KINEMATICS OF MOTION

Kinematics

If the rigid body produce the resultant and couple.

① If the resultant of the force System remains constant i.e direction of resultant is constant throughout the motion with zero couple then the motion of the System along a straight line path is called as Rectilinear motion.

② If the resultant of the force system is constant but its direction is Variable with zero couple then motion along a curved path called as Curvilinear motion

③ If the force System consists of only couple with zero resultant then motion of system in a circular manner called as centroidal axis rotation.

④ If the system having a resultant as well as couple and force system lie in ~~catpl~~ coplanar, then motion called translation - rotational motion.

⑤ If the force - system is 3-D, then motion is spatial motion or Satellite motion.

$V = \frac{ds}{dt}$ $\int ds = \int V dt$

$a = \frac{dv}{dt}$ $\int dv = \int a dt$

$dt = \frac{ds}{v} = \frac{dv}{a} \Rightarrow \boxed{\int v dv = \int a ds}$

\# $x = (2t^3 + t^2 + 2t)\,m$ Particle starts with :- $\boxed{at\ t=0}$ $\boxed{x=0 \quad V = 2\,m/sec \quad a = 2\,m/sec^2}$

$V_x = 6t^2 + 2t + 2$ m/sec

$a_x = 12t + 2$. m/sec²

\# $a = 6\sqrt{v}$ when $t = 2$ $v = 36\,m/sec$ $\boxed{s = 30m}$. Find s at $t = 38$

$\frac{dv}{dt} = 6\sqrt{v}$

~~$dv = 6\sqrt{v}dt$~~

$\int \frac{dv}{\sqrt{v}} = \int 6dt$

~~$v = \frac{6}{\sqrt{v}}\left(\frac{1}{2}\right) + C = \frac{3}{\sqrt{v}} + C$~~

$2\sqrt{v} = t + c$

$v = 36$ at $t = 2$

$\therefore 2\sqrt{36} = 2 + C \Rightarrow 2 \times 6 = 2 + C$

$12 - 2 = C$

$\boxed{C = 10}$ 10.

$\therefore \boxed{2\sqrt{v} = t + 10}$

TRICK

$\frac{(v)^{\frac{1}{2}+1}}{\frac{1}{2}}$

$\frac{dv}{dt} = 6\sqrt{v}$

$\int_{36}^{v} \frac{dv}{\sqrt{v}} = \int_{2}^{t} 6\,dt$

Always use give Values $V = 36$

V given, $t = 2$

$(2\sqrt{v})_{36}^{v} = (6t)_{2}^{t}$

$2[\sqrt{v} - \sqrt{36}] = 6\left(t\right)_{2}^{t}$

$2(\sqrt{v} - 6) = 6(t - 2)$

$\sqrt{v} - \cancel{6} = 3t - \cancel{6}$

$\sqrt{v} = 3t$

$\boxed{v = 9t^2}$

$v = 9t^2$

$\frac{ds}{dt} = 9t^2$

$\int_{30}^{s} ds = \int_{2}^{t} 9t^2\,dt$

$(S - 30) = 3t^3 - 24$

$\boxed{S = 3t^3 + 6}$ then use it to find!

$V = Kx^3 - 4x^2 + 6x$ Find 'a' when $x = 2m$ and $K = 1$.

$$a_x = \frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = \frac{dv}{dx} \times V.$$

$$a_x = (3Kx^2 - 8x + 6)[Kx^3 - 4x^2 + 6x]$$

$$= [3(1)(4) - 8(2) + 6][K(2)^3 - 4(2)^2 + 6(2)]$$

$$= [12 - 16 + 6][8 - 16 + 12] = 2 \times [4] = 8 m/sec^2.$$

$a = 4t^2 - 2$ When $t = 0$, Position is 2m to the left of origin. When $t = 2 sec$, Position is 20m to the left of origin. When $t = 4$ secs find the position.

$t = 0 \quad S = -2$

$t = 2 \quad S = -20m$

$t = 4 \quad ?$

$$a = \frac{dv}{dt} = \frac{4t^2 - 2}{1}$$

$$\int dv = \int (4t^2 - 2)dt \Rightarrow \int_u^v dv = \int_0^t (4t^2 - 2)dt$$

$$(v - u) = \left(\frac{4t^3}{3} - 2t\right)_0^t$$

$$(v - u) = (2t^2 - 2t)_0^t$$

$$v - u = \left(\frac{4t^3}{3} - 2t\right)$$

$$v = u + \left(\frac{4t^3}{3} - 2t\right).$$

$$v = \frac{ds}{dt} = \left(\frac{4t^3}{3} - 2t\right) + u.$$

$$\int_{-2}^{s} ds = \int_0^t \left(\frac{4t^3}{3} - 2t + u\right)dt$$

$$(S)_{-2}^{s} = \left(\frac{t^4}{3} - t^2 + ut\right)$$

$S + 2$

$$\boxed{S = \frac{t^4}{3} - t^2 + ut - 2}$$

at $t = 2 \quad s = -20.$

$$-20 = \frac{16}{3} - 4 + 2u - 2$$

$$-20 + 6 - \frac{16}{3} = 2u.$$

$$-6 - \frac{8}{3} = u$$

$$\boxed{u = \frac{-26}{3}} = -9.67.$$

→ If anything is max^m means it is a constant and its derivative is always zero.

$$S = \frac{t^4}{3} - t^2 - 9.67t - 2.$$

$S(t=4) \Rightarrow \boxed{28.65m}$

$a = 21 - 12x^2$ Find > x when v is max. Particle starts from rest from origin.

> v when $x = 1.5m$.

$$a = \frac{dv}{dt} = 21 - 12x^2.$$

When V is max^m means it is a constant ∴ derivative of any max^m Qty will be zero.

$$\frac{dv}{dt} = \frac{dv}{dx} \times \frac{dx}{dt} = 21 - 12x^2$$

∴ V is max^m ∴ $\frac{dv}{dt} = 0 \Rightarrow a = 0$ ∴ $21 = 12x^2 \Rightarrow x = \pm 1.32m$

(II) V when $x = 1.5m$.

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v \cdot \frac{dv}{dx}.$$

$$V\frac{dv}{dx} = 21 - 12x^2$$

$$\int_0^V V\,dv = \int_0^x (21 - 12x^2)\,dx$$

$$\left(\frac{V^2}{2}\right)_0^V = \left(21x - 4x^3\right)_0^x$$

$$\frac{V^2}{2} = 21x - 6x^2$$

$$\left(V^2 = 42x - 12x^2\right)$$

$x = 1.5 \Rightarrow \boxed{V = \pm 6 m/sec}$

\# $a = -8x^{-2}$ when $t = 1sec$ $x = 4m$
$v = 2m/s$ Find acl^n when $t = 2sec$

$$\frac{dv}{dt} = \frac{dv}{dx}\cdot\frac{dx}{dt} = v\frac{dv}{dx} = -8x^{-2}$$

$$v\,dv = -8x^{-2}\,dx.$$

$$\int_2^v v\,dv = -\int_4^x 8x^{-2}$$

$$\left(\frac{v^2}{2}\right)_2^v = \left(\frac{16}{x}\right)_4^x$$

$$\frac{v^2-4}{2} = \frac{16}{x} - \frac{16}{4}$$

$$\frac{v^2-4}{2} = \frac{32}{x} - 4$$

$$v^2 - 4 = 16\left(\frac{1}{x} - \frac{1}{4}\right)$$

$$(v^2 - \cancel{4}) = \frac{16}{x} - \cancel{4}$$

$$v^2 = 16/x.$$

$$v = \frac{4}{\sqrt{x}} = \frac{dx}{dt}$$

$$\int_4^x dx\sqrt{x} = \int_1^t 4dt$$

$$\frac{2}{3}\left[x^{3/2}\right]_4^x = 4(t-1)$$

$$\frac{2}{3}\left[x^{3/2} - 4^{3/2}\right] = 4(t-1)$$

$$x^{3/2} - 8 = \frac{4 \times 3}{2}(t-1)$$

$$x^{3/2} = 8 + 6t - 6.$$

$$\boxed{x^{3/2} = 6t+2} \quad x = (6t+2)^{2/3}$$

$$acl^n \Rightarrow a = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \frac{d}{dt}(6t+2)^{2/3} = \frac{2}{3}(6t+2)^{-1/3}(6)$$

$$\frac{2}{3} \quad v = 4(6t+2)^{-1/3}$$

~~at t = 2~~

$$\cancel{a = 4(12+2)^{-1/3}} = 4(14)^{-1/3}$$

$$(a = \cancel{-0.237 m/sec^2})$$

$$a = -8x^{-2}$$

$$a = -8\left[\frac{1}{(6t+2)^{2/3}}\right]^2$$

$$a = -0.237 m/sec^2$$

(at $t = 2$)

TRICK

Q7 (wrong in Book)

$y = x^2 - 4x + 100$ $\quad V_0 = (4\hat{i} - 6\hat{j})$ is V_x is constant, Find V_y, a_y when $x = 16m$.

$$V_y = \frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$$V_y = V_x \cdot \frac{dy}{dx} = V_x(2x-4).$$

$$V_y = V_x \cdot (2x-4)$$

at $V_x = 4$ and $x = 16$

$$V_y = V_x(32)$$

$$\boxed{V_y = 4(32) = 128 \text{ m/sec}}$$

$$a_y = \frac{dV_y}{dt} = \frac{dV_y}{dx} \times \frac{dx}{dt}$$

$$\cancel{a_y = V_x \cdot \frac{dV_y}{dt} = \frac{d(V_x^2(2x-4))}{dx}}$$

$$a_y = 2V_x^2.$$

$$\cancel{a_y = 2(16) = 32 m/sec^2}$$

$$a_y = \frac{dV_y}{dt} = \frac{dV_y}{dx} \times \frac{dx}{dt} = V_x \cdot \frac{d(V_y)}{dx}.$$

$$a_y = V_x \frac{d}{dx}\left[V_x(2x-4)\right] = V_x\left[V_x(2) + (2x-4)\frac{dV_x}{dt}\right]$$

$$a_y = V_x\left[2V_x + (2x-4)a_x\right]$$

$\hookrightarrow a_x = 0$ as V_x = Constant

$$a_y = 2V_x^2 = 2(16) = 32 m/sec^2$$

Rectilinear Motion

Motion of Two types

(i) URM :- Uniformly rectilinear motion (Means velocity is constant means accl^n = 0)

(ii) UARM :- Uniformly accelerated rectilinear motion (accl^n is constant).

$S = ut + \frac{1}{2}at^2$

$V^2 = U^2 + 2aS$

$V = u + at$

Balls drops freely from the top of the tower 36 m High. At the same instant second ball is thrown vertically up from ground with an initial velocity 18 m/sec. When and where both balls crosses each other.

$$\boxed{S_1 + S_2 = 36}$$

$V_1 = u_1 + at$

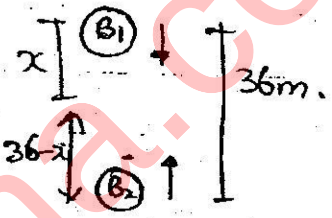
$V_1 = a_1 t$

$\boxed{V_1 = gt}$

$S_1 = u_1 t + \frac{1}{2} a_1 t^2$

$\boxed{S_1 = \frac{1}{2} g t^2}$

$x = \frac{1}{2} g t^2$



$S_2 = u_2 t + \frac{1}{2} a_2 t^2$

$S_2 = 18t + \frac{1}{2}(a)t^2$

$36 - \frac{1}{2} g t^2 = 18t + \frac{1}{2} a t^2$

$36 - \cancel{\frac{1}{2} g t^2} = 18t - \cancel{\frac{1}{2} g t^2}$

$\boxed{t = 2 sec}$

$V = u + at$ $\quad V^2 = 18^2 + 2as$

$V = 18 + at$

$S_1 = \frac{1}{2} \times (10)(4) = \boxed{20m}$ from top

$S_2 = 36 - 20 = \boxed{16m}$ from bottom.

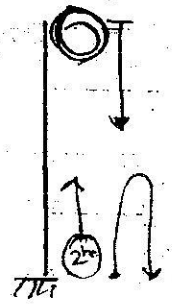
If the second ball which thrown vertically up by 10m/s then when and where both balls meet eachother.

$ut = 36$

$t = \frac{36}{10} = 3.6 secs.$

$S_1 = \frac{1}{2} \times g \times t^2 = \frac{1}{2} \times 10 \times (3.6)^2 = 64.8m$

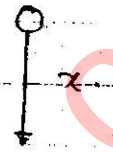
[It means that the first ball was thrown up and returned back to the ground and the upper ball has not reached the ground and thus the 2 balls never meet]



Stone is dropped through a certain Height. After 5 Seconds, it breaks the glass and in breaking it Losses 20% Velocity. Find the position in the next 1 second.

$u = 0$. $V = gt$.

at $(t = 5\text{secs})$ $\boxed{V = 5g}$



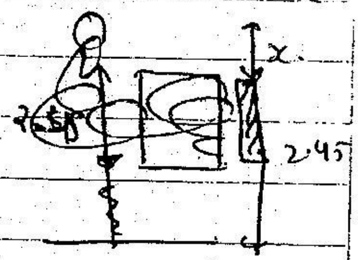
20% Loss $\Rightarrow$ $V' = 0.8 \times 5g = 4g$.

$S = ut + \frac{1}{2}gt^2$

$S = 4gt + \frac{1}{2}gt^2 = \frac{9}{2}gt^2 = \frac{3}{2} \times 9.81 \times 1 = 14.715m \times 3 = 44.145m$

Stone dropped through a certain Height and it crosses the window 2.45m Heigh in 0.5sec. Find the certain Height through which thrown stone

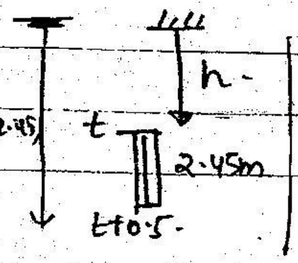
$\boxed{H = 2.45 + x}$ $t = 0.5secs.$



$(S = 2.45 + h)$

$\boxed{h = ut + \frac{1}{2}at^2}$ Top $\rightarrow$ h

$\boxed{h = \frac{1}{2}gt^2}$



$\boxed{S_2 = 2.45 + h = \cancel{\frac{1}{2}}(\cancel{ut}) + \frac{1}{2}gt^2}$ $\rightarrow$ Top - (h+2.45)

$2.45 + h = \frac{1}{2}g(t + 0.5)^2$

$2.45 + \cancel{\frac{1}{2}gt^2} = \frac{1}{2}g(\cancel{t^2} + 0.25 + t)$

$2.45 = \frac{0.25}{2} + \frac{t}{2}$.

$4.9 = 0.25 + t$

$\boxed{t = 4.65}$

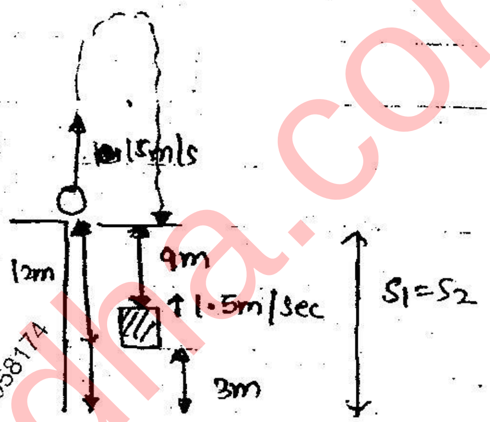
$h = \frac{1}{2}(9.81)(4.65)^2$

$\boxed{h = 106.05m}$

(Imp) If Both particles are direction in the same direction, then Height of Each particle is considered as initial displacement.

Imp

Ball is thrown vertically up by 15m/sec from 12m Level and at the same time open platform elevator passes the three metre Level vertically up with constant velocity 1.5m/sec. When and where ball will hit the elevator.



Ball

$V = u + at$

$V = 15 - gt$

$S = ut + \frac{1}{2}gt^2 + 12$

$s_1 = 15t - \frac{1}{2}gt^2 + 12$ (Imp)

Elevator

$\boxed{s_2 = 1.5t - \frac{1}{2}gt^2} + 3$

Velocity = constant

$\boxed{a = 0}$

$\boxed{s_2 = 1.5t + 3}$

then

$s_1 = s_2$

$\cancel{15t - \frac{1}{2}gt^2 + 12 = 1.5t - \frac{1}{2}gt^2 + 9}$

$\cancel{13.5t =}$

$15t - \frac{1}{2}gt^2 + 12 = 1.5t + 3$

$13.5t = -9 + \frac{1}{2}gt^2$

$\boxed{13.5t - 4.905t^2 + 9 = 0}$ Solve.

$t = 3.31 secs$ $s_1 = s_2 = 1.5t + 3 = 7.965m$.

Stone is dropped, after 5secs sound of splash is heard. What is depth of the well. Velocity of sound 340 m/sec.

$S = \frac{1}{2}gt^2$

$\boxed{h_2 = \frac{1}{2}(9.81) \times t_2^2}$ Stone

$\boxed{h_1 = 340(t)_1}$

$\cancel{340(t) = \frac{1}{2}(9.81)(t)^2}$

$h_1 = h_2$

$\boxed{t_1 + t_2 = 5 secs}$

$h = 340(t_1)$ Sound

$4.905t_2^2 = 340t_1$

$4.905t_2^2 = 340(5 - t_1)$

$\boxed{t = 4.68 sec}$

$\sqrt{\frac{2h}{g}} + \frac{h}{340} = 5$

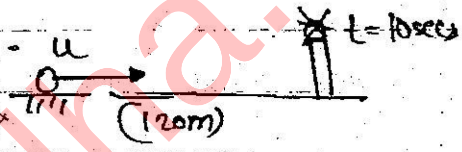
$0.451\sqrt{h} + 0.0029h = 5$

$\boxed{153 - 34\sqrt{h} + h = 1700}$

Motorcist drives a cars by 45 miles/hr. and he observes the traffic light which turns just Red, when the Light is 120m ahead of him and red becomes to green in 10 secs. So He ~~[illegible]~~ wish to pass traffic light without stopping there, ~~diffe~~ determine constant accl^n and Velocity of car at the crossing time.

$u = \frac{\cancel{7.76}}{20.125}$ m/sec. $S = 120m$ $t = 10secs$ (Red → green).

a = constant



$S = ut + \frac{1}{2}at^2$

$120 = 7.76(10) + \frac{1}{2}(a)(10)^2$

$120 = \cancel{[illegible]}\ 20.125(10) + \frac{1}{2}(a)(10)^2$

$-81.25 = \frac{1}{2}(a)(100)$

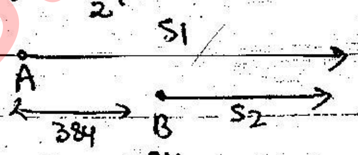
$\boxed{a = -1.62 m/sec^2}$ Retardation

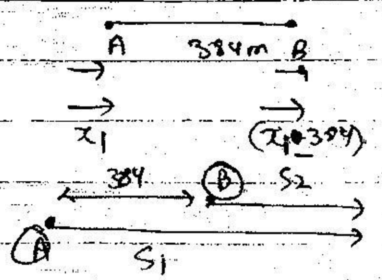
$V = u + at$

$V = 20.125 + (-1.62 \times 10) = 3.875 m/sec = 8.66$ Miles/hr.

Auto (A) is moving with 20 m/sec with constant accl^n. 5m/sec^2 and Auto (B) moving with 60m/sec with constant decel^n 3m/sec^2. Initially B is ahead of A by 384m. How soon will A pass B

$S_1 = 20(t) + \frac{1}{2}(5)t^2$ $S_2 = 60(t) - \frac{3}{2}(t)^2$





$S_1 - S_2 = 384$

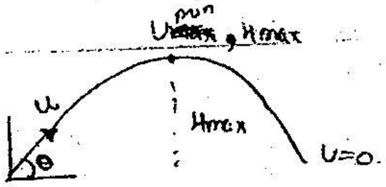
$20(t) + 2.5t^2 - 60t + 1.5t^2 = 384$

$\boxed{t = 16 sec}$

$\boxed{S_1 - S_2 = 384}$

$S_1 = S_2 + 384$

X-direction – Velocity constant
Y-direction – Accl^n constant



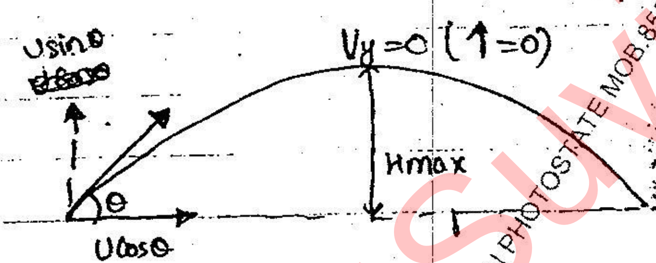
Projectile Motion

Soln divided into 2-parts:-

(1) $\boxed{\text{x-motion (direction)}}$:- Motion with constant Velocity i.e accl^n=0

(2) $\boxed{\text{y-motion (direction)}}$:- Motion with constant accl^n i.e velocity is not constant.

Imp
* Standard formulaes only valid if body moves from ground to ground. If the body stop within any position, then they cannot be used.



x motion

$V_x = U_x + a_x t$

$V_x = U\cos\theta + 0$

$\boxed{V_x = U\cos\theta}$

$S_x = U_x t + \frac{1}{2} \cancel{a_x} t^2$ (→0)

$\boxed{S_x = U\cos\theta\, t}$

At topmost position ($V_y=0$)

$\therefore$ $U\sin\theta - gt = 0$

$\boxed{t = \frac{U\sin\theta}{g}}$ Time to reach Max^m Height

y-motion

$V_y = U_y + a_y t$

$\boxed{V_y = U\sin\theta - gt}$

$S_y = U_y t + \frac{1}{2} a_y t^2$

$\boxed{S_y = U\sin\theta (t) - \frac{1}{2} g t^2}$

$\therefore$ $S_{y(max)}$ (Max^m height)

$S_{y(max)} = U\sin\theta \left(\frac{U\sin\theta}{g}\right) - \frac{1}{2} g \left(\frac{U\sin\theta}{g}\right)^2$

$\boxed{S_{y(max)} = \frac{U^2\sin^2\theta}{2g}}$

Max^m Range: (Vertically) $\Rightarrow$ $\boxed{\delta y = \frac{u^2\sin^2\theta}{2g}}$ → Height

(Horizontal) $\Rightarrow$ $\boxed{\delta x = \frac{u^2 \sin 2\theta}{g}}$

* ~~at the end of flight~~ ($\delta y = 0$).

$\therefore$ $u\sin\theta(t) - \frac{1}{2}g(t)^2 = 0$.

$\boxed{t = \frac{2u\sin\theta}{g}}$ Time of flight (Total time).

The total Time of flight = Twice the time to reach Max^m Height.

$\delta x(max) = u\cos\theta \times t = u\cos\theta \times \frac{2u\sin\theta}{g} = \frac{2u^2\cos\theta\sin\theta}{g}$.

$\therefore$ Total (Horizontal displacemt) $\boxed{\delta x = }$ $\boxed{\delta x = \frac{u^2\sin 2\theta}{g}}$

Find the angle of projection if Max^m Range and Max^m Height are equal.

$$\frac{\cancel{u^2}\sin 2\theta}{\cancel{g}} = \frac{\cancel{u^2}\sin^2\theta}{2\cancel{g}}$$

$$2\sin\theta\cos\theta = \frac{\sin^2\theta}{2}$$

$\boxed{\tan\theta = 4}$ $\boxed{\theta = \cancel{63.47}^\circ \; 75^\circ}$

Find angle of projection when range is max^m.

Range $\delta(x) = \frac{u^2\sin 2\theta}{g}$ (max^m) $\therefore$ $\sin 2\theta = 1$

$\therefore \boxed{\theta = 45^\circ}$

1st projectile is fired by 60°. 2nd projectile is fired by α degree knowing that range of the both projectile are same. Find firing angle of second projectile and Height ratio.

Soln

$R_1 = \frac{u^2\sin 2\theta}{g}$

$R_2 = \frac{u^2\sin 2\alpha}{g}$

$R_1 = R_2$

$\sin 2\theta = \sin 2\alpha$

$\sin(2\times 60^\circ) = \sin 2\alpha$

$0.866 = \sin 2\alpha$

$2\alpha = 60^\circ$

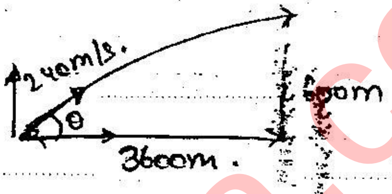
$\boxed{\alpha = 30^\circ}$

$\boxed{\frac{H_1}{H_2} = 3}$

* Projectile is fired from the ground by 240m/s on the target which is located at 600 m above the ground and 3600m Horizontally away from the gun. Determine the firing angle.

$u_x = 240\cos\theta$

$u_y = 240\sin\theta$.



$S_x = 3600m$ $\qquad S_y = 600m$.

$(3600 = 240\cos\theta\,(t) + 0)$

$t = \frac{\cancel{3600}\,30}{\cancel{240}\,\cos\theta} = \frac{\cancel{30}\,15}{\cancel{2}\cos\theta}$

$600 = 240\sin\theta\,(t) - \frac{1}{2}g(t)^2$

$600 = 240\sin\theta\left(\frac{15}{\cos\theta}\right) - \frac{1}{2}g\left(\frac{15}{\cos\theta}\right)^2$

$\overset{40}{\cancel{600}} = 240\frac{\sin\theta}{\cos\theta}(\cancel{15}) - \frac{1}{2}g\left(\frac{15\times15}{\cos^2\theta}\right)$

$\overset{8}{40} = \overset{48}{\cancel{240}}\tan\theta - \frac{1}{\cancel{2}}g\frac{(15)}{\cos^2\theta}$.

$\frac{15}{\cos^2\theta} + 8 = 48\tan\theta$

$15\sec^2\theta + 8 = 48\tan\theta$

$15(1+\tan^2\theta) + 8 = 48\tan\theta$

$23 = 48\tan\theta - 15\tan^2\theta$.

$15\tan^2\theta - 48\tan\theta + 23 = 0$

$(15x^2 - 48x + 23 = 0)$

$x = \frac{48 \pm \sqrt{48^2 - 4(15)(23)}}{2(15)}$

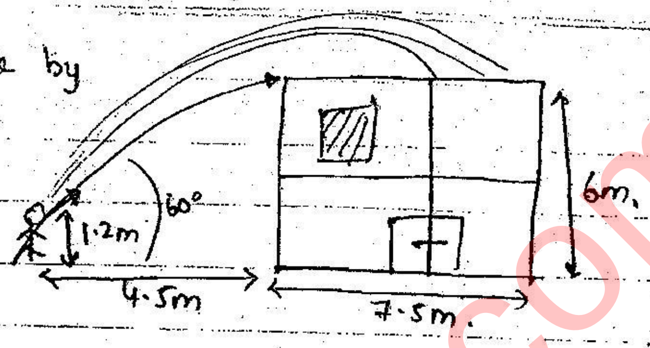
$x_1 \Rightarrow \tan\alpha_1$ $\qquad \alpha_1 =$

$x_2 \Rightarrow \tan\alpha_2$ $\qquad \alpha_2 =$

Both angles

(ii) where it hits on the roof.

Water discharged through the nozzle by 12m/s at 60° with horizontal. Whether the water will clear the edge of roof or not



~~$Sy(max) = \frac{u^2\sin^2\theta}{2g} = \frac{12\times12\times0.75}{2(g)} = 5.0504m$. (Yes) it will hit the wall.~~

(I) For clearing the edge of roof S_x must be $\boxed{4.5m}$ means water need to travel (4.5m) to hit the roof.

x-direction

$S_x = u_x t$

$4.5 = 12\cos(60°)\times t$

$\boxed{t = 0.75sec}$

y-direction

$S_y = u_y(t) - \frac{1}{2}g(t)^2$

$S_y = 12\sin 60°(0.75) - \frac{1}{2}g(0.75)^2$

$\boxed{S_y = 5.03m} > 4.8m$

$\therefore$ Yes

(II) To make the water fall on the roof only, then $S_y = 4.8m$ only.

$\therefore S_y = 4.8$

$4.8 = ut - \frac{1}{2}g(t)^2$

$4.8 = 12\sin 60°\, t - \frac{1}{2}(g)(t)^2$

$4.8 = 10.391(t) - 4.905t^2 \longrightarrow t_1$ and t_2

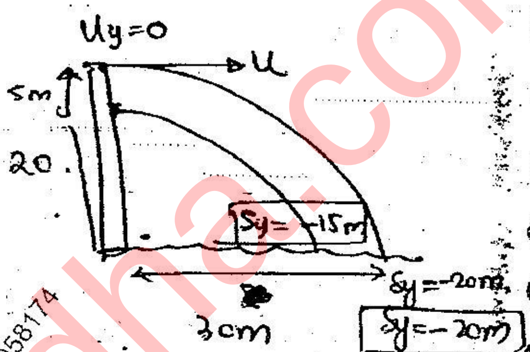
use them to get $S_x = u_x(t)$

$\rightarrow \left.\begin{matrix} S_{x_1} \\ S_{x_2} \end{matrix}\right]$ correct Ans $\boxed{4.5 < S_x < 12m}$

$S_{x_1} = 4.02m$ (X)

$S_{x_2} = 8.6m$ ✓

* Stone throws Horizontally from the bridge 20m High. It hits on the water 30 m away. Second stone throws with same initial velocity from 5 m below the bridge where it hits on the water.



1st case

x-motion

$S_x = 30m$

$S_x = U_x(t)$

$30 = U_x(2)$

$\boxed{U_x = 15m/sec}$

y-motion

$S_y = -20$

$-20 = U_y(t) + \frac{1}{2}a_y t^2$

$-20 = -\frac{1}{2}g(t)^2$

$40 = g(t)^2$

$t = \sqrt{\frac{40}{g}} = 2secs.$

2nd case

x-motion

$S_x = U_x(t)$

$S_x = 15 \times 1.74$

$\boxed{S_x = 26.23m}$

y-motion

$S_y = -15m$

$-15 = 0 - \frac{1}{2}g(t)^2$

$t = \sqrt{\frac{30}{g}} = 1.74 secs.$

~~$s = r\theta$~~

~~$r\theta = \omega_0 r t + \frac{1}{2} r \alpha t^2$~~

$V = u + at$	$S = r\theta$	$\omega = \omega_0 + \alpha t$
$S = ut + \frac{1}{2}at^2$	$V = r\omega$	$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$
$V^2 = u^2 + 2as$	$u = r\omega_0$	$r^2\omega^2 = \omega_0^2 r^2 + 2\alpha r \theta (r)$
	$a = r\alpha$	$\boxed{\omega^2 = \omega_0^2 + 2\alpha\theta}$
Linear motion	Relation	Angular motion

$V = \frac{ds}{dt}$ $\qquad \omega = \frac{d\theta}{dt}$

$a = \frac{dv}{dt}$ $\qquad \alpha = \frac{d\omega}{dt}$

$$\boxed{a^2 = a_x^2 + a_y^2}$$ Linear

So $\boxed{a^2 = a_n^2 + a_t^2}$ Linear

a_n = Normal Accl^n $\quad a_n = \frac{v^2}{r} = r\omega^2 = V\times\omega$

a_t = Tangential Accl^n $\quad a_t = r\alpha$

$\omega = 4\sqrt{t}$ when $t = 1$ sec $\theta = 2$ radians find θ and α when $t = 3$ secs

$$\omega = \frac{d\theta}{dt}$$

$$\omega dt = d\theta$$

$$\int_1^t 4\sqrt{t}\,dt = \int_2^\theta d\theta$$

$$4\times\frac{2}{3}\left[t^{3/2}\right]_1^t = [\theta]_2^\theta$$

$$\frac{8}{3}\left[t^{3/2} - 1\right] = \theta - 2.$$

$$\theta = 2 + \frac{8}{3}t^{3/2} - \frac{8}{3}.$$

$$\frac{t^{1/2+1}}{\frac{3}{2}}$$

$t = 3$ secs $\quad \theta = 2 + \frac{8}{3}(3)^{3/2} - \frac{8}{3} = 2.47$ rad

$$\alpha = \frac{d\omega}{dt} = \frac{d(4\sqrt{t})}{dt} = \frac{+4}{2\sqrt{t}} = +\frac{2}{\sqrt{t}}.$$

$$\alpha = \frac{2}{\sqrt{3}} = +1.15 \text{ rad/sec}^2.$$

Q4. $\alpha = \pi$ rad/sec^2 $\quad \theta = \pi$ rad.

$$\omega = \omega_0 + \alpha t.$$

$$\cancel{2\pi = \omega_0 + (\pi)(t)}.$$

$$\boxed{\omega^2 = \omega_0^2 + 2\alpha\theta}.\quad \omega^2 = \omega_0^2 + 2\alpha(\pi)$$

$$\omega_0^2 = \omega^2 - 2\alpha\pi = (2\pi)^2 - 2(\pi)(\pi)$$

$$= 4\pi^2 - 2\pi^2 = 2\pi^2$$

$$\boxed{\omega_0 = \sqrt{2}\,\pi}$$

x^{x-1} $\quad x.$ $\quad z^x$ $\quad \cancel{x^{x-1}}$ $\quad t^n$ $\quad \frac{t^{n+1}}{n+1}$ $\quad nt^{n-1}$

Q5 $\omega_0 = 27$ rad/s $\quad \alpha = -3t^2.$

$$\omega = \omega_0 + \alpha t$$

$$\cancel{\omega = 27 - 3t^2(t)}.$$

$$\cancel{0 = 27 - 3t^3}$$

$$\cancel{27 = 3t^3}$$

$$\cancel{t^3 = 9}$$

$$t =$$

$$\alpha = \frac{d\omega}{dt}$$

$$-3t^2 = \frac{d\omega}{dt}$$

$$-3t^2 dt = d\omega.$$

$$\left(-\frac{3t^3}{3}\right)_0^t = (\omega)_{\omega_0}^0$$

$$-\frac{3t^4}{4} = 0 - \omega_0$$

$$t^4 = \frac{4(27)}{3}$$

$$\cancel{t^4 = 36}$$

$$t^3 = 27$$

$$\boxed{t = 3\text{ sec}}$$

Q3 $\omega_0 = 3$ rad/sec $\alpha = 30$ cm/sec^2 $= \frac{V^2}{R}$ ~~r = 2cm~~

~~$V = r\omega$~~

~~$V = 3(3) = 9$ cm/sec~~

$(\alpha)^2 = (\alpha n)^2 + (\alpha t)^2$

$a = 30$ cm/sec^2

$a^2 = a_n^2 + a_t^2$ $a_n = r\omega^2 = 2 \times (3)^2 = 18$ cm/sec^2

$\boxed{a^2 - a_n^2 = a_t^2}$ $\boxed{a_t = r\alpha}$

$\therefore \alpha = \frac{a_t}{r}$.

Q6 $\omega = 5$ rev/sec $= 5 \times (2\pi)$/sec $= 10\pi$ rad/sec

$a^2 = a_n^2 + a_t^2$ $a_n = r\omega^2 = 0.1 \times (10\pi)^2 = 10\pi^2$

$a_t = r\alpha = r(0) = 0$.

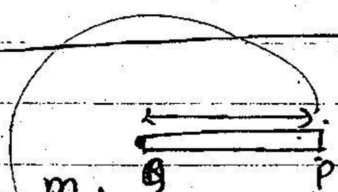
(ω = constant)

$\therefore \alpha = 0$. $\left(\frac{d\omega}{dt} = 0\right)$

$a = \sqrt{a_n^2 + a_t^2}$

$a = \sqrt{a_n^2} = a_n = 10\pi^2$

Q7 $\alpha = 12$ rad/sec^2 $\omega = 4$ rad/sec.



$a_n = r\omega^2$

$a_n = 2 \times (4)^2 = 32$ m/sec^2.

$a = \sqrt{a_n^2 + a_t^2}$ $a_t = r\alpha = 2 \times 12 = 24$ m/sec^2

$= 40$ m/sec^2

~~$a_t = 4$ m/sec^2 $= r\alpha$.~~

$\theta = \left(\frac{3}{4}\right)$

$V_r =$

$\boxed{a_t = 4\text{m/sec}^2}$

~~$a_n = \frac{V^2}{r}$~~

$\boxed{\cancel{\alpha = \frac{4}{2} = 2\text{m/sec}^2}}$

$v = r\omega$

$\omega = \frac{V}{r}$

$= \frac{2}{10}$

$= 0.2$ rad/s

$a = \sqrt{a_n^2 + a_t^2}$

~~$\alpha = 20$.~~

$\omega = r\alpha$

$\alpha = \frac{0.2}{10}$.

if not then can be converted also.

Radius of curvature (only when given in terms of $\hat{i},\hat{j},\hat{k}$).

S: (position) $\Rightarrow (x\hat{i}+y\hat{j}+z\hat{k})$ m

$$v = \frac{ds}{dt} = (v_x\hat{i}+v_y\hat{j}+v_z\hat{k})\ m/sec$$

$$a = \frac{dv}{dt} = (a_x\hat{i}+a_y\hat{j}+a_z\hat{k})\ m/sec^2$$

Tangential component of acc^n. i.e Component of acc^n linear to Linear (or parallel) Velocity.

$$\boxed{a_t = \bar{a}\cdot\hat{v}}$$ (dot product) $\bar{a}$ = Linear acc^n.

Normal acc^n (component) $\Rightarrow$ $\boxed{a_n = \sqrt{a^2 - a_t^2}}$ (But $a_n = \frac{v^2}{R}$)

i.e component of acc^n ($\perp$) to the Linear velocity. also a_n = ~~$(\bar{a}\times\hat{v})$ ...~~

Find radius of curvature at t = 1sec. $\bar{v} = (2t\hat{i}+t^2\hat{j})$ m/sec.

$\bar{v} = 2t\hat{i}+t^2\hat{j}$ when t = 1sec $\bar{v} = 2\hat{i}+1\hat{j}$. $v = \sqrt{5}$ m/sec.

$\bar{a} = 2\hat{i}+2t\hat{j}$ when t = 1sec $\bar{a} = 2\hat{i}+2\hat{j} \Rightarrow a = \sqrt{8}$ m/sec^2.
$= 2\sqrt{2}$ m/sec^2.

$$a_t = \bar{a}\cdot\hat{v} = (2\hat{i}+2\hat{j})\frac{(2\hat{i}+1\hat{j})}{\sqrt{5}} = \frac{4+2}{\sqrt{5}} = \frac{6}{\sqrt{5}}\ m/sec^2$$

$$a_n = \sqrt{a^2 - a_t^2} = \sqrt{8 - \frac{36}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}}\ \text{rad/sec}^2$$

$$a_n = \frac{V^2}{R}.$$

$$\frac{2}{\sqrt{5}} = \frac{5}{R}$$

$$\boxed{R = \frac{5\sqrt{5}}{2}\ m}$$

$V_x = 100 - t^{3/2}$

$V_y = 100 + 10t - 2t^2$

Find principle radius of curvature at the top of the path of particle.

[at the top $V_y = 0$].

$\bar{V} = (100 - t^{3/2})\hat{i} + (100 + 10t - 2t^2)\hat{j}$

$\bar{a} = \left(-\frac{3}{2}\sqrt{t}\right)\hat{i} + (10 - 4t)\hat{j}$ m/sec²

at the to $\bar{v}_y = 0$. $0 = 100 + 10t - 2t^2$ $t = 10$ sec.

at $t = 10$ sec

$\bar{V} = [100 - 10^{3/2}]\hat{i} + [100 + 10(10) - 2(10)^2]\hat{j}$

$= (68.3)\hat{i} + (0)\hat{j} = 68.3\hat{i}$ $\Rightarrow \bar{V} = 68.3$ m/sec²

$\bar{a} = (-4.74\hat{i}) + (-30)\hat{j}$ $\Rightarrow \bar{a} = 30.37$ m/sec

$a_t = \bar{a} \cdot \hat{V} = \frac{(-4.74\hat{i} + -30\hat{j})(68.3\hat{i})}{68.3}$

$a_t = \frac{323.742}{68.3} = 4.74$ m/sec²

$a_n = \sqrt{a^2 - a_t^2} = \sqrt{(30.37)^2 - (4.74)^2} = 29.99$ m/sec².

$29.99 = \frac{V^2}{R}$

$R = \frac{V^2}{30} = \frac{(68.3)^2}{30} = \underline{155.49\,m}$.

Find radius of curvature, particle starts from rest ($u = 0$)

$a_x = 3t$ $a_y = 30 - 10t$ at $t = 2$ secs

$a = \frac{dv}{dt}$

$\vec{a} = \int_0^t 3t\hat{i} + \int_0^t (30 - 10t)\hat{j}$

$\vec{v} = \left(\frac{3t^2}{2}\right)\hat{i} + (30t - 5t^2)\hat{j}$

at $t = 0$ $\vec{V} = 0$

* If particles crosses x-axis again :- put $y=0$ in ~~......~~ position (S) eqn.
* If particle passes y-axis, $\boxed{\text{then put } x=0}$ in (S) eqn and then get the time
* If particle Starts from rest, then put ~~velocity~~ ~~......~~ $\boxed{t=0}$ time

$a_x = -3.6\,m/sec^2$ and $a_y = -6\,m/sec^2$ $\underline{U = 60\,m/s}$ at slope 0.75 upto the right R at the top.

$\bar{a} = (-3.6\hat{i} - 6\hat{j})$

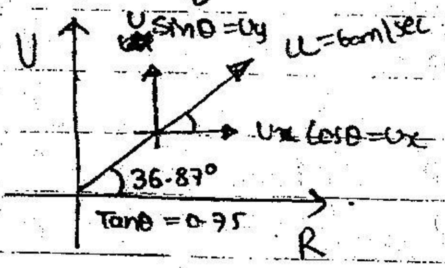
$a = \frac{dv}{dt}$

$\int_{60}^{v} dv = \int_{0}^{t} a\,dt$

$\int_{u_x}^{v_x} dv_x = \int_{0}^{t} a_x\,dt$

$\int_{u_y}^{v_y} dv_y = \int_{0}^{t} a_y\,dt$

$\boxed{v_y = u_y + a_y t}$



$v_x - u_x = a_x t$

$\boxed{v_x = u_x + a_x t}$

$(v_x = 48 - 3.6t)$ $(v_y = 36 - 6t)$

$u_x = u\cos\theta = 48$

$u_y = u\sin\theta = 36$

$\bar{v} = (48 - 3.6t)\hat{i} + (36 - 6t)\hat{j}$

at the top $v_y = 0$

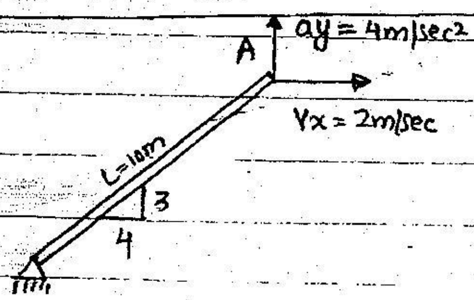
$36 - 6t = 0 \Rightarrow \boxed{t = 6\,secs}$

$\bar{v} = 26.4\hat{i} + 0\hat{j}$ $(a_t = \bar{a}\cdot\hat{v})$

$\bar{a} = -3.6\hat{i} - 6\hat{j}$ $(a_n = \sqrt{a^2 - a_t^2})$

Pg-35

~~Q.~~



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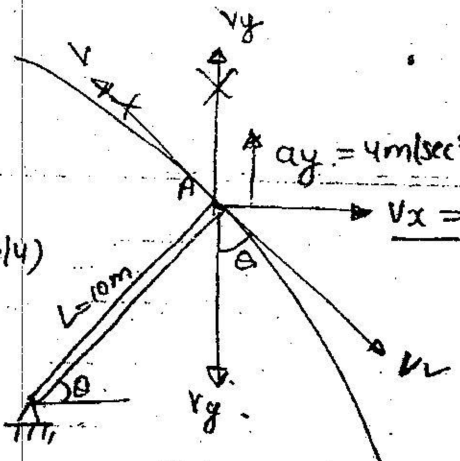


\# $V = \frac{V_x}{\sin\theta} = \frac{2}{\sin(\theta)} = \frac{2}{\sin(3/4)}$

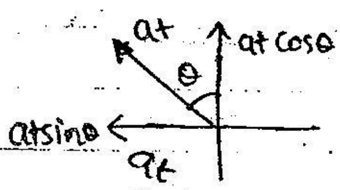
$a_n = \frac{V^2}{R}$

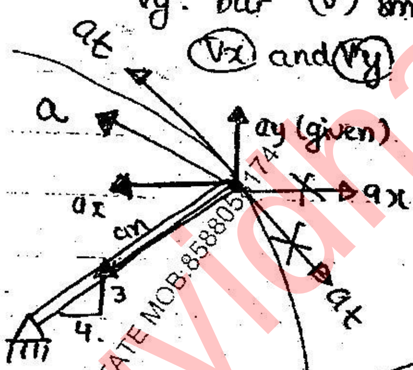
$$\boxed{\left[a_n = \left(\frac{2}{\sin\theta}\right)^2 \times \frac{1}{10}\right]}$$

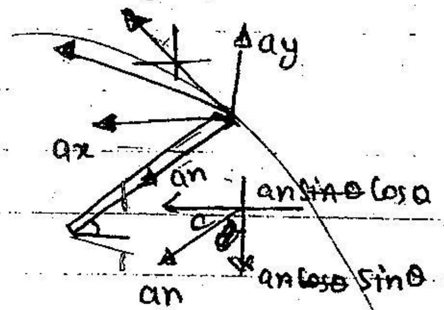
as V is b/w Vx and Vy only.

There are two options of V and Vy. but (V) must always be in (Vx) and (Vy).

$\left(a_n = \frac{3.33^2}{10}\right)$







Here two options of ax and at.

So, we need to choose final (a) such that it should be in b/w $\boxed{a_y, a_x}$ and also $\boxed{a_t, a_n}$.

as $\boxed{a = \sqrt{a_x^2 + a_y^2} \quad \text{or} \quad a = \sqrt{a_n^2 + a_t^2}}$

In y-direction.

$$\boxed{a_y = a_t\cos\theta - a_n\sin\theta}$$

given

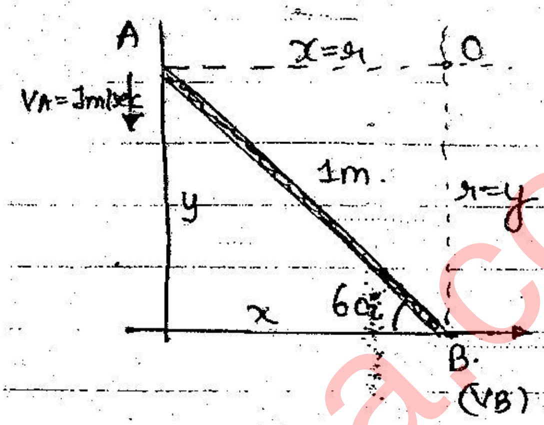
from Here find (at) then $\boxed{a_t = r\alpha}$

then get (α).

$(\alpha = 0.583\ rad/sec^2)$

means
(sab ka malik ek) :- means at rest. (only angular velocity, ~~linear~~ Velocity = 0 ~~at rest~~.)

* INSTANEOUS - CENTRE. (only one instantenous is there always)



V_A (⊥) distance from O = x.

$V_A = x\omega$.

$V_A = 1.\cos 60° \times \omega_{AB}$

$\omega = \frac{1}{\cos 60°} = 2 rad/sec$

V_B (⊥) distance → (y)

$V_B = r\,\omega_{AB}$.

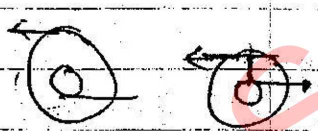
$V_B = (1.\sin 60°)(\omega)$

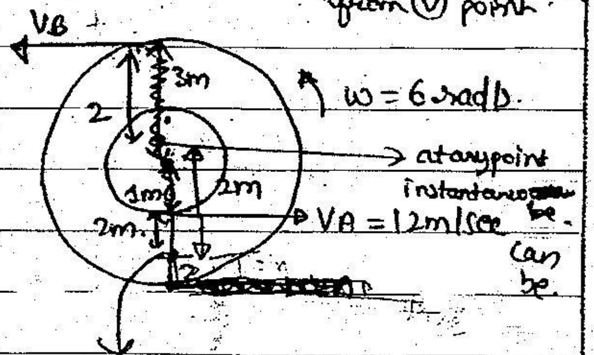
$V_B = \frac{\sqrt{3}}{2} \times (2) = \sqrt{3}\ m/sec$.

To find instantaneous centre, draw the perpendiculars from the points where the Linear Velocities are given.

And [the point where the (⊥)s meet is instaneous centre with distance = r]

$V = r\omega$. r = distance of int. centre from Ⓥ point.





$V_A = r \times \omega$.

$12 = r \times 6$.

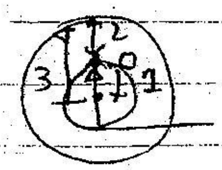
$\boxed{r = 2m} \rightarrow$ (2)m

$V_B = r\omega$

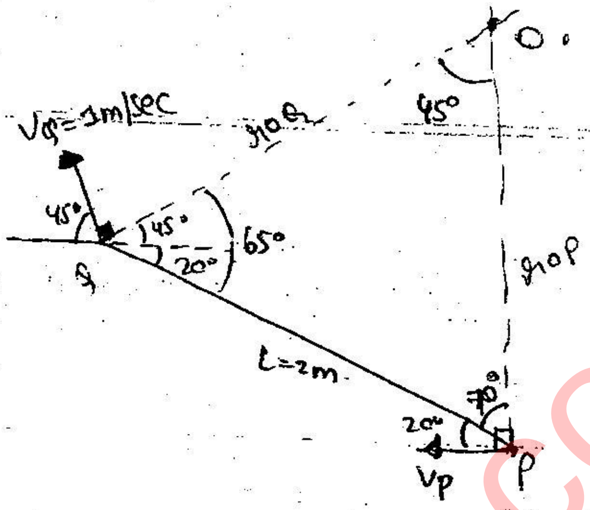
$V_B = 2(\omega)$

[$V_B = 2(6) = 12m/sec$]

Will have two instantaneous centres, one above 2 metres of VA and one below.



So check for the options, which matches



$V_Q = r_{OQ} \times \omega_{PQ}$

$\omega = \frac{V_Q}{r_{OQ}} = 0.377 rads.$

$V_P = r_{OP} \times \omega_{PQ}$

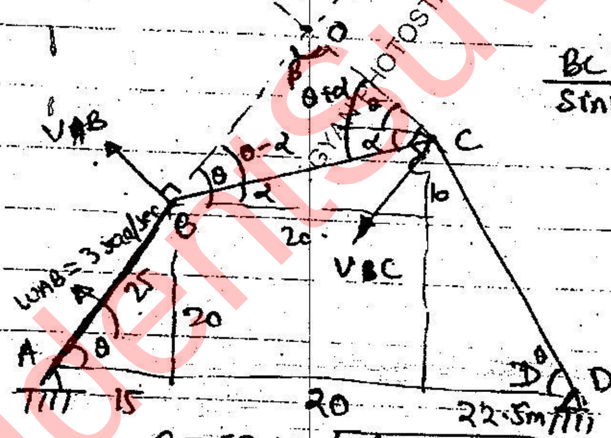
$V_P = 0.377 \times 2.56$

$V_P = 0.966 m/sec.$

$\frac{PQ}{\sin 45°} = \frac{OQ}{\sin 70°} = \frac{OP}{\sin 65°}$

$\boxed{OQ = \frac{2 \times \sin 70°}{\sin 45°}} = 2.65m$

$OP = \frac{2}{\sin 45°} \sin 65° = 2.36m$



$\theta = 53.13$ $\boxed{\alpha = 26.56°}$

$\frac{BC}{\sin \beta} = \frac{OB}{\sin(\theta+\alpha)} = \frac{OC}{\sin(\theta-\alpha)}$

$BC = \sqrt{10^2 + 20^2}$

$BC = ?$

$V_B = r\omega_{AB}$

$V_B = 25 \times 3 = 75 m/sec.$

also $V_B = r_{OB} \cdot \omega_{BC}$

$[75 = r_{OB} \times \omega_{BC}]$

$[\text{get } r_{OB} = OB]$

$V_C = r_{CD} \times \omega_{CD}$

$(V_C = r_{OC} \times \omega_{BC})$