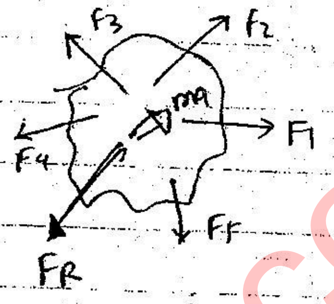
KINECTICS OF PARTICLE AND RIGID Body

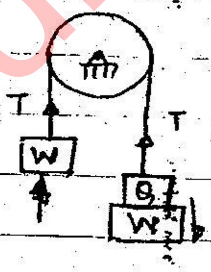
(1) D'Alembert's principle



$$\boxed{\Sigma F = m\vec{a}}$$

When system consists of No. of forces produce the resultant causes an accln in direction of resultant to maintain such body, is in dynamic equilibrium, inertia force having magnitude ($M\vec{a}$) is applied in opposite direction.

Find Q.



$\Sigma F_y = 0$

$$T - W = \left(\frac{W}{g}\right)a.$$

$$+\quad (Q+W) - T = \frac{(Q+W)}{g}.a$$

$$(Q+\cancel{W}-\cancel{W}) = a\left[\frac{W}{g} + \frac{Q+W}{g}\right].$$

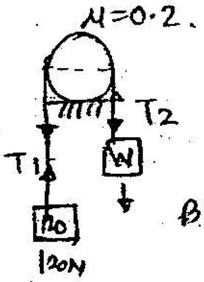
$$Q = \frac{aW}{g} + \frac{a(Q)}{g} + \frac{a(W)}{g}.$$

$$Q\left[1-\frac{a}{g}\right] = a\left(\frac{2W}{g}\right). \Rightarrow Q = \frac{2aW}{g\left[1-\frac{a}{g}\right]}$$

$$Q = \frac{2aW}{(g-a)}$$

Find W to start motion if $a = 0.1g$ m/sec² in direction of W.

$\mu = 0.2$.



$\beta = 180° \times \frac{\pi}{180}$

$\beta = \pi$ radians

$W - T_2 = \left(\frac{W}{g}\right) a.$

$T_1 - 120 = \left(\frac{120}{g}\right) \times a$

$T_1 = 120 + 12 = 132N.$

$(W - T_2) + (T_1 - 120) = W(0.1) + (120)(0.1)$

$(W - 120) - 0.87 T_1 = 0.1W + 12.$

$(W - 120) - 0.87(132) = 0.1W + 12.$

$0.99W = 246.84$

$\boxed{W = 249.33N}$

$T_2 > T_1$

$\frac{T_2}{T_1} = e^{\mu\beta}$

$T_2 = T_1 e^{0.2(\pi)}$

$\boxed{T_2 = 1.87 T_1}$

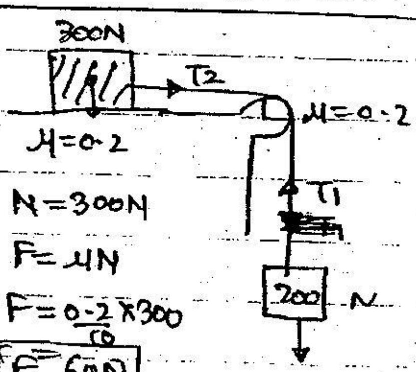
$= T_1 - T_2$

$= 0.87$

$(T_1 - T_2)$

$T_1 - 1.87 \cdot T_1.$

Find the accl^n of the system.



$\mu N \leftarrow \boxed{300} \rightarrow T_2$

$T_2 - \mu N = 30a.$

$\boxed{200 - T_1 = 20a.} \rightarrow (1)$

$(T_2 - T_1) + (200 - 60) = 50a.$

$(T_2 - T_1) + 140 = 50a$

$\boxed{-0.36 T_1 + 140 = 50a} \rightarrow (2).$

$N = 300N$

$F = \mu N$

$F = \frac{0.2}{10} \times 300$

$\boxed{F = 60N}$

$T_1 > T_2$

$T_1 = T_2 e^{\mu\beta}$

$[T_1 = 1.36 T_2]$

(1) and (2).

$T_1 = 200 - 20a = \frac{50a - 140}{-0.36}$

$-72 + 7.2a = 50a - 140$

$T_1 -$

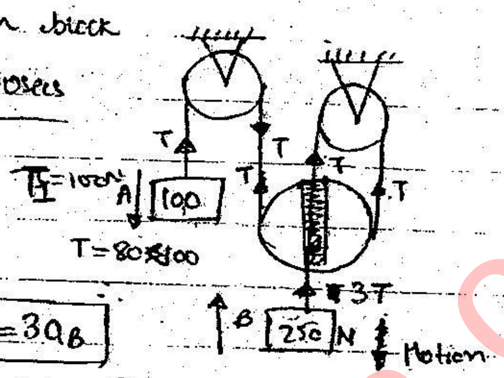
$42.8a = 68$

$[a = 1.58 \text{ m/sec}^2]$

Trick to check direction of motion | Always find the value of Tension (T) in each portion with weights at the location. Then use the min^m value of Tension T and then use that Tension (T) to determine the direction

~~3T−250 = ma~~

How far and in what direction block B will move starting from rest in 10 secs



3T=250
T_2=83N
out of T_2 and T_1
T_2 is smaller
∴ use T_2 to find direction of motion

By Inspection

Displacement (B) = S_B A = δ_A

$1\,\delta_A = 3\,\delta_B \Rightarrow \boxed{a_A = 3a_B}$

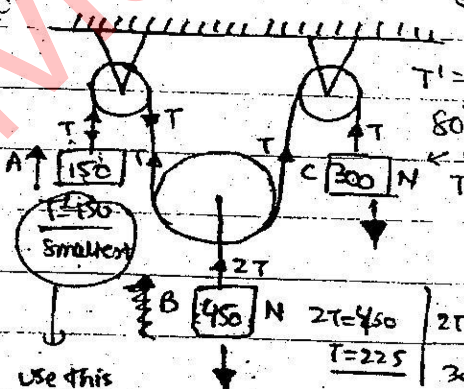
Tension at A = 1T ⋮ ① Tension at B = 3T ⋮ ③

$\Sigma F_{y_A} = 0$ $T = 100 - \frac{100}{g}\,3a_B$ → as $\boxed{a_A = 3a_B}$

$\Sigma F_{y_B} = 0$ $T = 250 + \frac{250}{g}(a_B)$

$S_B = \cancel{ut} + \frac{1}{2}a_B t^2$

$S_B = \frac{1}{2} \times a_B \times t^2$



use this to find the direction of the masses.

Sum (Upward) = Downward (sum)

$\boxed{1.\delta_A = 2\delta_B + 1\delta_C}$

∴ $\boxed{a_A = 2a_B + a_C}$ as double differentiating we will get $[a_A = 2a_B + a_C]$

T ↑ 150 A ↓ 150N δ_A upward

$\boxed{T - 150 = \frac{150}{g}(a_A)}$

2T ↑ 450 B ↓ 450N down δ_B

$\boxed{450 - 2T = \frac{450}{g} \times a_B}$

$100 - T = m \times a_A$

100 −

Find tension in String supporting block C.

T ↑ 300 ↓ 300N down δ_C.

$300 - T = \frac{300}{g} \times a_C$

also $\boxed{a_A = 2a_B + a_C}$ we get ans.

$a_A = 2a_B + a_C.$

$$(T-150)\frac{\cancel{g}}{150} = 2(450-2T)\frac{\cancel{g}}{450} + (300-T)\left(\frac{g}{300}\right)$$

$$T-150 = \frac{(300-4T)}{3} + \left(\frac{300-T}{2}\right)$$

$$6T-900 = 1800-8T+900-3T$$

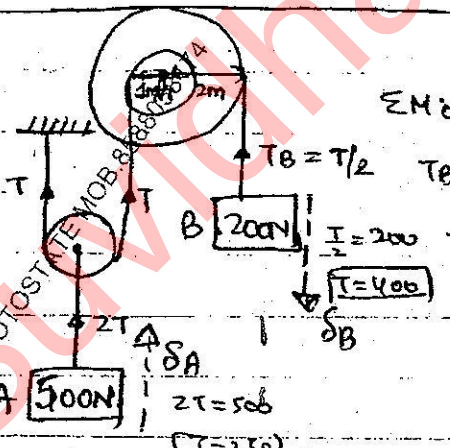
$$\boxed{T = 212\,N}$$

Find acc^n of block B

$(2)\,\delta_A = \left(\frac{1}{2}\right)\delta_B$

$4\,\delta_A = \delta_B$

$\boxed{4A_a = A_B} \rightarrow (1)$



$\Sigma M_O = 0$

$T_B(2) = T(1)$

$\frac{T}{2} = 200$ $\boxed{T = 400}$ $\boxed{T_B = \frac{T}{2}}$

$2T = 500$ $(T = 250)$

Min^m (use this)

$T_B' = \frac{250}{2} = 125$

$\therefore T_B' < 200$

$\therefore B \downarrow$

and $T_A' = 250(2)$
$= 500$
$\therefore A\uparrow.$

$\uparrow 2T$

500 $\uparrow \delta_A$

$\downarrow 500N$

$$2T - 500 = \left(\frac{500}{g}\right) \times A_a.$$

$$A_a = \frac{(2T-500)g}{500}.$$

$$200 - \frac{T}{2} = \left(\frac{200}{g}\right) \times A_b.$$

$$A_b = \frac{\left(200 - T/2\right) \times g}{200}.$$

$4A_a = A_B$

$$4\frac{(2T-500)\cancel{g}}{\cancel{500}} = \frac{(200-T/2)\cancel{g}}{\cancel{200}}$$

$$16T - 4000 = 1000 - 5T/2$$

$$16T + 5T/2 = 5000.$$

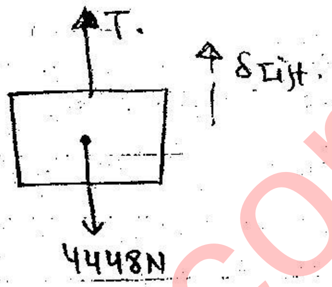
$\frac{37T}{2} = 5000$

$T = 270.27N$

$\boxed{A_b = +3.18}$

Pg – 39

Q2 $u = 0 \quad V = 1.828\,m/s \quad s = 1.828\,m$



$V = U + at \qquad V^2 = u^2 + 2as$

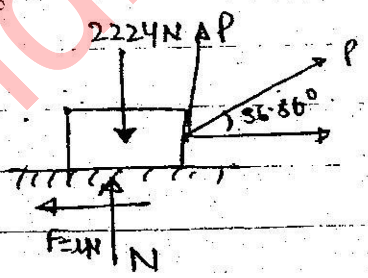
$\cancel{V = at} \qquad (1.828)^2 = 0 + 2 \times a \times (1.828)$

$\cancel{1.828 = a t} \qquad \boxed{a = 0.914\,m/sec}$

$T - 4448 = \frac{4448}{g} \times 0.914.$

$T = 4590N.$

Q3 $\boxed{N = 2224 + P\sin\theta}$



$P\cos\theta - uN = m \times a.$

$P\cos\theta - uN = m \times 0.2g.$

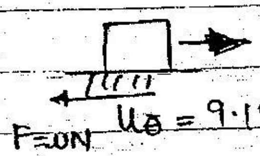
$P\cos\theta - u(2224 + P\sin\theta) = 2224 \times 0.2$

$P(\cos\theta + u\sin\theta) = 0.2 \times 2 \times 2224$

$P(\sim) = 889.6$

$\cancel{P = 1303.66N.} \quad \boxed{P = 966.87N}$

Q4



$F = uN = u(44.8N).$ $\qquad u_0 = 9.126\,m/s.$

$V^2 = u^2 + 2as$

$0 = U^2 + 2as$

$2as = -u^2$

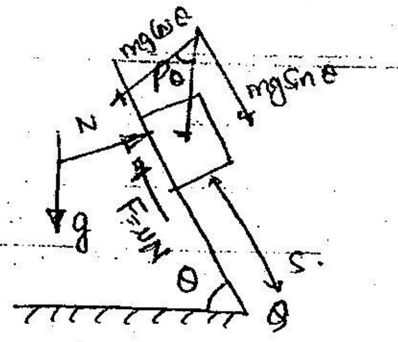
$a = -\frac{(9.126)^2}{2 \times 13.689} = -3.042\,m/sec^2$

$F = uN = m \times a.$

$\mu = \frac{44.8}{g} \times \frac{(3.042)}{(44.8)} = 0.310$

$$\boxed{I_{xx} = \frac{mL^2}{12}}$$

Q5



~~mg sin θ~~

$mg\sin\theta - \mu N = m \times \vec{a}$

$\cancel{m}g\sin\theta - \mu(\cancel{m}g\cos\theta) = \cancel{m} \times \vec{a}$

$g(\sin\theta - \mu\cos\theta) = \vec{a}$.

$$\boxed{a = g(\sin\theta - \mu\cos\theta)}$$

~~Tan θ~~

~~Tan θ = μ~~.

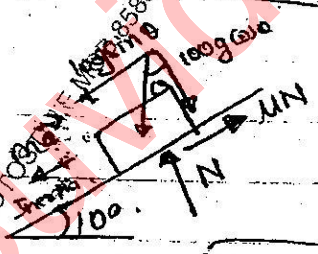
$S = \cancel{ut} + \frac{1}{2}at^2$.

$S = \frac{1}{2}a(t)^2$

$\frac{2S}{a} = t^2$

$t^2 = \frac{2S}{g(\sin\theta - \mu\cos\theta)}$ $\quad t = \sqrt{\frac{2S}{g\cos\theta(\tan\theta - \mu)}}$

Q8



$100g\sin\theta - \mu N = -m \times a$.

$100g\sin\theta - \mu(100g\cos\theta) = -m \times a$.

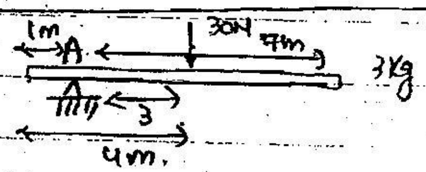
$g[100 \times \sin 10° - 0.3\cos 10°] = -a$.

100

$$\boxed{a = +1.19\ m/sec^2}$$

$$\boxed{Net = m\vec{a}}$$

Q10 $\boxed{M = I\alpha}$



$M = 30(3) = I\alpha$.

$90 = I\alpha$

$\alpha = \frac{90}{I}$

$I_x = I_{\bar{x}} + A\bar{y}^2$

$$\boxed{I_{xx} = I_{\bar{x}} + M\bar{y}^2}$$ formed with mass (M)

$$I_{\bar{x}} = \frac{ML^2}{12}$$

$I_{xx} = \frac{mL^2}{12} + M(b)^2$

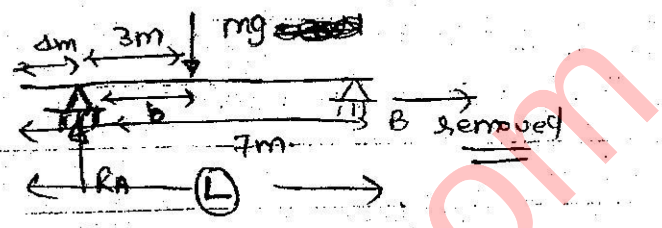
$I_{xx} = \frac{mL^2}{12} + 3 \times 3^2 \Rightarrow I = 43$

$\alpha = \frac{90}{43} = 2.09$ rad/sec^2.

$\frac{WL^2}{L^2+12(\frac{L^2}{4})}$

$\frac{W^2}{4L^2}$

Find Angular Accl^n and Vertical reaction of A.
If reaction B is suddenly removed then what happens.



$M = I\alpha$.

$M = mg \times b$.

$mg \times b = I \times \alpha$.

$\alpha = \frac{mg \times b}{I}$.

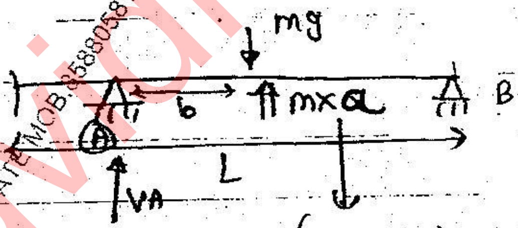
$I_{xx} = I_{\bar{x}} + m\bar{y}^2$

$= \frac{ML^2}{12} + M \times (b)^2$

$= M(\frac{L^2}{12} + b^2)$.

$$\boxed{\alpha = \frac{mg \times b}{M[\frac{L^2}{12}+b^2]} = \frac{gb}{(\frac{L^2}{12}+b^2)}}$$

If B is removed, then only VA Left.



(Inertia is opp. direction)

$\boxed{a = r\alpha}$

distance of mg from Hinge

$\boxed{r = b}$

$\Sigma F_y = 0$

$VA + m\alpha b = mg$

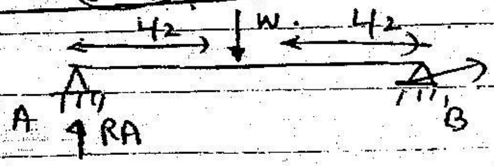
$VA = m(g - \alpha b) = m(g - \frac{gb^2}{\frac{L^2}{12}+b^2})$.

$VA = m\left[\frac{g\frac{L^2}{12} + gb^2 - gb^2}{\frac{L^2}{12}+b^2}\right] = m\left(\frac{gL^2 + 12gb^2 - 12gb^2}{L^2+12b^2}\right)$

$= \frac{mgL^2}{L^2+12b^2}$

Bar AB of Weight W N

Q4 Book



$M = I\alpha$

$I = I_{\bar{x}} + m\bar{x}^2 = \frac{ML^2}{12} + M \times (\frac{L}{2})^2 = \frac{ML^2}{12} + \frac{ML^2}{4} = \frac{ML^2}{3}$.

$W \times (l/2) = \frac{ML^2}{3} \times \alpha$

$$\boxed{\alpha = \frac{3W}{2ML}} = \frac{3mg}{2mg\,L} = \frac{3g}{2L}$$

$RA + ma = mg$

$RA = m(g - a) = m(g - l/2\,\alpha)$

$RA = m(g - \frac{l}{2}(\frac{3W}{ML}))$

$= m(g - \frac{3W}{4M}) = M(\frac{2gM - 3W}{2M})$

$= \frac{2W - 3W}{}$. $W/4$

Q13

Soln $M = 60\,N\text{-}m$ = Torque = $I\alpha$

$$60 = \frac{Ml^2}{12} \times \alpha \qquad \alpha = \frac{\overset{15}{\cancel{60}} \times \cancel{12}\,3}{\cancel{48} \times 4\,10} = 4.5\ rad/sec^2$$

$N = 200\ rpm$

$$\omega = \frac{2\pi N}{60} = \frac{2 \times \pi \times 200}{6\cancel{0}}$$

$$= \frac{40\pi}{6} = \frac{20\pi}{3}$$

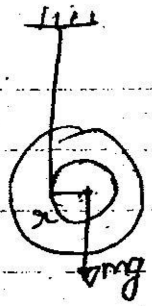
$\omega = \omega_0 + \alpha t$

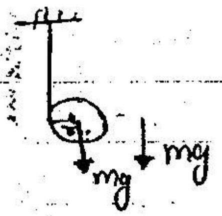
$\frac{20\pi}{3} = 0 + 4.5(t)$

$t = \frac{20\pi}{3} \times \frac{1}{4.5} = 4.6\ secs.$

Q14

$a = r\alpha$





$M = I\alpha$

$mg \times r = I\alpha$

$\alpha = \frac{mgr}{I}$

$\alpha = \frac{\cancel{m}gr}{\cancel{m}(K^2 + r^2)}$

$\alpha = \frac{gr}{K^2 + r^2}$

$a = r\alpha = \frac{gr^2}{K^2 + r^2}$

$\boxed{I = mK^2 + mr^2}$

(mass continously varying).

$I = m(K^2 + r^2)$

$F \times r = T.$

$mg \times r = I \times \alpha$

$\alpha = \frac{mgr}{I}$

$= \frac{mgr}{mK^2}$

$\boxed{\alpha = \frac{gr}{K^2}}$

$I = K^2 A$

$I = K^2 m$

$I = Ay^2$

$I = my^2$