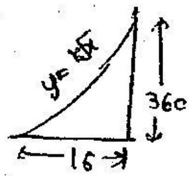
$$\text{Area} = \frac{b \times h}{n+1} = \frac{16 \times 360}{\left(\frac{1}{2}+1\right)}$$



$$\text{Centroid} = \left(\frac{n+1}{n+2}\right)(b) = \frac{\left(\frac{1}{2}+1\right)}{\left(\frac{1}{2}+2\right)} \times 16.$$

CHAPTER-2

Equilibrium of a Co-planar System

When the body have a zero resultant and zero couple then such body is said to be in equilibrium i.e body is at rest condition or having a constant velocity during motion.

Equilibrium Conditions / Principle equation for Co-planar force System

(i) For concurrent force system:-

$\Sigma F_x = 0$ and $\Sigma F_y = 0$ $(R=0)$

(ii) For parallel Force System

$\Sigma F = 0$ and $\Sigma M = 0$ (couple) $(R = \Sigma F_x$ or $\Sigma F_y)$

(iii) Non-Concurrent, Non-parallel force System

$\Sigma M = 0$ (couple) $\therefore$ $\Sigma F_x = 0$ and $\Sigma F_y = 0$ $(R=0)$

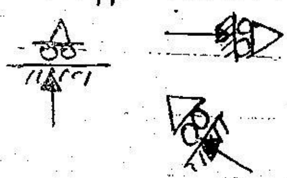
(iv) For point $\Sigma F_x = 0$ ΣF_y (Like concurrent)

(v) For Rigid body $\Sigma F_x = 0$, $\Sigma F_y = 0$ and $\Sigma M = 0$ (Non-concurrent & Non-par)

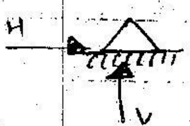
FBD (Free body diagram) :- An Isolated view of a body which shows external forces acting on body including self weight.

FBDS

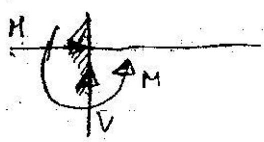
Roller support



Hinged Support



Fixed support



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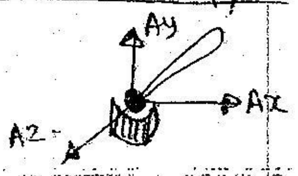
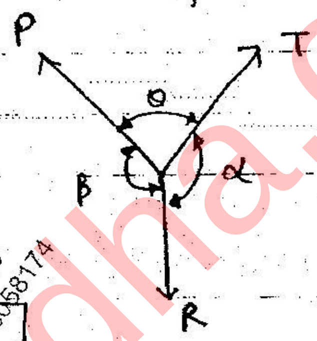
Ball and Socket support



Lami's Theorem (only for 3 coinciding forces) or can be compressive also.

When the system consists of 3 concurrent pulling forces (Tensile forces) and are in equilibrium then the magnitude of anyone force is directly proportional to sine of angle b/w the other two forces.



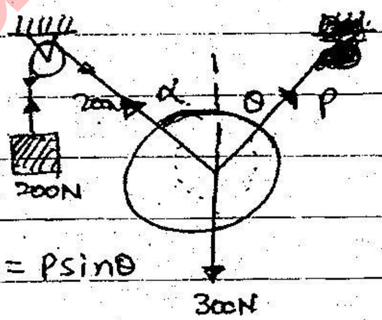
$P \propto \sin\alpha$, $T \propto \sin\beta$, $R \propto \sin\theta$

$P = K\sin\alpha$

$K = \dfrac{P}{\sin\alpha}$ $\quad K = \dfrac{T}{\sin\beta}$ $\quad K = \dfrac{R}{\sin\theta}$

Mathematically :- $\boxed{\dfrac{P}{\sin\alpha} = \dfrac{T}{\sin\beta} = \dfrac{R}{\sin\theta}}$

Find P and θ if $\alpha = 30°$.



$T - mg = 0$

$\boxed{T = 200N}$

$200\cos\alpha + P\cos\theta = 300$ $\qquad 200\sin\alpha = P\sin\theta$

$200\cos\alpha + P\cos\theta = 300$

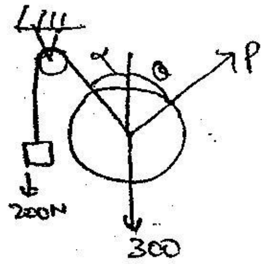
$P\cos\theta = 300 - 200\cos(30)$

$P\cos\theta = 126.79$ $\qquad P\sin\theta = 100N.$

$\tan\theta = 0.78$

$\boxed{\theta = 38.26°}$ $\qquad [P = 162.26N]$

Second part :- If $P = 160N$, Find θ and α.



$300 = P\cos\theta + 200\cos\alpha$

$P\sin\theta = 200\sin\alpha.$

$160\sin\theta = 200\sin\alpha$

$\frac{\sin\theta}{\sin\alpha} = \frac{5\cancel{0}}{4\cancel{0}} = \frac{5}{4}.$

$\sin\theta = \frac{5}{4}\sin\alpha.$

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - (\frac{5}{4}\sin\alpha)^2$

$\cos^2\theta = 1 - \frac{25}{16}$

$\boxed{\cos^2\theta = -\frac{9}{16}\sin^2\alpha}$

$300 = 160\cos\theta + 200\cos\alpha$

$30\cancel{0} = 16\cancel{0}\left(\frac{-9}{16}\sin^2\alpha\right) + 2\cancel{0}\cancel{0}\cos\alpha.$

$30 = -9\sin^2\alpha + 20\cos\alpha$

$20\cos\alpha - 9\sin^2\alpha = 30$

$20\cos\alpha = 9\sin^2\alpha + 30$

$20\cos\alpha = 9(1-\cos^2\alpha) + 30$

$20\cos\alpha = 9 - 9\cos^2\alpha + 30$

$\boxed{\cos\alpha = x}$ 13, 3, 3, 3

$9x^2 + 20x - 39 = 0$

$9x^2 + 26x - 6x - 39 = 0$

$3x(3x+13) - 2(3x+13) = 0$

$x = \frac{2}{3}$ $x = -\frac{13}{3}$

$\boxed{\cos\alpha = \frac{2}{3}}$ $\boxed{\cos\alpha = \frac{-13}{3}}$ $(-1 \le \cos x \le 1)$

$(\alpha = 48.18^\circ)$ $(\alpha = \text{Invalid})$

$\cos^2\theta = -\frac{9}{16}\sin^2 = -\frac{9}{16}\sin^2(48.18^\circ) =$

Method Short

$\sin\theta = \frac{5}{4}\sin\alpha$

$\boxed{\sin\theta = 1.25\sin\alpha}$

$160\cos\theta + 200\cos\alpha = 300$

$\boxed{\cos\theta = 1.875 - 1.25\cos\alpha}$ square and add

$1 = (1.25\sin\alpha)^2$

$+ (1.5625 \times 1)$

$+ 3.5156 - 4.6875\cos\alpha$

$\boxed{\alpha = 29.19^\circ}$

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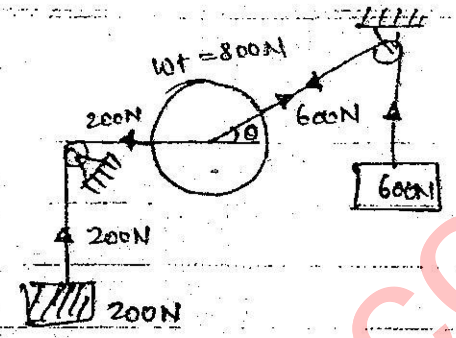
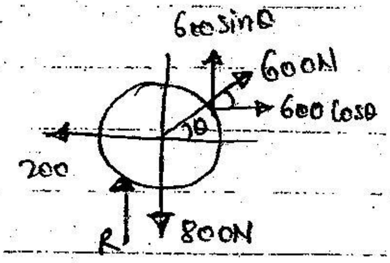
Find reaction b/w cylinder and Surface



FBD



$\Sigma F_x = 0 \Rightarrow 600\cos\theta - 200 = 0$

$\boxed{\cos\theta = \frac{1}{3}} = 70.52^\circ$

$\Sigma F_y = 0 \quad R + 600\sin\theta = 800$

$R = \frac{800}{600\sin(70)} = 234.3\,N.$

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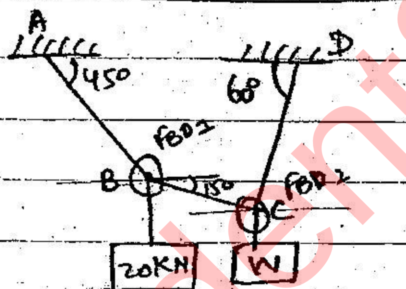
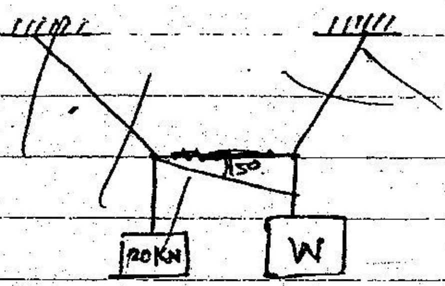
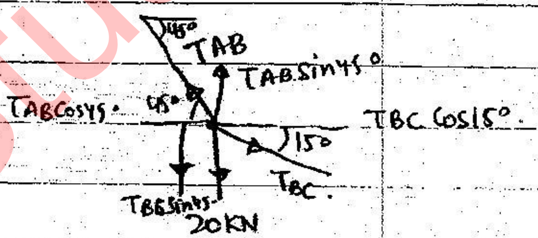
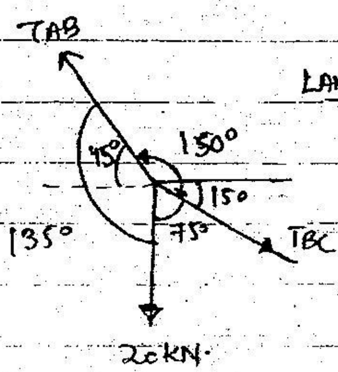




FBD of B point

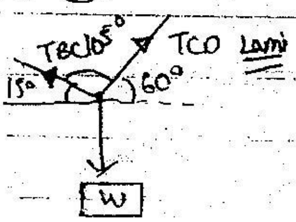




Lami's Theorm

Joint B.

Joint C



$\frac{T_{BC}}{\sin(135)} = \frac{T_{AB}}{\sin 75^\circ} = \frac{20}{\sin(150)}$

$T_{BC} = 20 \times \frac{\sin(135)}{\sin(150)} = 28.2\,KN.$

$\frac{W}{\sin(105)^\circ} = \frac{T_{BC}}{\sin(150^\circ)} \quad \therefore \boxed{W = 54.6\,N}$

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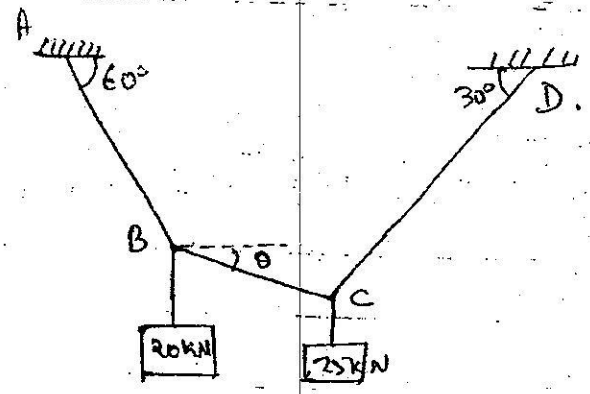
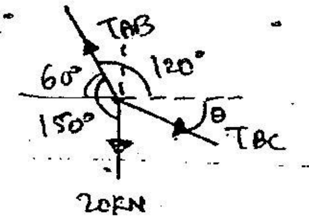
#



Point B



Point C

$(180° - (30+\theta))$

$$\frac{T_{BC}}{\sin(150)} = \frac{20}{\sin(120°+\theta)} \qquad \frac{25}{\sin(150°-\theta)} = \frac{T_{BC}}{\sin(120°)}.$$

$$\frac{20(\sin)(150)}{\sin(120+\theta)} = \frac{25\sin(120)}{\sin(150°-\theta)}$$

$$\frac{10}{\sin(120+\theta)} = \frac{21.65}{\sin(150-\theta)}$$

$$\sin(150-\theta) = 2.16\sin(120+\theta)$$

$$\frac{\sin(150-\theta)}{\sin(120+\theta)} = 2.16$$

$$\frac{\sin 150°\cos\theta - \cos 150\sin\theta}{\sin 120\cos\theta + \cos 120\sin\theta} = 2.16.$$

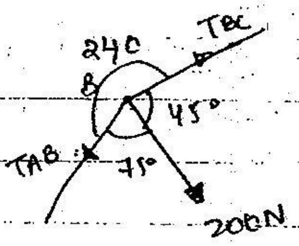
$$\frac{0.5\cos\theta + 0.86\sin\theta}{0.86\cos\theta - 0.5\sin\theta} = 2.16$$

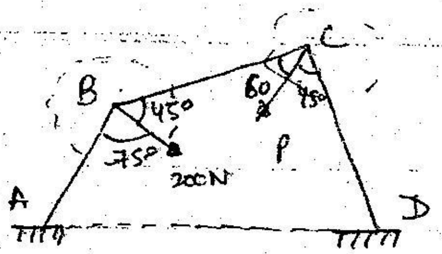
$$0.5\cos\theta + 0.86\sin\theta = -1.08\sin\theta + 1.8576\cos\theta$$

$$1.96\sin\theta = 1.3576\cos\theta$$

$$\boxed{\theta = 34.70}$$

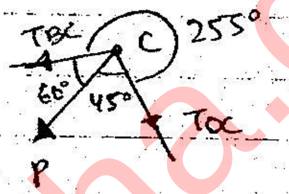
Find P





$$\frac{T_{BC}}{\sin(75°)} = \frac{200}{\sin(240°)}$$

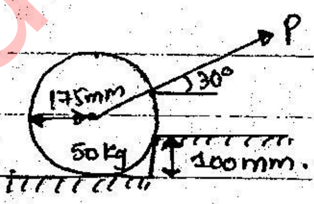
Point C



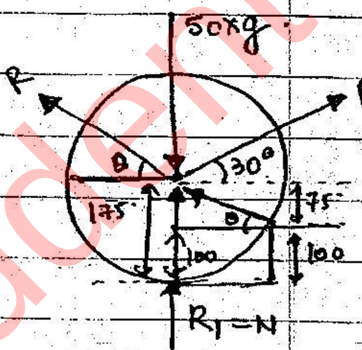
$$\frac{200\sin(75°)}{\sin(240°)} = \frac{P(\cancel{\ })(\sin 45°)}{\sin(\cancel{\ }\,255°)} \qquad \frac{P}{\sin(255°)} = \frac{T_{BC}}{\sin(45°)}.$$

$$\frac{-223.071 \times \sin(255°)}{\sin(45°)} = P \Rightarrow 304.72\,N.$$

Find P when the wheel is just to start over the ~~colour~~ corner



Sol^n

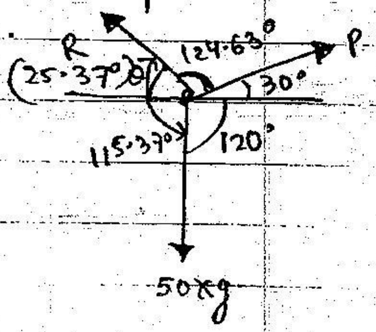


(As the wheel just start to move over corner therefore the Horizontal reaction $R_1 = N$ will be zero.) $(N=0)$

$$\therefore \sin\theta = \frac{75}{175}$$

$$\boxed{\theta = 25.37°}$$

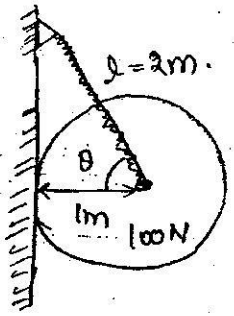
by lami's theorm.

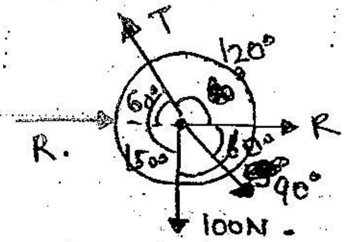


$$\therefore \frac{P}{\sin(115.37°)} = \frac{490.5}{\sin(124.63°)}$$

$$\boxed{P = 538\,N}$$

Find the reaction b/w vertical wall and cylinder.





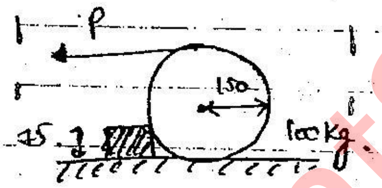
$\cos\theta = \frac{1}{2}$

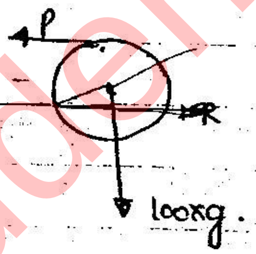
$\boxed{\theta = 60°}$

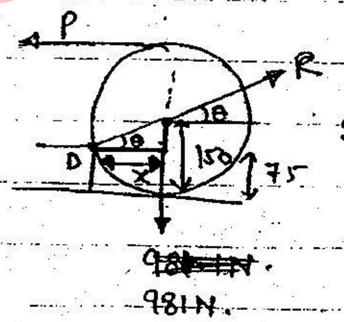
$$\frac{100}{\sin(120°)} = \frac{R}{\sin(150°)}$$

$$R = \frac{100\sin(150°)}{\sin(120°)} = 57.73N$$

Find P when wheel just start over the block.







$\sin\theta = \frac{75}{150} = \frac{1}{2}$

$\boxed{\theta = 60°}$

$\cos\theta = \frac{x}{150}$

$(x = 129.9\ mm)$

Method ①

$\Sigma M_D = 0$

$981 \times (129.9) = P(225)$

$\boxed{P = 566.36\ N}$

Method ② CHARLIES RULE

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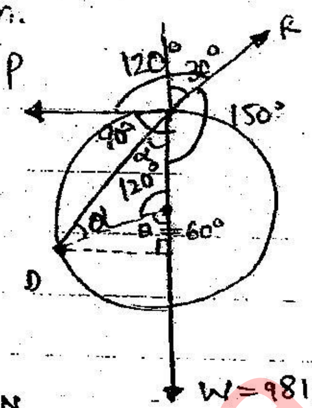
CHARLIES RULE

If the system consists of 3 Non-Concurrent forces, out of 3 one is self weight, then all the three forces must be intersect somewhere to form a concurrent force system.

Method②



$$\therefore \frac{P}{\sin(150^\circ)} = \frac{R}{\sin 90^\circ}$$

$$R = \frac{P}{\sin(150^\circ)} = \frac{W}{\sin(120^\circ)}$$

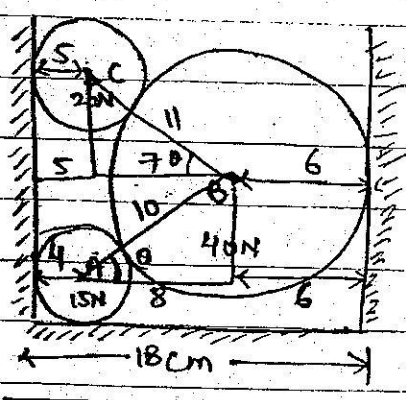
$$P = \frac{981 \times \sin(150)}{\sin(120)} = 566.38\,N$$

Chord attached to centre subtends equal angles.

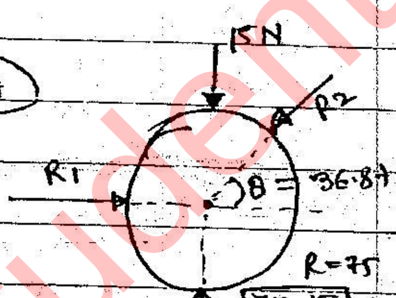
The Resultant (R) will pass through point of the intersection of weight (W) and the applied Load (P).

Thus R passes through (P) & (W) intersection.

Find reaction b/w vertical walls and cylinder A



Ball A



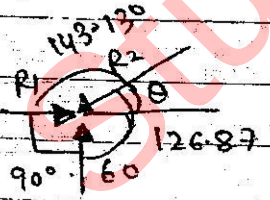
$R = 75$ $W = 15$

$\boxed{75-15}$ $\therefore \uparrow 60$

$\boxed{60N}$

$\cos\theta = \frac{8}{10}$

$(\theta = 36.87)$

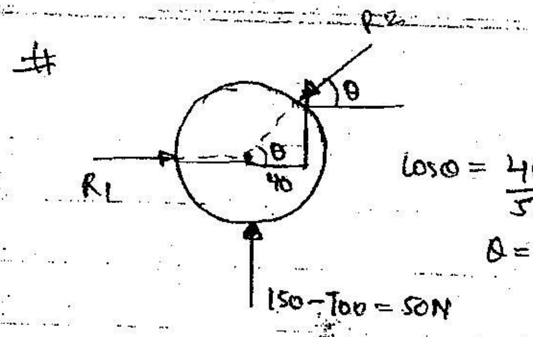


$$\frac{R_1}{\sin(126.87)} = \frac{60}{\sin(143.13)}$$

$$\boxed{R_1 = 80.01\,N}$$

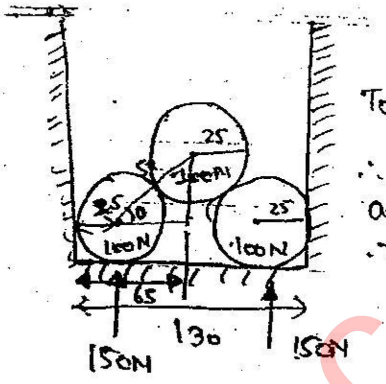
If the vertical boundaries are all straight and (⊥) then FBD of Lowest cylinder can be made directly only then ↓ The total Weight acts in the downward direction ↓ (75N) at Ball A

#



$\cos\theta = \frac{40}{50}$

$\theta = \cos^{-1}(4/5)$



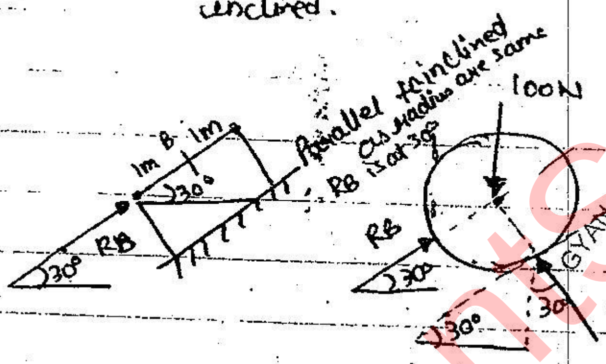
Total W = 300N

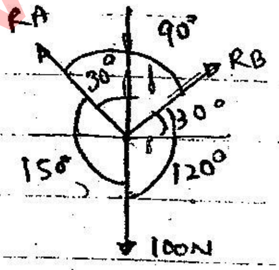
∴ Two balls at end

∴ Divided equally at end balls.

Find reaction at all contact Surfaces.

upper ball FBD first as surface is inclined.

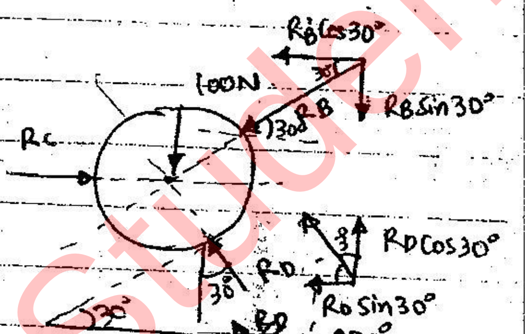


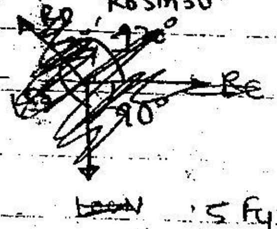


$$\frac{RA}{\sin(120°)} = \frac{100}{\sin 90°} = \frac{RB}{\sin(150°)}$$

$$RA = \frac{\sin 120° (100)}{1} = 86.60$$

$\boxed{RB = 50N}$





4 Forces

Lami's theorem

Not applicable.

~~$\frac{Rc}{\sin 150°} = \frac{100}{\sin 120°}$~~

$\Sigma F_y = 0$ $\quad R_D \cos 30° = 100 + R_B \sin 30°$

$R_D = 144.33$

$\Sigma F_x = 0$ $\quad R_C = R_D \sin 30° + R_B \cos 30°$

$= 115.46N.$

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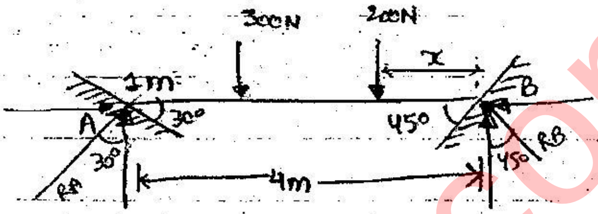
Find distance x to maintain the bar is in position (Equilibrium).

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$



$R_A \cos 30° + R_B \cos 45° = 300 + 200$ (ΣF_y)

$\boxed{0.86 R_A + 0.70 R_B = 500}$

$R_A \sin 30° = R_B \sin 45°$

$0.5 R_A = 0.707 R_B$

$\boxed{R_A = 1.414 R_B}$

$R_A = 366\,N$

$R_B = 258.84$.

$M_0^A = 300(1) + 200(4-x) - R_B \cos 45° (4)$

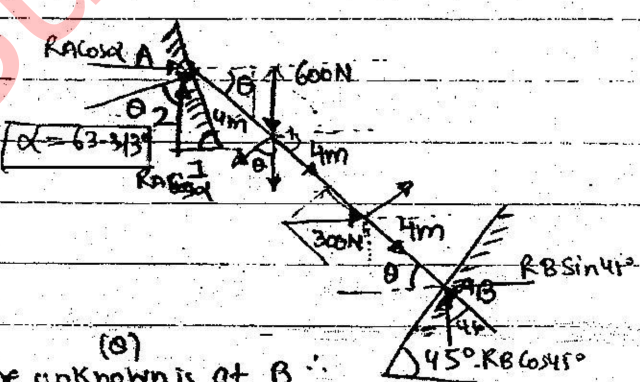
$0 = 300 + 200(4-x) - R_B(2.82)$

$732.11 = 300 + 200(4-x)$

$2.16 = 4 - x$

$\boxed{x = 1.83\,m}$

Find angle θ for Equilibrium.



$\boxed{\alpha = 63.43°}$

$\Sigma F_x = 0$

$R_A \sin\alpha = R_B \sin 45°$

$+ 560$.

$R_A \cos\alpha + R_B \cos 45° = 600$

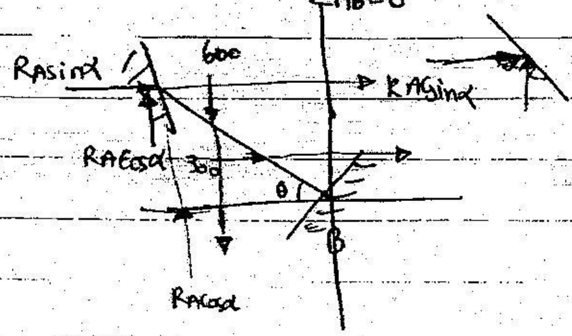
add

$R_A = \dfrac{300}{(\sin\alpha + \cos\alpha)}$

$\boxed{R_A = 223.60\,N}$

The unknown is at B ∴

$\Sigma M_B = 0$



$\Sigma M^B = 0$

$R_A \sin\alpha (12 \sin\theta) + R_A \cos\alpha (12 \cos\theta)$

$- 600(\cos\theta)(8) + 4 \sin\theta (300) = 0$

$3599.82 \sin\theta - 3596.89 \cos\theta = 0$

$\boxed{\theta = 44.27°}$

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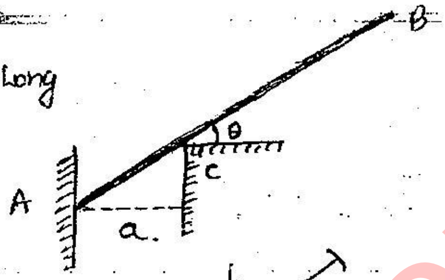
* If there is Hinge Support Always apply $\boxed{\text{moment}}$ term first.

~~Beca~~ Uniform Bar of mass m Kg, L metre Long rests as shown. Find angle θ.



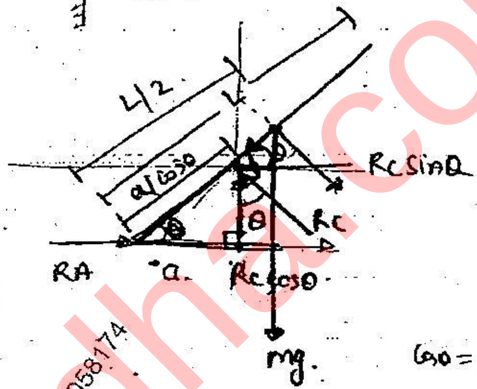
$\Sigma F_x = 0$

$\boxed{R_A = R_C \sin\theta}$

$\Sigma F_y = 0$

$-Mg + R_C \cos\theta = 0$

$\boxed{R_C \cos\theta = Mg}$



$\cos\theta = \frac{a}{x}$

$\Sigma M_O^C = 0$ (as θ is at C)

~~mg cos~~ $mg\left[\frac{L}{2\cos\theta} - a\right] - R_A(a\tan\theta)$

mg (x)

$\left(\cancel{\frac{L}{2} - \frac{a}{\cos\theta}}\right)$

$\left(\frac{L}{2\cos\theta} - a\right)$

$mg\left[\frac{L}{2\cos\theta} - a\right] = R_A(a\tan\theta)$

~~Sinθ~~ $\tan\theta = \frac{bc}{a}$

$mg\left[\frac{L}{\cos\theta} - a\right] = Mg\tan\theta(a\tan\theta)$

---x---

(2) $\Sigma M_A = 0$ (Shortcut)

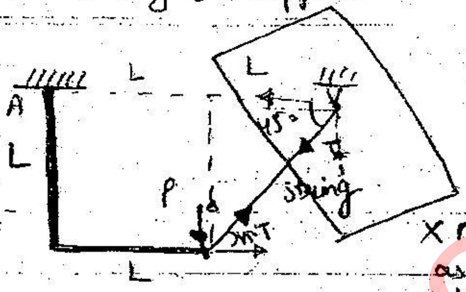
$mg \times \left(\frac{L}{2}\cos\theta\right) = R_C \times \left(\frac{a}{\cos\theta}\right)$

$\cos^3\theta = 2a/l$

$\boxed{\cos\theta = \sqrt[3]{2a/l}}$

Beam :-		~~Two~~ 1 Hinged support and 1 roller
Boom :-		1 Hinged and one free
Bar :-		Both ends resting on supports.

Find tension in the string



X neglect
as tension
will be same
at all Location.

$T\sin45°(2L)$

$- T\sin45°(L)$

$\Sigma M_A = 0.$

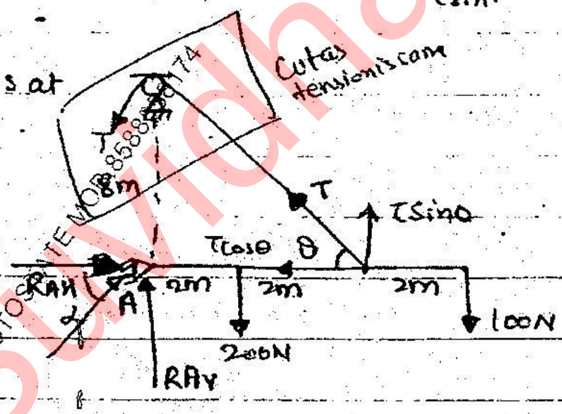
$P(\cancel{L}) - T\sin45°(\cancel{L}) - T\cos45°(\cancel{L}) = 0.$

$P - \frac{2T}{\sqrt{2}} = 0$

$P = \frac{T\sqrt{2}}{\cancel{\sqrt{2}}}$

$\boxed{T = \frac{P}{\sqrt{2}}}$

Find Horizontal & Vertical components at Hinge support A.



$\tan\theta = \frac{8}{4} = 2$

$\boxed{\theta = 63.43°}$

$\Sigma M^A = 0$

$200(2) + 100(6) - T\sin\theta(4) = 0.$

$\boxed{T = 279.52N}$

Hinge used
∴ Use Moment
eqⁿ first

$\Sigma F_y = 0 \quad T\sin\theta - 200 - 100 + RA(V) = 0$

$\boxed{RA(V) = 50N}$

$\Sigma F_x = 0 \quad RA(H) = T\cos\theta$

$\boxed{RA(H) = 125.026\ N}$

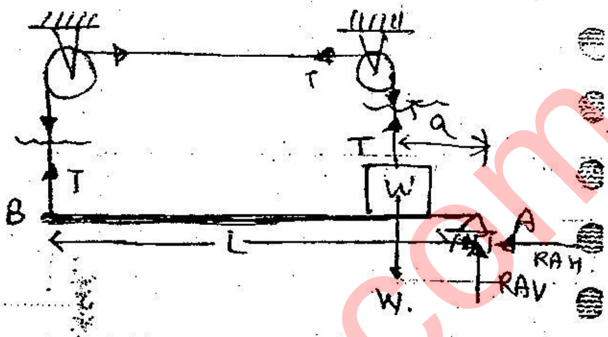
$R_A = \sqrt{V^2 + H^2} = 134.62N$

Reaction of (A)

$\alpha = \tan^{-1}\left(\frac{V}{H}\right) = 21.79°$

Tension in string remains constant throughout if pulley is not having restriction like string is not lifting a pulley.

Find reaction at A.



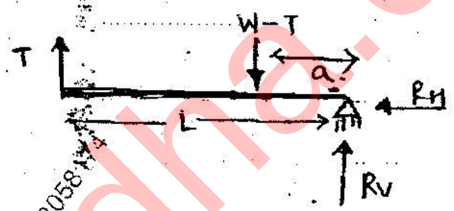
$\Sigma M^A = 0.$

$T(L) - (W-T)a = 0$

$T(L) = (W-T)a.$

$TL = Wa - Ta$

$$\boxed{T = \frac{Wa}{L+a}}$$



$\Sigma F_x = 0 \qquad R_H = 0$

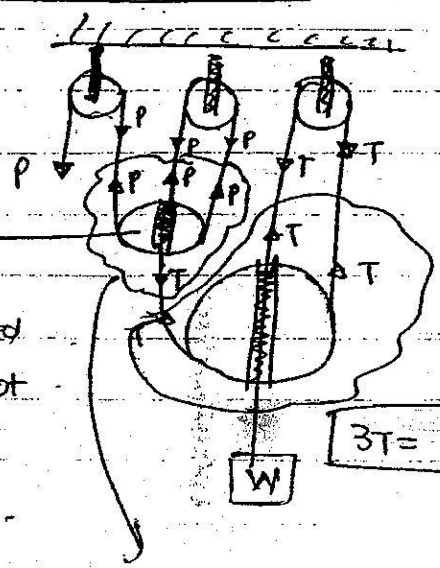
$\Sigma F_y = 0 \qquad R_V = W - T - T$

$R_V = W - 2T$

$$R_V = W - \frac{2Wa}{L+a} = \frac{WL + Wa - 2Wa}{L+a} = \frac{WL - Wa}{L+a} = \frac{W(L-a)}{L+a}$$

$$\boxed{R_N = \sqrt{R_V^2 + R_H^2} = \frac{W(L-a)}{(L+a)}}$$

If W = 720 N and weight of each pulley 36 N. Find P.



Here the string is not continued ∴ (P) will not be same in lower pulley.

$$\boxed{3T = W + 36}$$

$3P = T + 36$

$$\boxed{P = 96 N}$$

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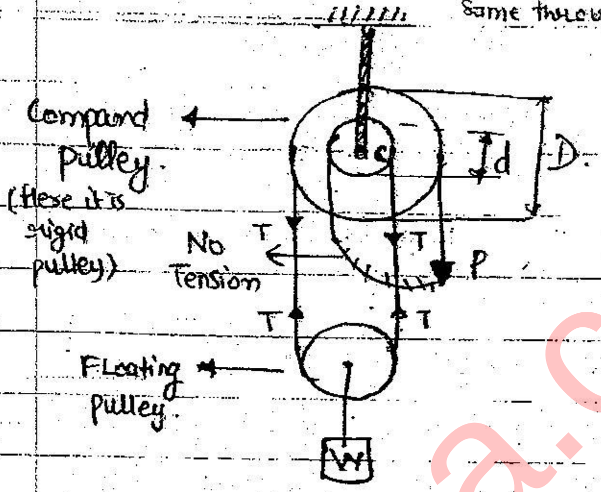
Find W in terms of P

Here compound pulley is rigid so the tension of the string won't be same throughout



$\Sigma M_C = 0$

$-T \times \frac{D}{2} + T\frac{d}{2} + P(D/2) = 0$

$$\boxed{T = \frac{PD}{D-d}}$$

$W = T + T = 2T = \frac{2PD}{D-d}$.

Floating pulley always is frictionless pulley means the pulley also moves along with the string.

If the pulley is rigid then pulley won't moves with string, that pulley is not frictionless.

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