Roll No. ....

91529

B. Sc. 2nd Semester Physics (Hons.)
(New Scheme)
Examination – May, 2016

MATHEMATICS - II

Paper : Phy-204

Time : Three Hours] *[Maximum Marks : 40*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting at least *two* questions from each Unit. All questions carry equal marks.

UNIT - I

1. (a) Discuss the continuity of the function $f(x)$ at $x = 0$, where :

$$f(x)=\begin{cases} \dfrac{x-|x|}{x}, \text{when } x \neq 0 \\ 2, \text{when } x=0 \end{cases}$$

(b) If a function f is continuous on a closed interval $[a, b]$, then prove that it is uniformly continuous on $[a, b]$.

2. (a) Show that for every value of x,

$$\sin x = x - \frac{x^3}{\lfloor 3} + \frac{x^5}{\lfloor 5} + \ldots\ldots\ldots\ldots + (-1)^{n-1}\frac{x^{2n-1}}{\lfloor 2n-1} + (-1)^n\frac{x^{2n}}{\lfloor 2n}\sin\theta x, 0 < \theta < 1.$$

(b) Find the approximate value of $\sqrt{17}$ to four decimal places by taking the first three terms of a Taylor's expansion.

3. (a) Show that the function :

$$f(x,y) = \begin{cases} \dfrac{2x^4 + 3y^4}{x^2 + y^2}, (x,y) \neq (0,0) \\ 0, \qquad (x,y) = (0,0) \end{cases}$$

is continuous at the point (0, 0)

(b) State and prove Schwarz theorem

4. (a) If a function f is continuous at a, then prove that $|f|$ is also continuous at a, but converse is not true.

(b) The function defined by

$$f(x) = \begin{cases} x^2 + 3x + a, if\ x \le 1 \\ bx + 2, \quad if\ x > 1 \end{cases}$$ is given to be derivable for every x. Find a and b.

UNIT - II

5. (a) Show that the function f defined by $f(x) = x, x \in [0, 1]$ is integrable and $\int_0^1 f(x)\,dx = \frac{1}{2}$

(b) If f is a bounded function on $[a, b]$ then prove that for each $\in > 0$, there corresponds $\delta > 0$, such that :

$$L(f,P) > \int_a^b f dx - \in$$

6. (a) Prove that a bounded function having a finite number of points of discontinuity on $[a, b]$ is integrabel on $[a, b]$.

(b) Evaluate $\int_1^3 (x^2 + 2x + 3)\,dx$ by using the limit of Riemann sums.

7. (a) State and prove the first mean value theorem of integral calculus.

(b) Prove that $\frac{1}{2} \le \int_0^1 \frac{dx}{\sqrt{4 - x^2 + x^3}} \le \frac{\pi}{6}$.

8. (a) Evaluate $\int \frac{dx}{a^2 - x^2}$.

(b) Find the reduction formula for $\int \sec^n x\ dx$ and evaluate $\int \sec^6 x\ dx$.
