CHAPTER-1

Coplanar System → Means same /single plane (2-D plane only)

Resultant : Replacement of all forces of the force System by a Single equivalent force is Known as resultant.

If the System will have some resultant Value, then the body is in motion in the direction of resultant due to resultant force.

Equilibrant :- The external opposite force having the same magnitude of resultant but its direction is exactly opposite to maintain the body again at rest condition. Such force is called as equilibrant.

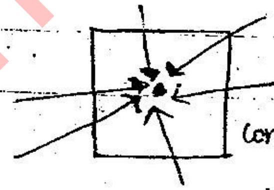
* Classification of co-planar force Systems :-

1) Concurrent force system
2) Parallel force system
3) Non-Concurrent, Non-parallel force System.

Concurrent force System

When all the forces of the given force system are intersecting at a single point then such force system is called as Concurrent force system.

All the forces will be inclined with x and y axis then they are resolved in the $(x-y)$ axis and then resultant is calculated.



ΣF_x and ΣF_y.

$R_x = \Sigma F_x \qquad R_y = \Sigma F_y$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\boxed{\tan\theta = \left|\frac{\Sigma F_y}{\Sigma F_x}\right|}$$

(Direction of resultant)
(angle with x-axis)

NOTE

- If $\Sigma F_x = 0$, then $R = \Sigma F_y$, i.e. R is acting vertically or parallel to the y-axis or along the y-axis.
- If $\Sigma F_y = 0$, then $R = \Sigma F_x$, i.e. R is acting Horizontally or parallel to the x-axis.
- If $\Sigma F_x = 0$ and $\Sigma F_y = 0$, then $R = 0$ (Body is at Rest). such body is said to be in Equilibrium.

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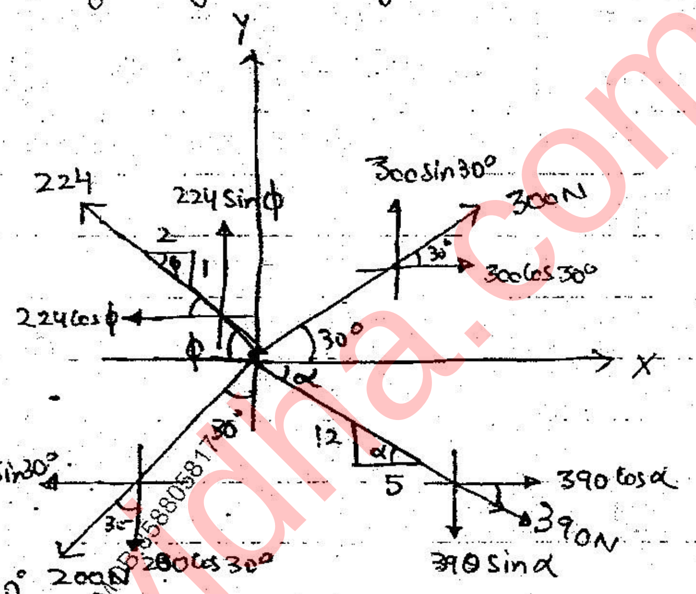
Q Find the Resultant and its direction of the given Force system?



$\overset{-}{\longleftarrow}\overset{+}{\longrightarrow}$ Fx

$\Sigma Fx = 300\cos 30° + 390\cos(\alpha)$
$\quad - 224\cos\phi - 200\sin 30°$
$\quad = 109.457\,N.$

$\Sigma Fx = +109.45\,N\ (\longrightarrow)$ $\boxed{\text{Right}}$

$\uparrow^{+}\ \downarrow^{-}$

$\Sigma Fy = 300\sin 30° + 224\sin\phi - 200\cos 30°$
$\quad - 390\sin\alpha$

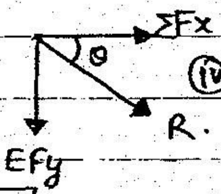
$\Sigma Fy = (-)283.029\,N.\ (\downarrow)$ $\boxed{\text{Down}}$

$\alpha = \tan^{-1}\left(\frac{12}{5}\right) = 67.38°$

$\phi = \tan^{-1}\left(\frac{1}{2}\right) = 26.565°$

$R = \sqrt{(\Sigma Fx)^2 + (\Sigma Fy)^2} = \sqrt{(109.45)^2 + (-283.029)^2} = 303.4\,N.$

$\tan\theta = \left|\frac{\Sigma Fy}{\Sigma Fx}\right|$



IV Quad.

$\tan\theta = \left|\frac{-283.029}{109.45}\right| \Rightarrow \boxed{\theta = 68.815°}$

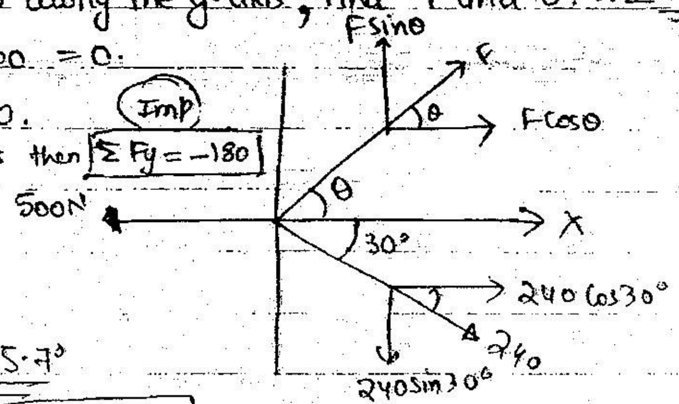
The resultant acts down to the right.

Q If R = 180N is directed up along the y-axis, Find F and θ. $\therefore \Sigma Fx = 0$

$\Sigma Fx = 240\cos 30° + F\cos\theta - 500 = 0.$

$\Sigma Fy = F\sin\theta - 240\sin 30° = 180.$ Imp

If it was given R acting down y-axis then $\boxed{\Sigma Fy = -180}$



$F\cos\theta = 292.153$

$F\sin\theta = 300$

$\tan\theta = \frac{300}{292.15}$ $\quad \underline{\underline{\theta = 45.7°}}$

$\boxed{F = 424.26\,N}$

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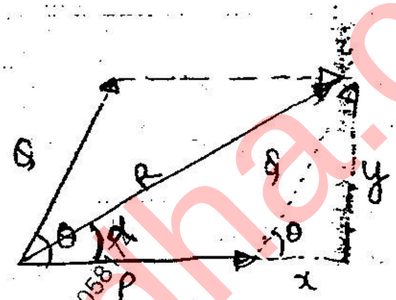
Law of parallelogram of forces (only 2 forces concurrent)

When the system consists of two concurrent pulling forces (Tensile forces) are drawn graphically, represents the adjacent sides of the parallelogram then resultant of the force system represent the diagonal of the parallelogram.

$\sin\theta = \frac{y}{Q}$ $\quad y = Q\sin\theta$

$\cos\theta = \frac{x}{Q} \Rightarrow x = Q\cos\theta$



$$\boxed{x^2 + y^2 = Q^2}$$

$$R^2 = y^2 + (P+x)^2 = y^2 + P^2 + x^2 + 2Px$$

$$= y^2 + x^2 + P^2 + 2PQ\cos\theta = Q^2 + P^2 + 2PQ\cos\theta$$

$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

$$\boxed{\tan\alpha = \frac{y}{P+x} = \frac{Q\sin\theta}{P + Q\cos\theta}}$$

This is used if the Resultant is making some angle with P or Q (other than 90°)

$$\boxed{\cos\alpha = \frac{P+x}{R} = \frac{P + Q\cos\theta}{\sqrt{P^2+Q^2+2PQ\cos\theta}}}$$

This formula used if it is given than (R) is ⊥ to any one of the P or Q.

$$\boxed{\sin\alpha = \frac{y}{R} = \frac{Q\sin\theta}{\sqrt{P^2+Q^2+2PQ\cos\theta}}}$$

Q Two forces 2000N and 2400N at 60° b/w them. Find the Resultant and its direction.

Soln $P = 2000N$ $Q = 2400$ $\theta = 60^\circ$

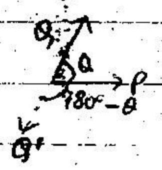
$$R = \sqrt{P^2+Q^2+2PQ\cos\theta} = 3815.75N$$

$$\tan\alpha = \frac{Q\sin\theta}{\sqrt{P^2+Q^2+2PQ\cos\theta}} = \frac{2400\sin 60^\circ}{R} = 33^\circ.$$

$\boxed{\alpha = 33^\circ}$ with the P Force

To Find angle with Q :- $[\alpha' = \theta - \alpha = 60^\circ - 33^\circ = 27^\circ]$ with Force Q.

A System Consists of two force. 2^{nd} is double of First. Then resultant is 260N. If the Larger force is in reverse direction then Resultant is 180 N. Find the value of both forces and angle b/w them.



Soln. 1st Force $= P$

2nd Force $\Rightarrow Q = 2P$

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta}$$

$$260 = \sqrt{P^2+2P^2+2P(2P)\cos\theta} = \sqrt{5P^2+4P^2\cos\theta} \rightarrow ①$$

$$180 = \sqrt{P^2+(2P)^2+4P^2\cos(180^\circ-\theta)} = \sqrt{5P^2-4P^2\cos\theta} \rightarrow ②.$$

$(260)^2 = 5P^2+4P^2\cos\theta$

$(180)^2 = 5P^2-4P^2\cos\theta$

$\Rightarrow \left(\frac{260}{180}\right)^2 = \frac{5+4\cos\theta}{5-4\cos\theta}$

$(2.086)(5-4\cos\theta) = 5+4\cos\theta$

$10.43 - 8.344\cos\theta = 5+4\cos\theta$

$5.43 = 12.34\cos\theta$

$\boxed{\theta = 63.89^\circ}$

$(180)^2 = 5P^2 - 4P^2(\cos\theta)$

$(180)^2 = 5P^2 - 1.76P^2$

$(180)^2 = 3.23P^2$

$\boxed{P = 100.1N}$

$Q = 2P = \underline{200.2N}$

A System Consists of 2 Forces at 120°. Bigger force is 40 N at resultant is (⊥) to the smaller one. Find the value of smaller force.

Solⁿ: α is with P Force. ∴ Smaller Force = P

$Q = 40N \qquad \theta = 120°$

$\cos\alpha = \dfrac{P + Q\cos\theta}{R} \qquad \alpha = 90°.$

∴ $\cos\alpha = 0$ ∴ $P + Q\cos\theta = 0$.

$P + 40\cos 120° = 0$ ∴ $\boxed{P = 20N}$

Two forces P and Q, produce Resultant R. If Q is double then R is (⊥) to P, then what happen?

Solⁿ

1) $\left.\begin{matrix}P\\Q\end{matrix}\right\}R \qquad R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

2) $\left.\begin{matrix}P\\2Q\end{matrix}\right\}R \qquad R\,(\perp)\,P \qquad \cos\alpha = \dfrac{P + Q\cos\theta}{R} \Rightarrow \cos 90° = 0.$

$P + Q\cos\theta = 0 \Rightarrow P = -Q\cos\theta$

$\boxed{P = -2Q\cos\theta} \qquad Q \to 2Q$

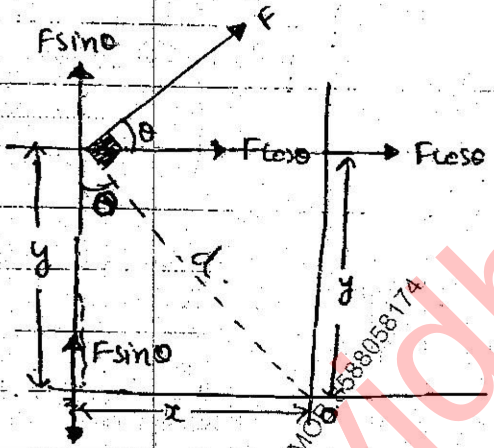
∴ $R' = \sqrt{(-2PQ\cos\theta) + (2Q)^2 + 2(-2Q\cos\theta)(2Q)\cos\theta}$

$R' = \sqrt{-4Q^2\cos^2\theta + 4Q^2 - \cancel{4Q^2\cos^2\theta}} = \sqrt{Q^2} = Q$

∴ $\boxed{R_{new} = Q}$

Varignon's Theorm

Moment due to inclined force about a particular point is the algebraic sum of Moment due to its components about the Same particular point.



$x = d\sin\theta$

$y = d\cos\theta$

Moment of F about Origin O $\quad M_o^F = F \times d.$

Moment of Components of F about O: $M_o^F = F\cos\theta(y) + F\sin\theta \times x$

$= F\cos\theta(d\cos\theta) + F\sin\theta\,(d\sin\theta)$

$= F\cos^2\theta\,(d) + F\sin^2\theta\,(d)$

$\boxed{M_o^F = F \times d}$ Same

Statement

The Moment due to resultant of the force System about a particular point is the algebraic Sum of moments of the all the forces at different distances about the Same particular point.

Imp RULE: Certain force (mean unknown) must be draw :-

#

1) Where its moment = 0 is given or
2) from which the ~~fo~~ Certain force is passed or
3) Where its X-intercept or Y-intercept is given

Moment of Force components

- Moment = Vertical Force X Horizontal distance
- Moment = Horizontal Force X Vertical distance

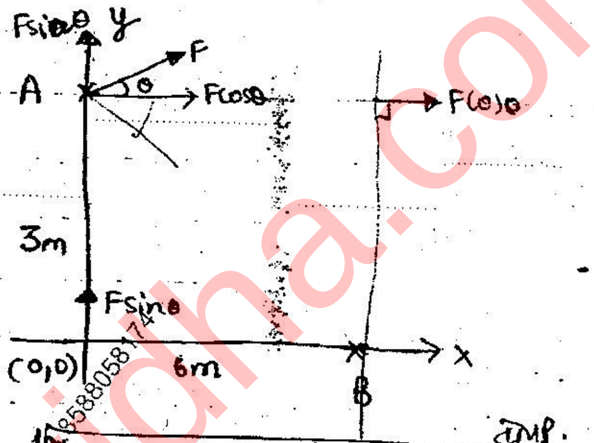
Moment due to certain force about origin is 180 N-m (clock wise) and about point B is 90 N-m (anti-clockwise). Find Magnitude of certain force and its direction if its moment about point A is zero

Soln

$M_O^F = +180$ N-m

$M_B^F = -90$ N-m

$M^F_A = 0.$



$M^F_O = F\sin\theta \times 0 + F\cos\theta(3) = +180$ Nm

$M^F_B = F\sin\theta \times 6 + F\cos\theta(3) = -90$ Nm

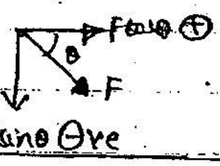
IMP. Always take the direction of F in towards the (+)ve 1st Quadrant only.

$\therefore$ $\boxed{F\cos\theta = 60 \text{ Nm}}$ Right.

$\boxed{F\sin\theta = -45 \text{ Nm}}$ down $\tan\theta = \left|\frac{y}{x}\right|$

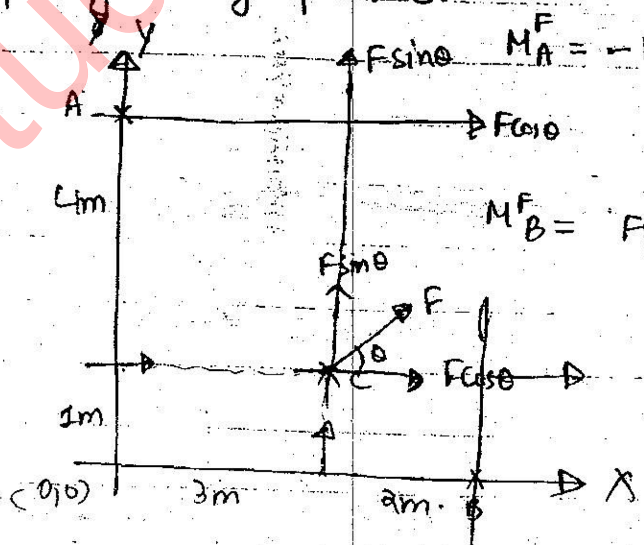
$\therefore$ F = 75 N $\theta = 36.8°$

Thus the resultant F goes in IV quadrant



Moment due to certain force about point A and B is 300 N-m and 600 N-m both clockwise. Find its moment about origin if it is passing through point C.

Soln



$M^F_A = -F\sin\theta(3) - F\cos\theta(4) = 300$

$M^F_B = F\sin\theta(2) + F\cos\theta(1) = 600.$

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Moment = Vertical Force × Horizontal distance (at x-axis)

Moment = Horizontal Force × Vertical distance (y-axis)

$-3F\sin\theta - 4F\cos\theta = 300$

$2F\sin\theta + F\cos\theta = 600 \quad \times 4$

$-3F\sin\theta - 4F\cos\theta = 300$

$\underline{8F\sin\theta + 4F\cos\theta = 2400}$

$5F\sin\theta = 2700$

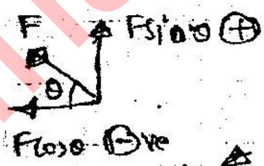
$F\sin\theta = 540.$

$2\times(540) + F\cos\theta = 600$

$F\cos\theta = 600 - 1080$

$= -480$

$\boxed{F\cos\theta = -480}$



$F^2 = (540)^2 + (480)^2$

$\boxed{F = 722.49\,N}$

$\tan\theta = \left|\frac{540}{-480}\right| = 1.125$

$\boxed{\theta = 48.36^\circ}$

Moment about origin (0,0)

$M_O^F = -F\sin\theta\,(3) + F\cos\theta\,(1)$

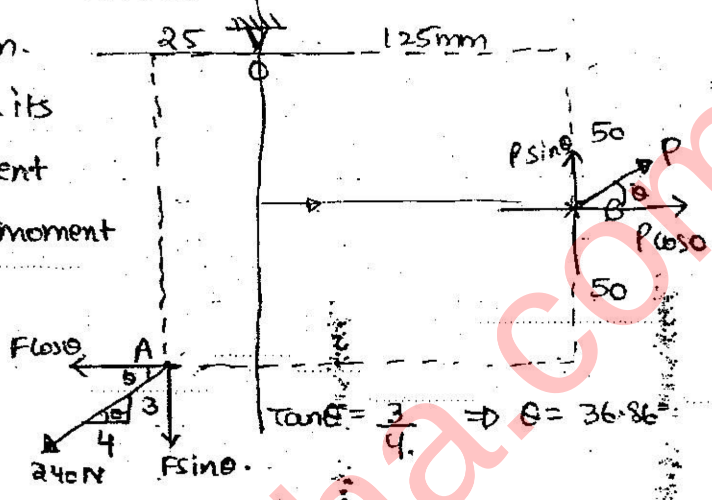
$= -540(3) - 480(1)$

$= -2100\,Nm$ (Anti-clock)

Always take the x-y axis of point where moment is to be found and the transfer the other forces on that axis and find the net moment.

1) Find Moment of 240N about origin.

2) Find the least value of Force P and its direction at B so that its moment is balanced by the given Force moment at origin.



$\tan\theta = \frac{3}{4} \Rightarrow \theta = 36.86°$

(i) $M_o^{240} = +240\cos\theta\ (\times 100) - F\sin\theta \times (25)$

$= 240(0.79)\times 100 - 240(0.59)(25)$

$\boxed{M_o^{240} = 15603.82\ \text{Nmm.}}$ $= \cancel{15.31}\ \text{N-m.}$ $= 15.60\ \text{N-m}$

(ii) $M_o^P + M_o^{F=240} = 0$ (Balance Condition ∴ $M_1 + M_2 = 0$)

$-P\sin\theta(125) - P\cos\theta(50) = 15603.31$

$P\sin\theta(125) + P\cos\theta(50) = +15603.$

$P(125\sin\theta + 50\cos\theta) = +15600$

$P = \dfrac{-15600}{125\sin\theta + 50\cos\theta} = \dfrac{+624}{5\sin\theta + 2\cos\theta}.$

~~at θ = 90° P min~~

~~∴ P = 624 / 5(1)+0 = +124.8N at θ = 90°~~

~~it direction will be horizontal (parallel to x-axis)~~ vertical.

Need to be Max^m ∴ $5\sin\theta + 2\cos\theta$ be Max^m

$\frac{d}{d\theta}(5\sin\theta + 2\cos\theta) = 0.$

$5\cos\theta - 2\sin\theta = 0.$

$\tan\theta = 5/2$ ∴ $\boxed{\theta = 68.2°}$

$P_{max} = \dfrac{624}{5\sin(\theta) + 2\cos\theta}$

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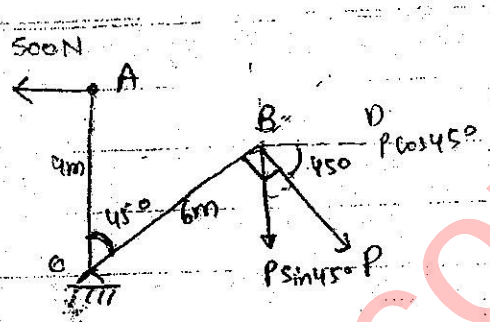
Find the P if its moment is balanced by 500N about O



$$M_o^{500} + M_o^{P} = 0$$

$$-500(4) + P(6) = 0$$

$$P = +\frac{500(4)^2}{\cancel{6}\,3} = \frac{1000}{3}\,N$$

Angle b/w AO and OB = 45°

and b/w BP and BD = 45°

$\therefore$ $\boxed{P \perp \text{ to } OB}$

Parallel force system

When all the forces never intersect each other.

Types:-

(i) Likely parallel force system (Means directions are same)

(ii) Unlikely parallel force system (Means diff./opposite directions)

$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$ general.

(1) Parallel force System

for parallel

$\boxed{R = \Sigma F}$ either ΣF_x or ΣF_y (as forces will be parallel)

$\boxed{R\bar{a} = \Sigma M_o}$ Moment of Resultant force (R) at distance $\bar{a}$ from point O = Sum of Moment of all forces about O.

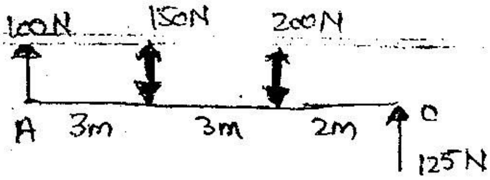
$\bar{a}$ = perpendicular distance, Location, position, Line of action, moment arm, shortest distance

NOTE :- 1) If $\Sigma F = 0$ then $R = 0$, no displacement but there maybe couple

$\boxed{C = \Sigma M}$

2) If $\Sigma F = 0$ and $\Sigma M = 0$, then $R = 0$ and $C = 0$.

Find Resultant and its line of action from point A



Soln

$R = \Sigma F$

$= 100 - 150 - 200 + 125$

$= 225 - 350 = -125\ N \quad \therefore\ R \downarrow$ (down)

$C = \Sigma M_A$ (Net couple)

$C = +150(3) + 200(3+3) - 125(8)$

$= 450 + 1200 - 1000 = 650\ Nm$ (Clockwise)

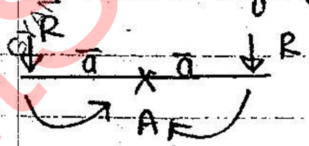
thus $R\bar{a} = \Sigma M$

$R(\bar{a}) = C$ $\qquad R\bar{a}$ = Moment due to (R) at A

$\bar{a} = \frac{650}{+125} = \boxed{5.20m}$ always ⊕ve $\qquad \therefore$ R is acting down

(I) distance $(\bar{a})$ will always be ⊕ve.

After that to Know (R) position either Left or Right of A, do this! –



Wrong To the left $\bar{R}$ is giving anticlockwise Moment

Right To the Right $\bar{R}$ is giving clockwise moment.

as $\Sigma M_A = 650$ (clock) $\therefore$ Correct

Non-concurrent & Non-parallel force System

Forces ~~nigh~~ neither intersect at a point and nor intersect at infinity.

$\boxed{R\bar{a} = \Sigma M_o}$ and Non-parallel means:– $R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

and $\tan\theta = \left|\frac{\Sigma F_y}{\Sigma F_x}\right|$

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* Resultant (R) is always an independent vector and couple (C) also an independent vector i.e they donot dependent upon position in a system.

NOTE
- if $\Sigma F_x = 0$ then $\boxed{R = \Sigma F_y}$ then R is acting vertically.
- If $\Sigma F_y = 0$ then $\boxed{R = \Sigma F_x}$ then R is acting Horizontally
- If $\Sigma F_x = 0$ and $\Sigma F_y = 0$ then $R = 0$, No displacement but they may be couple $\boxed{C = \Sigma M}$
- If $\Sigma F_x = 0$ and $\Sigma F_y = 0$ and $\Sigma M = 0$ then $R = 0$, $C = 0$, Body is at Rest.

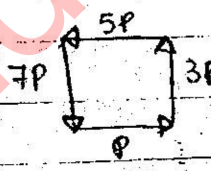
\# System has 4 forces P, 3P, 5P and 7P, each of on sides of the square directed back to back. Find the resultant.

$\Sigma F_x = P - 5P = -4P$

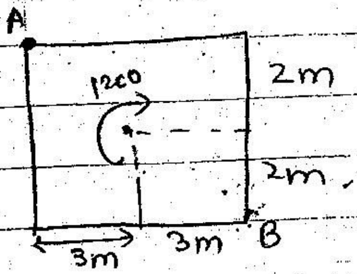
$\Sigma F_y = 3P - 7P = -4P$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{(-4P)^2 + (-4P)^2}$

$R = \sqrt{16P^2 + 16P^2} = 4\sqrt{2}P$



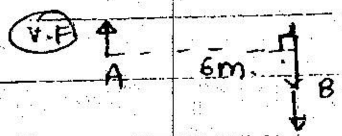
\# Find Horizontal and Vertical force moment at A and B produce the same couple.



$H.F \times 4 = 1200$

$\boxed{H.F = 300N}$

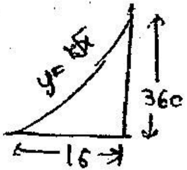
Only Equal and opp. forces produces a couple



$\therefore V.F \times (6) = 1200$

$\boxed{V.F = 200N}$

$$\text{Area} = \frac{b \times h}{n+1} = \frac{16 \times 360}{\left(\frac{1}{2}+1\right)}$$



$$\text{Centroid} = \left(\frac{n+1}{n+2}\right)(b) = \frac{\left(\frac{1}{2}+1\right)}{\left(\frac{1}{2}+2\right)} \times 16.$$