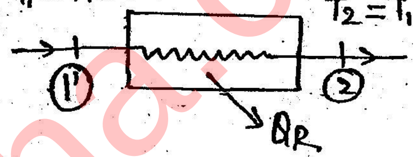
$W_{1-1'} = \frac{m R (T_1' - T_1)}{n-1} = \underline{\underline{9.016 \text{ kW}}}$

$Q_{1-1'} = \frac{\gamma - n}{\gamma - 1} \times W_{1-1'}$

$= \left(\frac{1.4 - 1.3}{1.4 - 1} \right) \times (9.016)$

$\boxed{Q_{1-1'} = -2.25 \text{ kW}}$

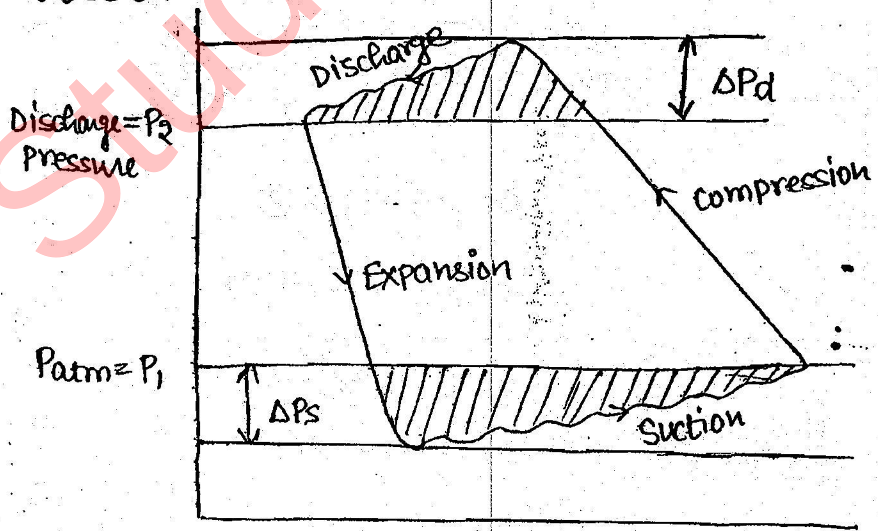
(4)



$Q_R = \dot{m} c_p (T_2 - T_1')$

$\boxed{Q_R = 9.47 \text{ kW}}$

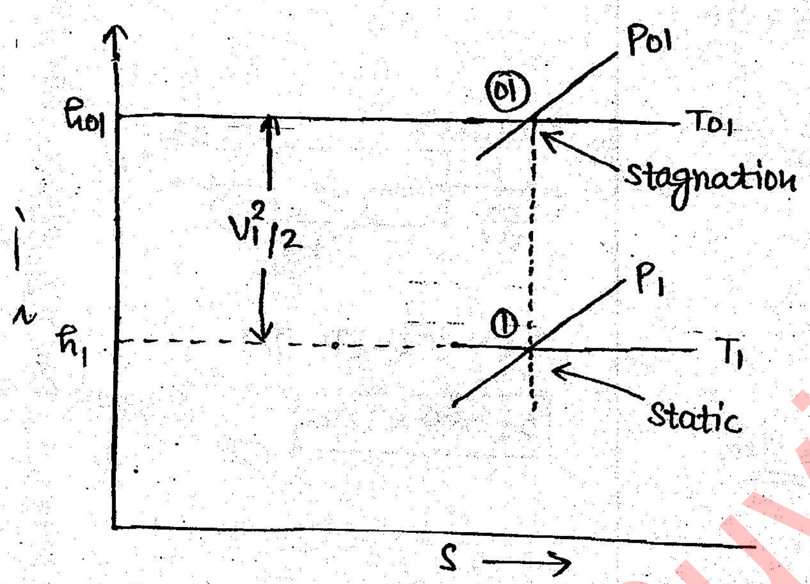
ACTUAL INDICATOR DIAGRAM :-



In actual working there is resistance offered by the suction and discharge valve due to which the actual workinput required by compressor increases by the area shown by shaded.

DYNAMIC COMPRESSOR :-

Stagnation state :-

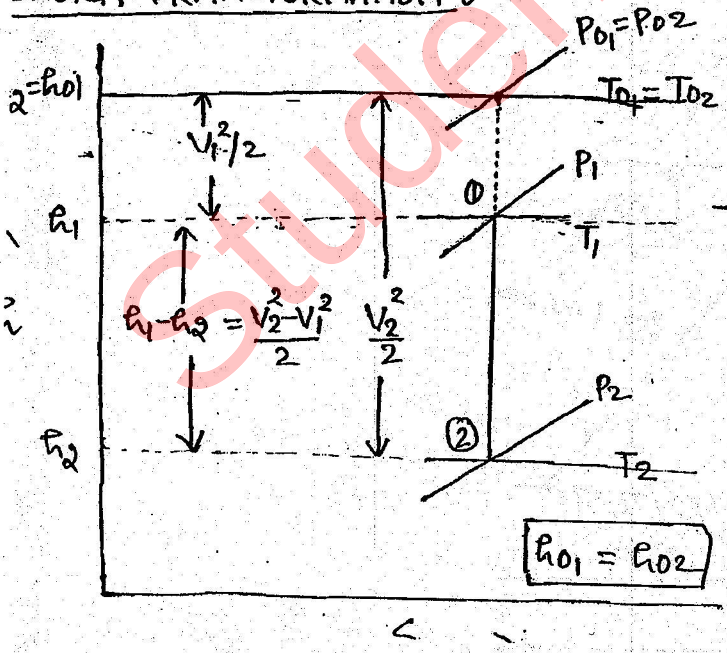


$$h_1 + \frac{V_1^2}{2} = 0 + h_{01}$$

$$\boxed{h_{01} = h_1 + \frac{V_1^2}{2}}$$

The state of a fluid obtained when it is isentropically deaccelerated to zero velocity state is reffered as stagnation state.

ENERGY TRANSFORMATION :-



$V_2 >>> V_1$

(1) h_1 → Nozzle → (2) h_2

P_1, V_1 P_2, V_2

BY SFEE (1) & (2)

$$h_1 + \frac{V_1^2}{2} = h_2 + \frac{V_2^2}{2}$$

$$\boxed{h_1 - h_2 = \frac{V_2^2 - V_1^2}{2}}$$