(b) If $\vec{f} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$; evaluate $\iint_S \vec{f}\cdot\hat{n}\,dS$, where S is the surface of the cube bounded by $x = 0, x = 2, y = 0, y = 2, z = 0, z = 2$. 6

8. (a) Prove that $\iiint_V \frac{1}{r^2}\,dV = \iint_S \frac{\vec{r}\cdot\hat{n}}{r^2}\,dS$, where $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$. 6

(b) State and prove Stoke's Theorem. 6

SECTION – V

9. (a) Define divergence of a vector function. $2 \times 6 = 12$

(b) Find unit tangent vector to any point on the curve $x = a\cos t, y = a\sin t, z = bt$.

(c) Find the value of λ so that the following vectors are coplanar :

$\vec{a} = 2\hat{i} - 7\hat{j} + \lambda\hat{k},\ \vec{b} = \hat{i} + 2\hat{j} - \hat{k},\ \vec{c} = 3\hat{i} - 5\hat{j} + 2\hat{k}$.

(d) Write position vector of a point (x, y, z) in cylindrical polar co-ordinates.

(e) Give the statement of Gauss-Divergence Theorem.

(f) Find the volume element dV in cylindrical co-ordinates.

Roll No. ....

91556

B. Sc. 2nd Sem. (Mathematics) (Hons.) (Old & New)

Examination – May, 2016

VECTOR CALCULUS

Paper : BHM-123

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question No. IX of Section-V is *compulsory*. All questions carry equal marks.

SECTION – I

1. (a) If $\vec{a}, \vec{b}, \vec{c}$ are the position vectors of A, B, C, prove that $\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}$ is a vector perpendicular to plane of ΔABC. 6

(b) If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$, show that three vectors $\vec{a}, \vec{b}, \vec{c}$ are orthogonal in pairs and $|\vec{b}| = 1, |\vec{c}| = |\vec{a}|$. 6

2. (a) Prove that the necessary and sufficient condition for the vector function $\vec{f}$ of a scalar variable t to have constant direction is $\vec{f} \times \frac{d\vec{f}}{dt} = 0$. 6

(b) Show that $\vec{a} \times (\vec{b} \times \vec{c}), \vec{b} \times (\vec{c} \times \vec{a}), \vec{c} \times (\vec{a} \times \vec{b})$ are coplanar. 6

SECTION – II

3. (a) Prove that grad ϕ is normal to the surface $\phi(x, y, z) = c$, where c is a constant. 6

(b) If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ and $r = |\vec{r}|$; prove that $div(r^n \vec{r}) = (n+3)r^n$. 6

4. (a) Prove that Curl $\frac{\vec{a} \times \vec{r}}{r^3} = -\frac{\vec{a}}{r^3} + \frac{3\vec{r}}{r^5}(\vec{a} \cdot \vec{r})$, where $\vec{a}$ is a constant vector. 6

(b) Find Curl (Curl $\vec{f}$) of the function $\vec{f} = y(x+z)\hat{i} + z(x+y)\hat{j} + x(y+z)\hat{k}$. 6

SECTION – III

5. (a) If u, v, w are orthogonal curvilinear co-ordinates, then $\frac{\partial \vec{r}}{\partial u}, \frac{\partial \vec{r}}{\partial v}, \frac{\partial \vec{r}}{\partial w}$ and $\nabla u, \nabla v, \nabla w$ are reciprocal system of vectors. 6

(b) If (r, θ, ϕ) are spherical co-ordinates, show that $\nabla\left(\frac{1}{r}\right) = \nabla \times (\cos\theta \nabla\phi)$. 6

6. (a) Express the acceleration of a particle in spherical co-ordinates. 6

(b) Show that : 6

$$\nabla^2\phi = \frac{1}{h_1h_2h_3}\left[\frac{\partial}{\partial u}\left(\frac{h_2h_3}{h_1}\frac{\partial\phi}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{h_3h_1}{h_2}\frac{\partial\phi}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{h_1h_2}{h_3}\frac{\partial\phi}{\partial w}\right)\right].$$

SECTION – IV

7. (a) Evaluate :

$$\int_C \vec{f} \cdot d\vec{r},$$

where $\vec{f} = (x^2 + y^2)\hat{i} - 2xy\hat{j}$, the curve C is the rectangle in xy-plane bounded by $y = 0$, $x = a$, $y = b$, $x = 0$. 6