FRICTION

Static friction

It is the tangential force which opposes the motion of one body relative to the other when the body is at rest condition.

Its value range from 0 to max^m value.

Limiting friction (F)

It is the max^m value of the static friction when the body is intending tends to move.

Kinetic friction (F_K)

The tangential force which opposes the motion of one body relative to other when the body is in motion. It is always less than static friction.

Coefficient of static friction (μ)

$$\boxed{\mu = \frac{F}{N}}$$ Ratio of limiting friction to the Normal reaction.

Coefficient of Kinetic friction (μ_K)

$$\boxed{\mu_K = \frac{F_K}{N}}$$ Ratio of Kinetic friction to Normal R^n

Angle of Friction (ϕ)

The angle of resultant of Normal and Frictional forces with respect to Normal force.

Angle of Repose (θ)

The max^m Angle of inclined Surface when the system impends.

Belt friction

The friction b/w rope, string and fixed drum

Laws of Friction / Laws of dry friction / Coloumb's friction

(i) Frictional force is independent on the surface contact, however it depends on normal reaction and nature of the surface.

(ii) $\boxed{\mu_s > \mu_k}$

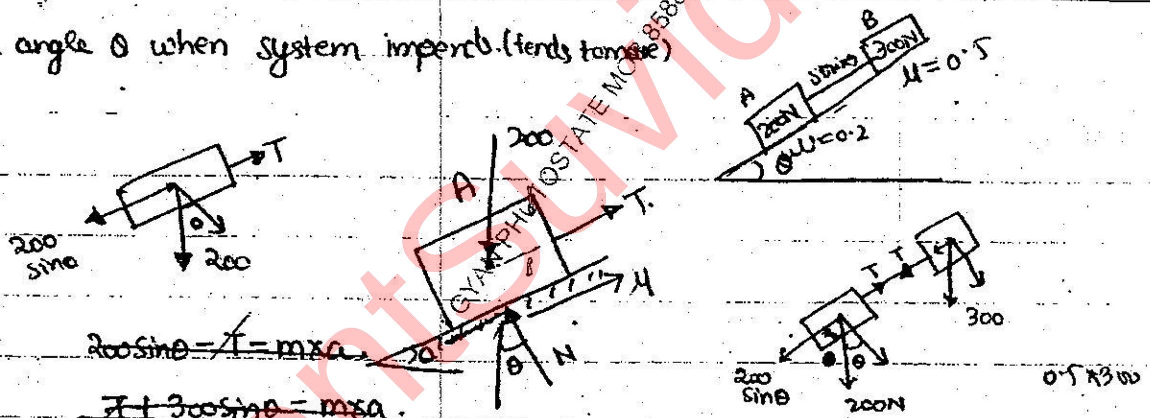
(iii) Speed increases, friction decreases upto a certain limit then it remain constant.

(iv) $\boxed{\mu = \tan\phi}$

(v) Frictional force always opposes the motion of one body relative to the other.

(vi) If the surface is smooth then frictional force is neglected; then in that case, resultant becomes the only Normal reaction.

Q Find angle θ when system impends (tends to move)



$200\sin\theta = \cancel{T} = m\times a$

$\cancel{T} + 300\sin\theta = m\times a$

$500\sin\theta = (m_1+m_2)a$

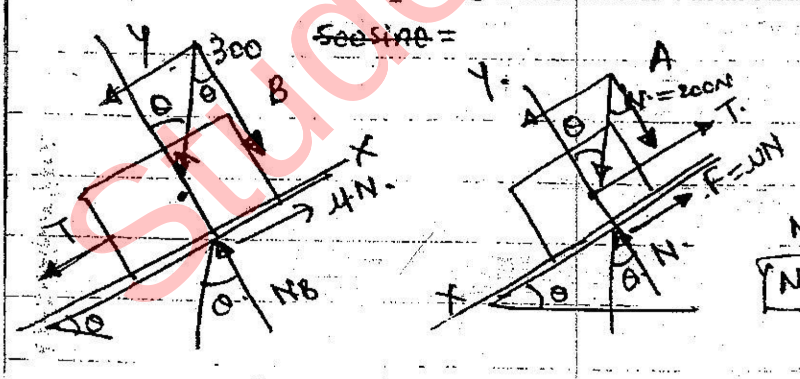
~~$500\sin\theta =$~~

$\mu N - 300\sin\theta = 200\sin\theta - 40\cos\theta$

$500\sin\theta = 150\cos\theta + 40\cos\theta$

$\tan\theta = \frac{190}{500}$

$\boxed{\theta = 20^\circ}$



$N = W\cos\theta$

$\boxed{N = 200\cos\theta}$

$NB = 300\cos\theta$

$T - \mu N + 300\sin\theta = 0$

$T = \mu N - 300\sin\theta$

$T + F = W\sin\theta$

$T = W\sin\theta - \mu N$

$T = W\sin\theta - 0.2\times 200\cos\theta$

$\boxed{T = W\sin\theta - 40\cos\theta}$

The Least value of P for the motion to start is :-

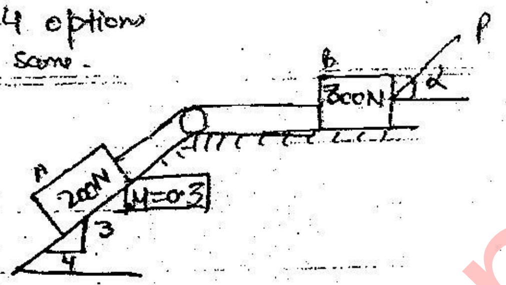
TRICK

$\boxed{\alpha = \tan^{-1}(\mu)}$ only if α has different values (Not for same)

μ for that block in all 4 options

Not if any of the 2 options only are same.

Find least value of P and its direction if the motion of system starts right wards



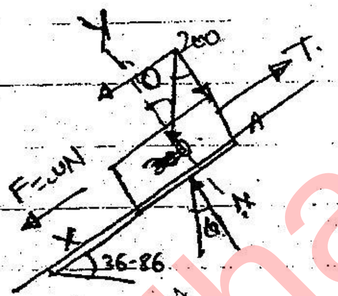
$N = 200\cos\theta$

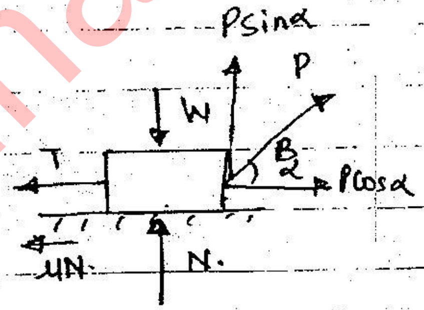
$T - \mu N - 200\sin\theta = 0.$

$T = \mu N + 200\sin\theta$

$T = 0.3(200)\cos\theta + 200\sin\theta$

$T = 60\cos\theta + 100\sin\theta$ — (1)





$\boxed{\cancel{W=N}}$ $\cancel{F=\mu N} = \cancel{\mu W}$

$\boxed{W = N + P\sin\alpha}$ $\boxed{N = W - P\sin\alpha}$

$P\cos\alpha - T - \mu N = 0$

$P\cos\alpha = T + \mu N.$

$P\cos\alpha = T + \mu(W - P\sin\alpha).$

$P\cos\alpha - \mu(W - P\sin\alpha) = T$ — (2).

$\frac{D(\vec{v})}{dt} = \frac{\partial}{\partial t}(\vec{v}\cdot\nabla)\cdot\vec{v}) + \frac{d}{dt}(\vec{v})$

$60\cos\theta + 200\sin\theta = P\cos\alpha - \mu(W - P\sin\alpha).$

$167.97 = P\cos\alpha - \mu W + \mu P\sin\alpha.$

$167.97 + (0.3)(300) = P\cos\alpha + \mu P\sin\alpha.$

$257.99 = P\cos\alpha + 0.3P\sin\alpha.$

$P(\cos\alpha + 0.3\sin\alpha) = 257.97$

$$\boxed{P = \frac{258}{\cos\alpha + 0.3\sin\alpha}}$$

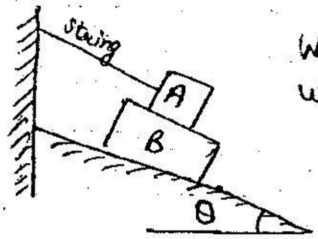
For least value of P_g denominator need to be max:-

$\therefore \cos\alpha + 0.3\sin\alpha = max^m$ (constant) $\therefore \frac{d}{d\alpha}() = 0$

$-\sin\alpha + 0.3\cos\alpha = 0$

$\boxed{\alpha = 16.7}$

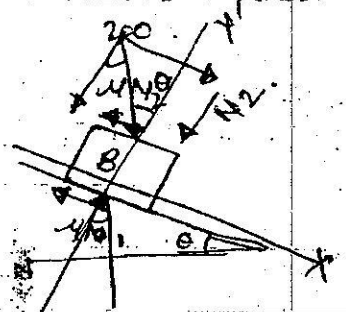
Find angle θ where Block B impends.

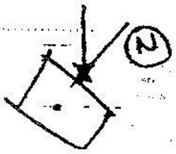


$W_A = 120\,N$

$W_B = 200\,N$

$\mu = 0.25\,N$





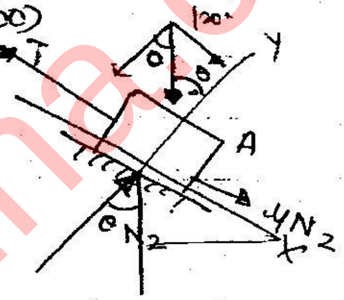
$N_1 = N_2 + 200\cos\theta$

$= (120 + 200)$

$= 320\cos\theta$

$\boxed{\cancel{N_1 = 320\cos\theta}}$

$\boxed{200\sin\theta - \mu N_1 - \mu N_2 = 0}$



$200\sin\theta - \mu(120\cos\theta) - \mu(320\cos\theta) = 0$

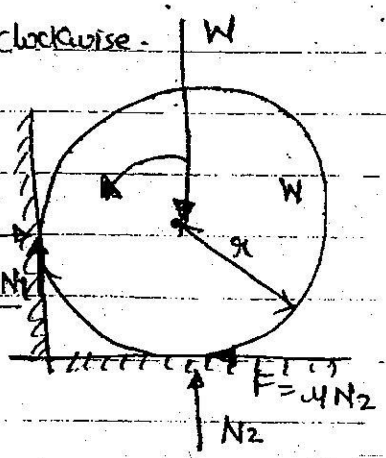
$200\sin\theta = \mu(440\cos\theta)$.

$\tan\theta = \frac{8}{5}(\mu) = \frac{8}{5}(0.25)$

$\boxed{N_2 = 120\cos\theta}$

$\boxed{\theta = 28.80^\circ}$

Find Couple "C" which starts rotating Anti-clockwise.



$\Sigma F_x = 0$.

$N_1 = \mu N_2$

$\Sigma F_y = 0$

$\mu N_1 + N_2 = W$

$\mu(\mu N_2) + N_2 = W$

$\boxed{N_2 = \frac{W}{1+\mu^2}}$

$\mu r(N_1 + N_2) = C$

$\mu r(\mu N_2 + N_2) = C$

$\mu N_2 r(1+\mu) = C$

$\boxed{\frac{\mu(1+\mu)W}{(1+\mu^2)} r = C}$

$\Sigma M(\text{centre}) = 0$ (As friction will stop and impend the motion)

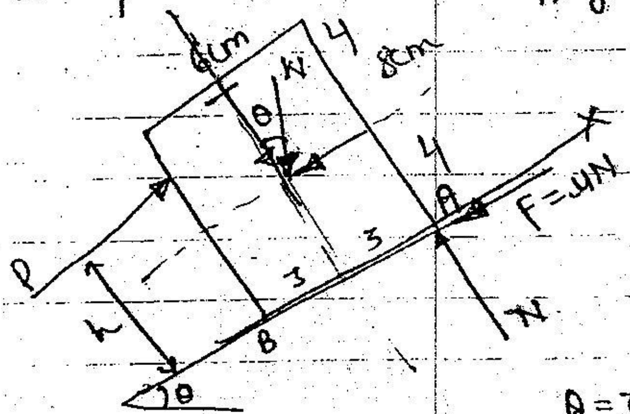
$\mu N_1(r) + \mu N_2(r) = C$

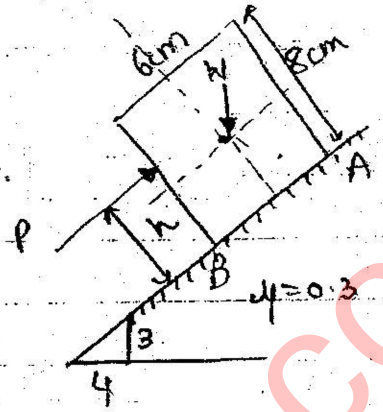
Clock wise   Anti-clockwise

Since motion of the roller is anti-clockwise, then frictional force will acts in opposite direction i.e clockwise.

Whenever the dimension of the block is given, always mark the weight through C.O.G.

Determine greatest height H when the block is slide up the inclined without tipping over.





greatest heigh (h) means the block will tend to tilt at the farthest point (A).

For min^m heights the block will tend to tilt at (B).

$\theta = \tan^{-1}\left(\frac{3}{4}\right)$

$\Sigma F_y = 0$

$N = W\cos\theta$

$\boxed{N = 0.8W}$

$F = \mu N = 0.3 \times 0.8W = 0.24W$

$\Sigma F_x = 0$, $P - F - W\sin\theta = 0$

$\boxed{P = 0.84W}$

$\Sigma M_A = 0$ $\quad P(h) - W\cos\theta(3) - W\sin\theta(4) = 0.$

$P(h) = W\cos\theta(3) + W\sin\theta(4)$

$0.84\cancel{W}(h) = 0.8\cancel{W}(3) + 0.6\cancel{W}(4)$

$h = \frac{4.8}{0.84}$ $\quad \boxed{h = 5.71\text{ cm}}$

BELT FRICTION

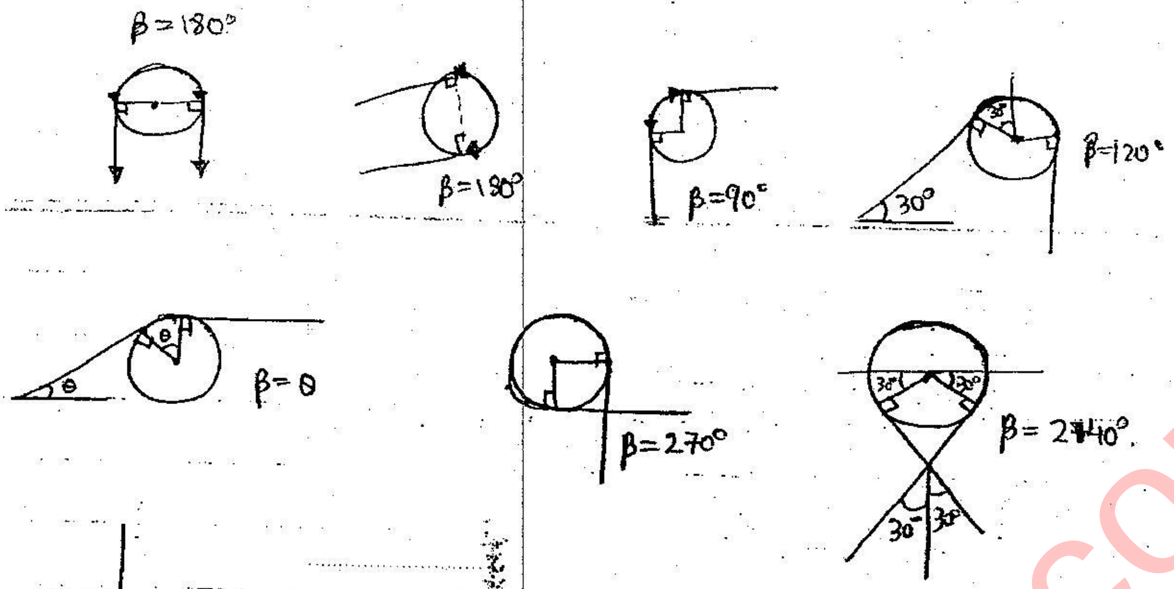
→ if $T_2 > T_1$

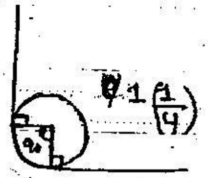
$$\boxed{\frac{T_2}{T_1} = e^{\mu\beta}}$$

β = Total Contact of rope on pulley measured in radians.

if $T_1 > T_2$

$$\boxed{\frac{T_1}{T_2} = e^{\mu\beta}}$$





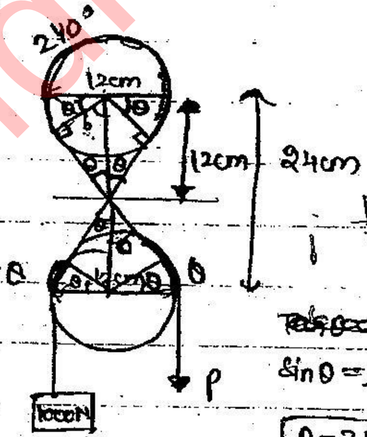
Means ~~[illegible]~~ $360(1) + \frac{360}{4}$

$\boxed{\beta = 360 + 90 = 450}$

$1\left(\frac{1}{4}\right)$ Means one complete revolution of rope $\oplus$ $\left(\frac{1}{4}\right)^{th}$ contact as shown

$\boxed{360 + 90}$

\# Find Max^m and Min^m value of Force P or Find P to start motion and prevent motion. or Find range of P.



$\mu = \frac{1}{\pi}$

$\sin\theta = \frac{6}{12}$

$\boxed{\theta = 30°}$

$\beta = 240 + 30 + 30$

$\boxed{\beta = 300°}$

Total Rope contact with pulleys (1+2)

CASE-1 Pmax

Means $P \geq 1000$

$$\frac{P}{1000} = e^{\mu\beta}$$

$$P = 1000\, e^{\frac{1}{\pi}(300)\left(\frac{\pi}{180}\right)}$$

$$\boxed{P = 5294\,N}$$

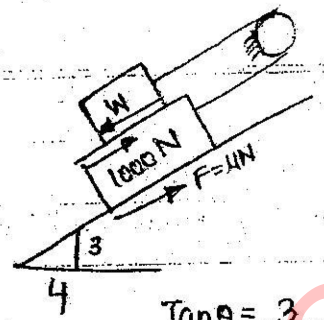
Case 2 Pmin Means $P < 1000$

$$\frac{1000}{P} = e^{\mu\beta}$$

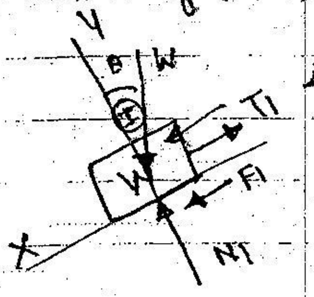
$$P = \frac{1000}{e^{\frac{1}{\pi}(300)(\pi/180)}}$$

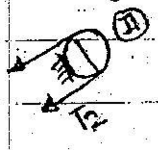
$$\boxed{P = 189\,0N}$$

$\mu = 0.2$ b/w rope and fixed drum at all contact surfaces also.



Find Min weight W to prevent downplay motion of (1000N) body.





$\tan\theta = \frac{3}{4}$

$\theta = \tan^{-1}(3/4)$

$\boxed{\theta = 36.86}$

$N_1 = W\cos\theta = 0.8W$

$F_1 = \mu N_1 = 0.16W$

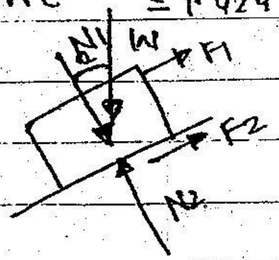
$T_1 = \mu N_1 + W\sin\theta$

$(T_1 = 0.76W)$

$T_2 > T_1$

$\frac{T_2}{T_1} = e^{\mu\beta}$

$T_2 = T_1 e^{\mu\beta} \Rightarrow T_1 e^{0.2 \times 180 \times \frac{\pi}{180}} = 0.76We^{(\ )} = 1.424W$



$F_2 = \mu N_2$

$F_2 = 0.2(800 + 0.8W)$

$F_2 = 160 + 0.16W$

$N_2 = W\cos\theta + N_1$

$N_2 = 800 + 0.8W$

$N_2 = 800 + 0.8W$

$\Sigma F_x = 0$

$F_1 + F_2 + T_2 - 1000\sin\theta = 0$

$0.16W + 160 + 0.16W + 1.429W - 600 = 0$

$\boxed{W = 252N}$

$\boxed{\theta = 2\pi}$ $\boxed{T = 200}$ $\boxed{1000}$ $\beta = \frac{\pi}{4}$

$\frac{P}{T} = e^{\mu\beta}$

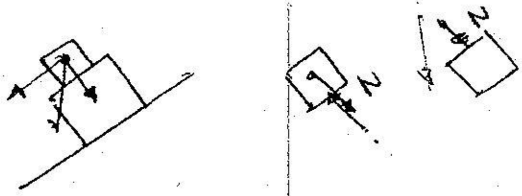
$P = 200\, e^{(0.3)(\pi/4)}$

$P = T$

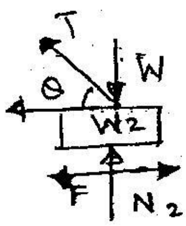
$(P > T)$

$\frac{P}{T} = e^{\mu\beta}$

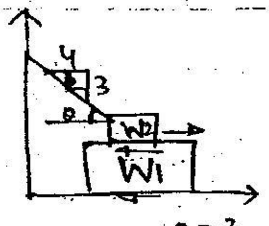
$\beta = 90 \times \frac{\pi}{180} \cdot 2$



Q2



$F = \mu N_2$



$\tan\theta = \frac{3}{4}$

$\boxed{\theta = 36.86^\circ}$

$N_2 = W - T\sin\theta$

$N_2 = 50 - T(0.59)$

$T\cos\theta - \mu N_2 = 0.$

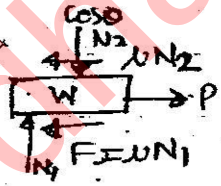
$N_2 = 50 - \mu N_2 (0.73)$

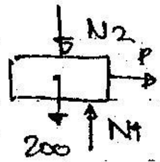
$T\cos\theta = \mu N_2$

$N_2 = \frac{50}{\ }$

$41N.$

$T = \frac{\mu N_2}{\cos\theta}$





~~$N_1 = N_2 = 40.94N$~~ 64.02.

~~$P = \mu N_1 + \mu N_1 = \mu(N_1)(2)$~~

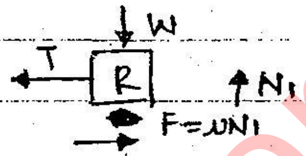
~~$P = 0.3(40.94)(2)$~~

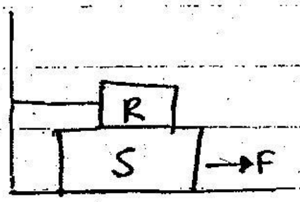
$N_1 = 200 + N_2 = 200 + 41 = 241$

$P = \mu N_1 + \mu N_2$

$= 0.3(241) + 0.3(41) = 84.6\,N.$

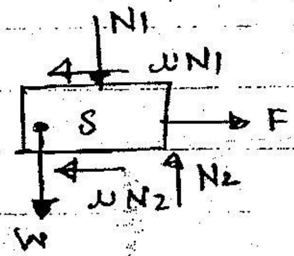
Q3





$N_1 = W = 1000N.$

$T = F = \mu N_1 =$



$N_2 = W + N_1$

$= 150(10) + 1000$

$= 2500N$

$F = \mu(N_1 + N_2) = 0.4(2500 + 1000)$

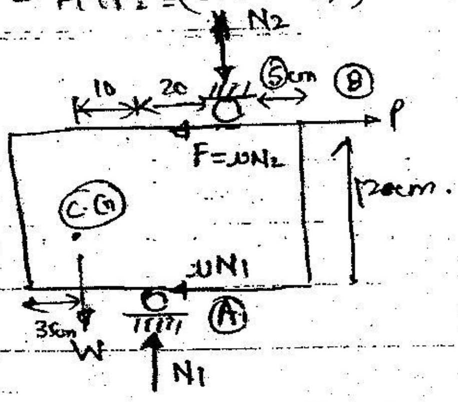
$=$

Q5 | For slipping to take place, Frictional force at both contact surface must be equal (out of both which one is Higher Value)

Q5

$\boxed{F_1 = F_2 = 0.6g}$ $\therefore$ $F = F_1 + F_2 = (0.6 + 0.6)g = 1.2g = \simeq 12N.$

Q4



$\Sigma M_B = 0$

$N_1(20) + \mu(N_1)(12)$

$- 100(30) = 0.$

$N_1(20) + 4N_1 = 100(30)$

$N_1 = \frac{\cancel{100(30)}}{\cancel{24}}$ 125 ... 10 ... 8

$\boxed{N_1 = 125}$

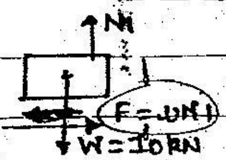
$\Sigma F_x = 0$ $P = \mu(N_1 + N_2)$

$\Sigma F_y = 0$ $N_1 = W + N_2$

$N_2 = 125 - 100 = 25N.$

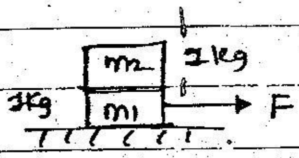
$P = \mu(25 + 125) = \frac{1}{3} \times 150 = 50N.$

Q4.

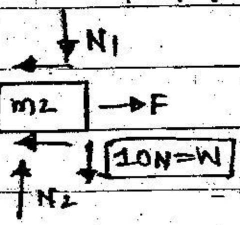


$\boxed{N_1 = W}$

$\boxed{N_1 = 10N}$



$F = \mu N_1 = 0.3(10) = 3N$



For slipping

$\boxed{F > 8.83N}$

$N_2 = 10 + N_1 = \cancel{[illegible]} = \cancel{[illegible]}\ 20N.$ (10+10)

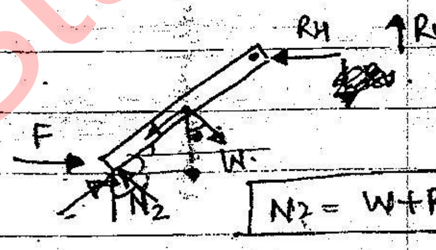
but at rest only

~~$F = \mu(N_1 + N_2) = 0.3(13) = 3.9N$~~

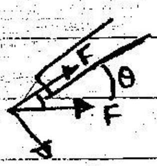
$\boxed{F = 8.83N}$

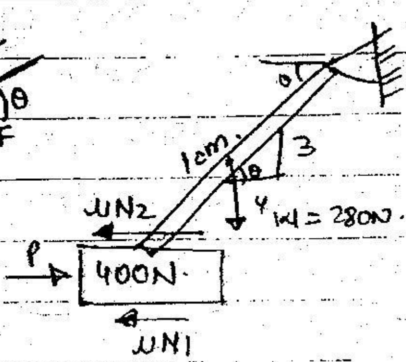
$F = 0.3(10 + 20) = 9N$

Q5



$\boxed{N_2 = W + RV}$





$\Sigma M^{(B)} = 0$

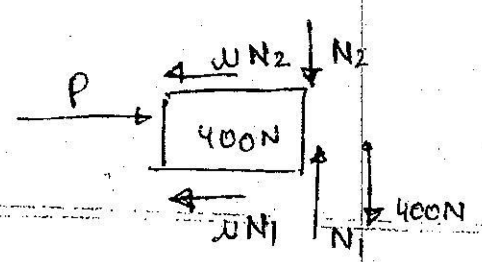
~~$N_2 \cos\theta - W\cos\theta$~~

$N_2 \cos\theta = \frac{W\cos\theta}{2} + F\sin\theta$

$N_2 \cos\theta = W\cos\theta + \mu N_2 \sin\theta$

$N_2(\cos\theta - \mu\sin\theta) = \frac{W\cos\theta}{2} = 200$

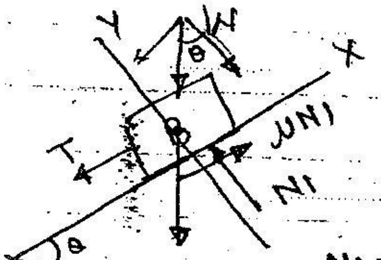
$N_2 = \frac{0.800}{0.56}(W) \Rightarrow 399.82N$

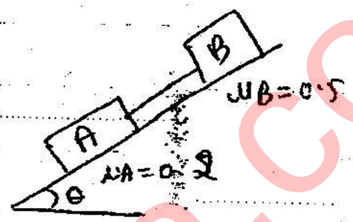


$N_1 = N_2 + 400$
$N_1 = \cancel{B}200 + 400$
$N = \cancel{800} 600N.$

$P = \mu(N_1 + N_2) = 0.4(600 + 200) = 320N.$

Q6.



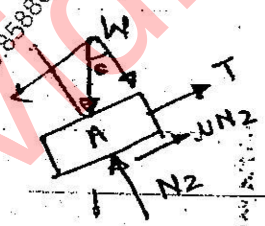


$N_1 = W\cos\theta$

$\boxed{N_1 = 200\cos\theta}$

$T + W\sin\theta = \mu N_1$

$\boxed{T = \mu_B(200\cos\theta) - 200\sin\theta}$



$T + \mu N_2 = W\sin\theta.$

$T = W\sin\theta - \mu N_2$

$N_2 = W\cos\theta$

$W\sin\theta - \mu N_2 = \cancel{W\sin} \mu N_1 + W\sin\theta.$

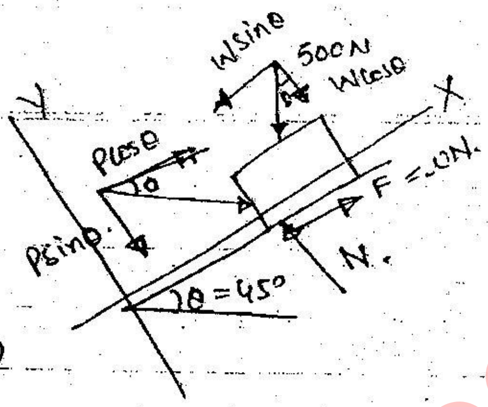
$200\sin\theta - \mu_A(\cancel{W}200\cos\theta) = \mu_B(200\cos\theta) - W\sin\theta$

$400\sin\theta = 400\cos\theta(0.7)\cancel{\cos\theta}$

$\tan\theta = 0.7$

$\boxed{\theta = 35°}$

Q7 P to prevent motio means block coming down. (Friction - up)
P to start motion mean block going up (Friction down)



$\Sigma F_y = 0.$

$N = P\sin\theta + W\cos\theta$

$N = (500 + P)\sin\theta \qquad (\theta = 45°)$

$\Sigma F_x = 0.$

$P\cos\theta + \mu N = W\sin\theta$

$\cancel{P = \frac{W\sin\theta - \mu N}{\cos\theta} = W(\tfrac{1}{\sqrt{2}}) - 0.25 \times [500}$

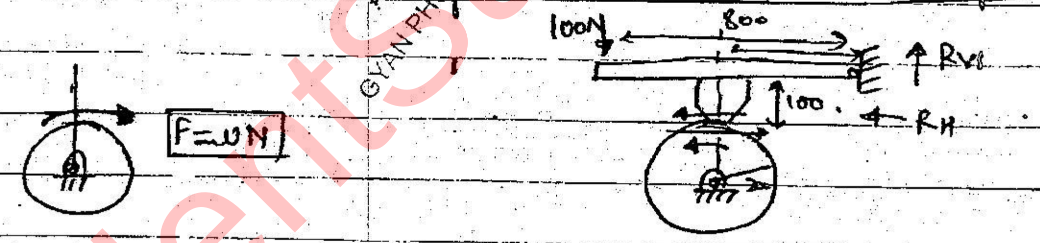
$P\cos\theta + \mu(500 + P)\sin\theta = W\sin\theta$

$P + \mu(500) + \mu P = W$

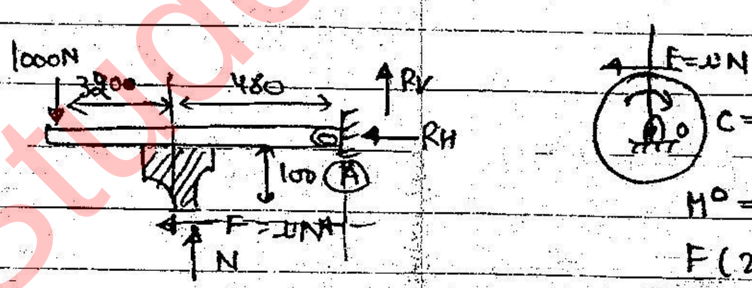
$P(1+\mu) = W - 500\mu$

$P = \frac{W - 500\mu}{1+\mu} = \frac{500(1-\mu)}{(1+\mu)} = 300\,N.$

Q9



$\cancel{1000 = R_V}$



$\cancel{F = \mu N = 1000}$

C = Brake torque.

$M^O = 0$

$F(200) = C$

$C = 320(200)$

$= 64000\,Nmm$

$= \underline{64\,N\text{-}m}$

$M^A = 0 \Rightarrow \quad N(480) + (0.2)N(100) = 1000(800)$

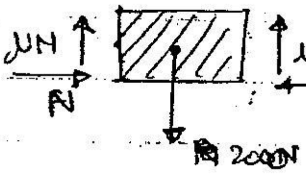
$N(500) = \cancel{1000}^{2}(800)$

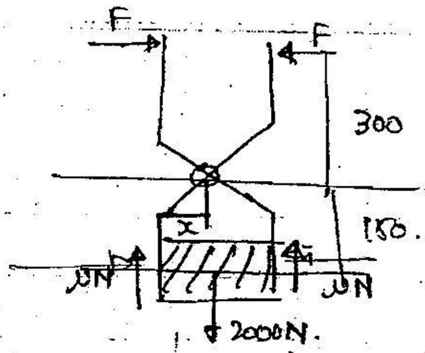
$\boxed{N = 1600\,N}$

$F = \frac{0.2 \times 160\cancel{0}}{1\cancel{0}} = 320\,N$

Since width of given block not given $\therefore x$ is neglected.

Q12





$\Sigma M_{pin} = 0$ $\quad N(150) = F(300) + \mu N(x)$

$N = F(2)$ $\quad$ $\mu N(x)$: Very small (neglected)

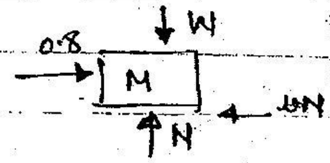
$\boxed{N = 2F}$

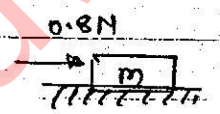
$2000 = 2\mu N$

~~2000~~ 500 $= \cancel{2}\mu(\cancel{2}F)$

$F = \frac{500}{\mu} = \frac{500}{0.1} = 5000\,N$

Q13





$\boxed{F = \mu N}$ is the max^m static frictional force

$\cancel{F = \mu N}$ always take the Min^m F

$\cancel{F = 0.1(10)}$

$\cancel{F = 1N}$

$\boxed{F = 0.98N}$ $(g = 9.81)$

~~0.8 − 0.98 = 0.18N~~. Take the lowest value of (F) $= \mu N$.

$N = 1g$

$\boxed{P = 0.8N}$ applied $\quad$ $\boxed{F = \mu N = 0.8g. = 0.981N} > 0.8N$ $\quad \therefore 0.8N$ can stop the body.

F (from applied) $= 0.8N$

$\therefore$ The actual friction from ΣFx can simply stop the body from moving,

~~$\Sigma Fx = 0.8 - 0.8g$~~

$\Sigma Fx = 0$

$\therefore \boxed{F = 0.8N}$ $\quad (P - F = 0$, $F = 0.8N)$ $\quad \therefore \boxed{F = 0.8N}$