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- (c) Write down the orthogonal property of Legendre polynomial. 2
- (d) If  $L\{f(t)\} = F(s)$  then prove that  
 $L\{e^{at} f(t)\} = F(s-a)$  2
- (e) State shifting property of inverse Laplace transformation. 2
- (f) State convolution theorem for inverse Laplace transformation. 2

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B.Sc. 4th Semester (Hons) Common with ID No. 60345  
B.Sc. (Old Scheme) Examination, May-2016

MATHEMATICS

Paper-BHH-242

Special Functions and Integral transforms

Time allowed : 3 hours ] [ Maximum marks : 60

Note : Attempt any five questions in all, selecting one question from each section. Section-V is compulsory.

Section-I

1. (a) Find the power series solution of following initial value problem :

$$(x^2 - 1) \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + xy = 0, y(0) = 2, y'(0) = 3$$

6

- (b) Find the series solution of the differential equation

$$(x - x^2) \frac{d^2y}{dx^2} + (1 - 5x) \frac{dy}{dx} - 4y = 0$$

about 0.

6

2. (a) Show that :  $\frac{d}{dx} (x^n J_n(x)) = x^n J_{n-1}(x)$ . 6

- (b) Show that :

$$J_1(x) = \left( \frac{48}{x^3} - \frac{8}{x} \right) J_1(x) + \left( 1 - \frac{24}{x^2} \right) J_0(x). \quad 6$$

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## Section-II

3. (a) Express  $x^3 + 2x^2 - x - 3$  in terms of Legendre's polynomials. 6
- (b) Verify that Legendre polynomial

$P_3(x) = \frac{1}{2}(5x^3 - 3x)$  satisfies the Legendre equation when the parameter  $n$  is equal to 3. 6

4. (a) Show that :

$$H_6(x) = 64x^6 - 480x^4 + 720x^2 - 120 \quad 6$$

- (b) Express  $H(x) = x^4 + 2x^3 + 2x^2 - x - 3$  in terms of the Hermite's polynomials. 6

## Section-III

5. (a) Find the Laplace transform of  $t^2 \cos at$ . 6
- (b) Find the inverse Laplace transform of

$$\frac{s}{(s^2 + a^2)^2} \quad 6$$

6. (a) Using convolution theorem, evaluate

$$L^{-1}\left(\frac{s}{(s^2 + a^2)^3}\right) \quad 6$$

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- (b) Solve the following equation by transform method:

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} = 2, x(0) = 3, x'(0) = 1. \quad 6$$

## Section-IV

7. (a) Find the Fourier cosine transform of  $e^{-x^2}$ . 6
- (b) Find the Fourier sine transform of the function

$$f(x) = \begin{cases} \sin x, & 0 < x < a \\ 0, & x > a \end{cases} \quad 6$$

8. (a) Using Parseval's identity, prove that

$$\int_0^{\infty} \frac{dx}{(x^2 + 1)^2} = \frac{\pi}{4} \quad 6$$

- (b) Using Parseval's identity, prove that

$$\int_0^{\infty} \frac{dx}{(a^2 + x^2)(b^2 + x^2)} = \frac{\pi}{2ab(a+b)} \quad 6$$

## Section-V

9. (a) Determine radius of convergence of the power

$$\text{series } \sum_{m=0}^{\infty} \left(\frac{x}{4}\right)^{m^2} \quad 2$$

- (b) Write down the expressions for the Bessel's function of first kind of order  $n$  as well as of order  $-n$ . 2

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