(b) Find the sum of n terms of the series :

$$\cot^{-1}(2.1^2)+\cot^{-1}(2.2^2)+\cot^{-1}(2.3^2)+\ldots\ldots \quad 6$$

SECTION – V

9. (a) Show that $3^{2n}+7$ is a multiple of 8. $2\times 6=12$

(b) Find the least positive integer (mod 11) to which 282 is congruent.

(c) If n is power of 2, then prove that $\sigma(n)$ is odd.

(d) Separate log $(\alpha + i\beta)$ into real and imaginary parts.

(e) Using De-moivre's theorem, solve $x^4-x^3+x^2-x+1=0$.

(f) Show that :

$$\tan h^{-1}(\cos\theta)=\cos h^{-1}(\operatorname{cosec}\theta)$$

Roll No. ....

91554

B. Sc. 2nd Sem. (Mathematics) (Hons.) (Old & New)

Examination – May, 2016

NUMBER THEORY AND TRIGONOMETRY

Paper : BHM-121

Time : Three Hours] *[Maximum Marks : 60*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question No. IX of Section-V is *compulsory*. All questions carry equal marks.

SECTION – I

1. (a) Prove that a number is divisible by 9 iff the sum of its digits is divisible by 9 (nine). 6

(b) Prove that the product of two positive integers is equal to the product of their L.C.M. and G.C.D. 6

2. (a) Find the remainder when $53^{103} + 103^{53}$ is divided by 39. 6

(b) If $(a, m) = d$ and d/b, prove that the linear congruence $ax \equiv b \bmod m$ has exactly d solutions which are incongruent mod m. 6

SECTION – II

3. (a) Find the highest power of 6 contained in 500! 6

(b) Prove that the product of two odd prime numbers cannot be a perfect number. 6

4. (a) Let p be an odd prime and $(a, p) = 1$. Then a is quadratic residue or a quadratic non-residue (mod p) according as :

$a^{\frac{p-1}{2}} \equiv 1 \pmod p$ or $a^{\frac{p-1}{2}} \equiv -1 \pmod p$ 6

(b) Is the congruence $x^2 \equiv 150 \pmod{1009}$ solvable ? 6

SECTION – III

5. (a) If the roots of the equation $t^2 - 2t + 2 = 0$ are α, β, show that $\frac{(x+\alpha)^n - (x+\beta)^n}{\alpha - \beta} = \frac{\sin(n\phi)}{\sin^n \phi}$, where $x + 1 = \cot\phi$. 6

(b) Show that roots of the equation $(1+x)^{2n} + (1-x)^{2n} = 0$ are given by $\pm i \tan\left(\frac{2r-1}{4n}\right)\pi$, where $r = 1, 2, 3, \ldots, n$. 6

6. (a) Show that : 6

$$\left[\sin(\alpha - \theta) + e^{\pm i\alpha} \sin\theta\right]^n = \sin^{n-1}\alpha \left[\sin(\alpha - n\theta) + e^{\pm i\alpha} \sin n\theta\right]$$

(b) If $\tan(\theta + i\phi) = \sin(x + iy)$, prove that $\cot hy . \sin h2\phi = \cot x . \sin 2\theta$. 6

SECTION – IV

7. (a) Separate $\log \sin(x + iy)$ into real and imaginary parts. 6

(b) Prove that $\tanh^{-1} z = \frac{1}{2} \cosh^{-1}\left(\frac{1+z^2}{1-z^2}\right)$. 6

8. (a) Find the sum of the series $\sin x + \sin 3x + \sin 5x + \ldots\ldots$ to n terms and deduce the sum of the series $1 + 3 + 5 + \ldots + (2n-1)$. 6