IMPULSE MOMENTUM & WORK ENERGY Method

1) L.O.C.O.M (Law of conservation of motion)

$$\boxed{m_A V_A + m_B V_B = m_A V_A' + m_B V_B'}$$

before impact — after impact

2) Coeff. of restitution

$$\boxed{e = \frac{V_B' - V_A'}{V_A - V_B}}$$

Range of e $\boxed{0 \le e \le 1}$

3) Resultant work = ΔKE

a) For Vertical Motion $V = \sqrt{2gh}$ — h = Max^m position or Max^m Height

b) For Horizontal motion $V = \sqrt{2gh\mu}$

4) $F_{avg} = m \times a = \frac{W}{g}\left(\frac{v' - v}{\Delta t}\right)$ — Δt = Impact lasts = Less in time during motion.

5) Energy Loss in System:- $\boxed{\Delta E = E_1 - E_2}$

$E_1 \Rightarrow$ B.I.K.E (Before Impact K.E) $= \frac{1}{2}(m_A V_A^2 + m_B V_B^2)$

$E_2 \Rightarrow$ A.I.K.E (After Impact K.E) $= \frac{1}{2}[m_A (V_A')^2 + m_B (V_B')^2]$

6) Linear Impulse = change in momentum

$$\boxed{F \times dt = m(v - u)}$$

III If e = 0, then $\boxed{V_B' = V_A'}$ (such impact called as perfectly plastic impact) means one particle embedded or stick to other.

If e = 1, then $\boxed{V_A - V_B = V_B' - V_A'}$

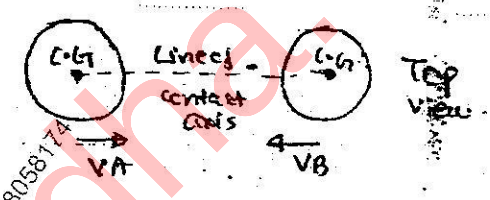
↳ called as perfectly elastic impact

If e > 1, then super elastic impact.

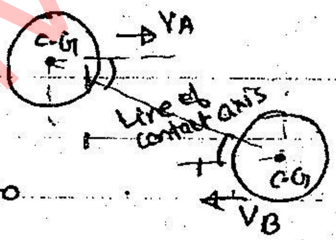
* Coefficient of restitution : The ratio of relative velocity after impact to the relative velocity before impact.

* Types of Impact

(i) Direct Central Impact : When velocity of both particles are collinear to each other (in same line) as well as collinear to line of contact axis (means the line of the C.G of both bodies).



(ii) Indirect / oblique Impact : When the velocity of both or anyone particle are not collinear to line of contact axis.



Soln of the oblique impact are divided into 2 motion

1) Y-Motion :- Motion with constant velocity i.e $V_{AY} = V_{AY}'$ and $V_{BY} = V_{BY}'$

2) X-motion :- To find V_{AX}' and V_{BX}' apply L.C.C.O.M and e formulae.

Therefore

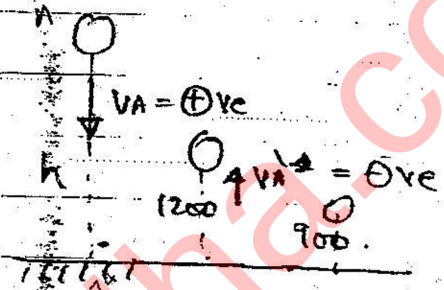
$$\boxed{V_A' = \sqrt{(V_{AX}')^2 + (V_{AY}')^2}}$$

Ball falls freely under gravity through a certain Height and it rebount twice by 1200 mm and 900 mm respectively. Determine certain Height and e.

$\cancel{m_A} V_A = \cancel{m_A} V_A'$ + but the ball does not reach same Height as surface is rigid

$+ m_A V_{A} = m_B V_B'$

$\therefore V_A' \neq V_A$.



$V_B = V_B' = 0$

surface.

$$e = \frac{\cancel{V_B'} - V_A'}{V_A - \cancel{V_B}} = \frac{-V_A'}{V_A} = \frac{V_A'}{V_A}$$

$$e = \frac{V_A'}{V_A}$$

$$e = \sqrt{\frac{2h'g}{2gh}} = \sqrt{\frac{h'}{h}} = \sqrt{\frac{h''}{h'}}$$

$$e = \sqrt{\frac{h'}{h}} = \sqrt{\frac{h''}{h'}} \Rightarrow h = 1600\text{ mm}.$$

$$e = \sqrt{\frac{h'}{h}} = \sqrt{\frac{1200}{1600}}$$

$$e = \sqrt{\frac{3}{4}}$$

Man Walking on the platform by 3 m/sec and he jump on the boat. Determine Velocity of the boat. Man weight = 160 N Boat = ~~6000~~ 640 N.

$m_1 v_1 + m_2 v_2 =$

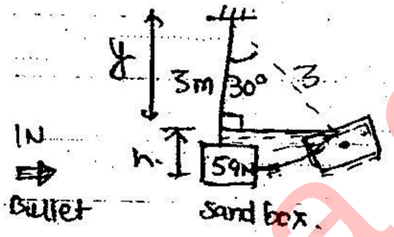
$$m_1 v_1 + m_2 v_2 = (m_1 + m_2) V.$$

$$\frac{160}{10}(3) + 0 = \frac{(160 + 640)}{10}(V)$$

$$48 = 80(V)$$

$$V = \frac{\cancel{48}\ \cancel{12}\ 3}{\cancel{80}\ \cancel{20}\ 5} = 0.6\text{ m/sec}.$$

Bullet is fired horizontally into the sand box which is suspended by the chord. Knowing that sand box is swing through 30° after impact. Determine velocity of the bullet just before impact.



$\frac{1}{2}mv^2 = mgh$

$m_1 v_1 = (m_1 + m_2) v$

$\frac{v_1}{g} = \frac{(59+1)}{g} V$

$\boxed{V = \frac{V_1}{60}}$

$\cos 30° = \frac{y}{3}$

$y = 3\cos 30°$

$\boxed{y = 2.6m}$

$\boxed{h = 3 - y = 0.4m}$

$\boxed{V = r\omega}$

$\boxed{\omega = \frac{d\theta}{dt}}$

$V^2 - U^2 = 2as$

The block moved upto height

$h = 0.4m$

$V = \sqrt{2gh} = \sqrt{2 \times 9.81 \times 0.4}$

$V = 2.801 m/s$

$V = \frac{V_1}{60} \Rightarrow V_1 = 60 \times 2.80 \Rightarrow 168.0 m/sec$

$\boxed{V_B' = \sqrt{2gh} = 2.801 m/s}$ Box

$\boxed{V_B = 0}$ Box

V_A = Bullet

Case 1: If Bullet remains embedded in the sand box.

$m_A V_A + m_B V_B = m_A V_A' + m_B V_B'$

$m_A V_A + 0 = 1 \times V_A' + 59 \times 2.801 \Rightarrow \boxed{V_A = 168 m/sec}$

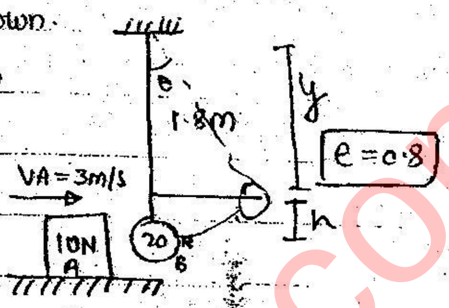
Case 2: If Bullet emerges through the sand box by 1m/sec

$V_A = V_A' + 59 \times 2.804$

The $V_A' = 1 + 2.801$ (after collision bullet speed was $2.801 + 1$).

$\therefore V_A = 3.801 + 59 \times 2.801$

Block A allow to impact on ball B as shown. determine after impact Velocity and max^m position of the block after impact. Also find max^m position of ball.



$$M_A V_A + \cancel{M_B V_B}^{\,0} = M_A V_A' + M_B V_B'$$

$$10 \times 3 = 10 \times V_A' + 20 \times V_B'$$

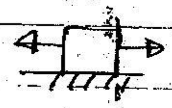
$$\boxed{3 = V_A' + V_B'}$$

$$e = \frac{V_B' - V_A'}{V_A - V_B} = \frac{V_B' - V_A'}{3} = 0.3 \Rightarrow V_B' - V_A' = 0.9$$

$$V_A' + V_B' = 3$$

$$\boxed{\begin{array}{l} V_A' = -0.6\,m/s \\ V_B' = 1.8\,m/s \end{array}}$$

$$F = \mu N = m \times a$$



$$0.2 \times 10 = \frac{10}{g} \times a$$

$$\boxed{0.2g = a}$$

$$\boxed{U^2 = 2as} \Rightarrow \boxed{U^2 = 2 \times 0.2g \times S.}$$

$$\frac{(0.6)^2}{2 \times 0.2g} = S \Rightarrow 0.0918m.$$

$$\boxed{K.E = P.E}$$

$$\frac{1}{2}\cancel{m}V^2 = \cancel{m}gh$$

$$\frac{V^2}{2g} = h$$

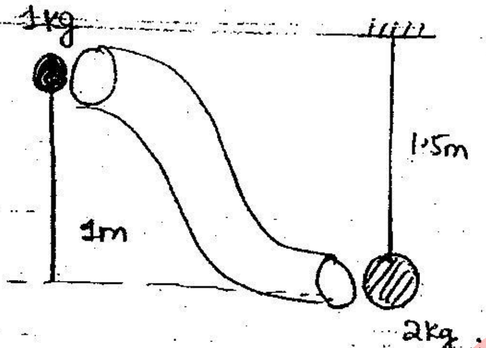
$$\boxed{h = 0.165m}$$

$$y = 1.8 - h = 1.8 - 0.165m =$$

$$\cos\theta = \frac{y_1}{l}$$

$$\boxed{\theta = 24.7^\circ}$$

1 kg ball passes through the bend pipe and allow to impact on 2 kg ball. Find after impact velocity.



$\boxed{e = 0.6}$

$m_1 v_1 + \cancel{m_2 v_2}^{0} = m_1 v_1' + m_2 v_2'$

$1 \times (\sqrt{2gh}) = 1 \times v_1' + 2 v_2'$

$\boxed{\sqrt{2g} = v_1' + v_2'}$

$\boxed{e = \dfrac{v_2' - v_1'}{v_1 - v_2}}$

1 kg ball passes in a Hemisphere and allow to impact on $\frac{1}{3}$ kg ball. Knowing that $\frac{1}{3}$ kg ball just bound over the hemisphere after impact find Height req. through which 1 kg ball is released.

$r = 2m$

$e = 0.9$

$mgh = \frac{1}{2} m v^2$

$v_1^2 = 2gh$

$\boxed{v_1 = \sqrt{2gh}}$

$m_1 v_1 = m_2 v_2' + m_1 v_1'$

$m_1 v_1 = m_1 v_1' + m_2 v_2'$

$\boxed{\frac{1}{3} v_1 = v_1' + 2 v_2'}$

$v_2' = \sqrt{2gh'} = \sqrt{2g(2)} = \sqrt{4g} = 2\sqrt{g}$.

$\sqrt{2gh} = 2\sqrt{g}\left(\frac{1}{3}\right) + v_1'$

$\boxed{e = \dfrac{v_2' - v_1'}{v_1 - v_2}}$

Two cars are moving in opposite direction, 16 and 12 m/sec. Impact on each other if $e = 0.8$, find after impact velocity.

$m_a = 200 kg$ $\quad m_b = 300 kg$.

$m_A v_A + m_B v_B = m_A v_A' + m_B v_B'$

$2\cancel{00}(16) - 3\cancel{00}(12) = 2\cancel{00}(v_A') + \overset{3}{\cancel{300}}(v_B')$

$32 - 36 = 2v_A' + 3v_B'$

$[-4 = 2v_A' + 3v_B']$

$e = \dfrac{v_B' - v_A'}{v_A - v_B}$

$0.8 = \dfrac{v_B' - v_A'}{16 + 12}$

Pg 46 Q2

$\frac{1}{2}Kx^2 = \frac{1}{2g}m(v^2 - \cancel{u^2})$ released $\therefore v = s\sqrt{\frac{Kg}{W}}$

Q3 $x = t + 5t^2 + 2t^3$ $x_1 = 0$ $x_2 = 3 + 45 + 54 = 102$

$v = 1 + 10t + 6t^2$

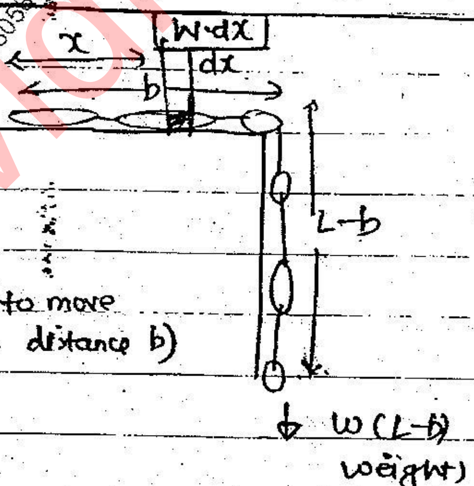
$a = 10 + 12t$ $\vec{a} = \frac{(0+10)+(10+12(3))}{2} = 46$.

$\boxed{F = m \times a = 2 \times 46 = 92N}$

$\boxed{W \cdot D = \Delta K \cdot E = \frac{1}{2}m(v^2 - u^2)}$

Q6

~~Force = $\int_0^b (W \cdot dx) \cdot x$~~



Force $= W \cdot dx$. (of the Shaft).

Work done $= \int_0^b W \cdot dx \cdot x$ (to move distance b)

$\boxed{W \cdot D = \Delta K \cdot E}$

Net work done $= \frac{1}{2}(m)[v^2 - u^2]$

Hanging length moves along length b

Total W·D $\Rightarrow \int_0^b (W \cdot dx) \cdot x + w(L-b) \times b = \frac{1}{2}m(v^2 - \cancel{u^2})$ initial $= 0$

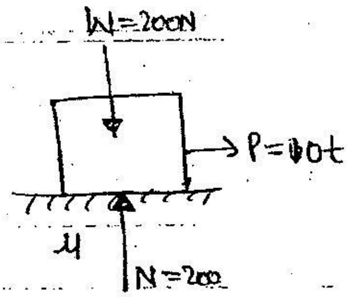
$$\int_0^b (W dx) \cdot x + w(L-b)b = \frac{1}{2}\frac{wL}{g}(v^2)$$

Pg = 40

Q37

$\mu_s = 0.4$

$\mu_k = 0.2$



$F_s = \mu N = 0.4 \times 200 = 80N$

Static $F_s = P = 10t$

$80 = 10t \Rightarrow t = 8 sec$ for 8 secs the body will be at rest.

after 8 secs body will move.

Kinetic $F_K = \mu_K \times N = 0.2 \times 200$
$= 40N.$

$$\int_8^{10} (P - F_K)\,dt = m(v-u)$$

Linear impulse = change in momentum.

$$\int_8^{10} (10t - 40)\,dt = m(v - \cancel{u}) = mv.$$

Pg - 48

Q44

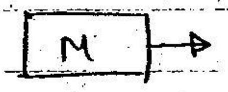
$mv = (m+M)v'$

$$\boxed{v' = \frac{mv}{m+M}}$$

also $v' = \sqrt{2g\mu 4}$.

$$\therefore \left[V = \left(\frac{m+M}{m}\right)\sqrt{2g\mu 4} \right]$$

$F = \mu N$
$= \mu mg.$



~~$\frac{1}{2}mv'^2 = mg s$~~

~~K.E = P.E~~

~~$v' = \sqrt{2gs}$~~

$$\boxed{\begin{array}{l}\frac{1}{2}\cancel{m}v'^2 = \mu \cancel{m}g.s \\ v' = \sqrt{2g\mu s}.\end{array}}$$

$W = mg$

$m = \frac{W}{g}$

Q12 $\frac{1}{2}Kx^2 = \frac{1}{2}m v_A'^2 + \frac{1}{2} m v_B'^2$

$\boxed{Kx^2 = m v_A'^2 + m v_B'^2}$

$m_A v_A$

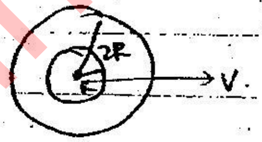
$\boxed{m_A v_A' + m_B v_B' = 0}$

$x = 0.15$

$10.6 \times 10^3 \times (0.15)^2 = \frac{222.4}{g} v_A'^2 + \frac{133.44}{g} . v_B'^2$

$133.44 . (2339.88) = 1.6 v_A'^2 + v_B'^2$

Q9 $K.E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$



$\omega = \frac{V}{\cancel{R}} = \frac{V}{2R} \rightarrow$ outer ~~r~~adius.

$I = \frac{m[(2R)^2 + R^2]}{2}$

$K.E = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{5mR^2}{2}\right) \times \frac{V^2}{4R^2}$

$I = \frac{m[5R^2]}{2}$

$= \frac{1}{2}mv^2 + \frac{5mv^2}{16} = \frac{13mv^2}{16}$

$I_{disc} = \frac{mr^2}{2}$

Q10 Initial momentum = Final momentum.

$mvr = I_c \times \omega$

$\boxed{mvr = \frac{3mr^2}{2} \times \omega}$