Section–V

9. (a) Give an example of a set which is neighbourhood of each of its points. 2

(b) Define open set and give an example of an open set. 2

(c) What do you mean by limit point of a set ? 2

(d) Show by an example that arbitrary intersection of the neighbourhoods of a point need not be a neighbourhood of that point. 2

(e) Show that set of integers Z is a closed set. 2

(f) Prove that a finite set has no limit point. 2

B.Sc. 4th Semester (Hons) Common with ID No. 60344
B.Sc. (Old Scheme) Examination, May–2016

MATHEMATICS

Paper–BHH–241

Sequences and Series

Time allowed : 3 hours] *[Maximum marks : 60*

Note : *Attempt any **five** questions in all, selecting **one** question from each section. **Section–V** is **compulsory**.*

Section–I

1. (a) If S and T are non-empty bounded subsets of R then prove that SUT is also bounded and Sup. (SUT) = Max {Sup. S, Sup. T}. 6

(b) Prove that intersection of a finite number of open sets is an open set. 6

2. (a) For any set A, $\overline{A}$ (closure of A) is a closed set. 6

(b) If a set A has Heine-Borel property then every closed subset B of A also has Heine-Borel property. 6

Section–II

3. (a) Show that

$$\lim_{n\to\infty}\left[\frac{1}{n^2}+\frac{1}{(n+1)^2}+\dots+\frac{1}{(2n)^2}\right]=0. \quad 6$$

(b) Using Cauchy's second theorem on limits prove that $\lim_{n\to\infty} \frac{(\angle n)^{1/n}}{n} = \frac{1}{e}$. 6

4. (a) Prove that the Geometric series $a + ar + ar^2 + ... \infty$

(i) converges to $\frac{a}{1-r}$ if $|r| < 1$

(ii) diverges if $r \geq 1$.

(iii) oscillates finitely if $r = -1$

(iv) oscillates infinitely if $r < -1$. 6

(b) Show that $\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{1}{n}$ is convergent. 6

Section–III

5. (a) State and prove Cauchy's root test. 6

(b) Test the convergence of the series

$$1 + \frac{1}{2}x + \frac{1.3}{2.4}x^2 + \frac{1.3.5}{2.4.6}x^3 + ...$$ 6

6. (a) If x, a, b, d are all positive, discuss the convergence of the series :

$$\frac{a}{b} + \frac{a(a+d)}{b(b+d)}x + \frac{a(a+d)(a+2d)}{b(b+d)(b+2d)}x^2 + ...$$ 6

(b) Using Cauchy's condensation test, discuss the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n(\log n)^p}$. 6

Section–IV

7. (a) Test the convergence and absolute convergence of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^p} = 1 - \frac{1}{2^p} + \frac{1}{3^p} - \frac{1}{4^p} + ..., p > 0.$$ 6

(b) If $\sum_{n=1}^{\infty} a_n$ is absolutely convergent series, then the series of its positive terms and the series of its negative terms are both convergent. 6

8. (a) Show that Cauchy product of the convergent series $\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n+1}}$ with itself is divergent. 6

(b) Show that Cauchy product of two divergent series

$$\sum_{n=0}^{\infty} a_n = 1 - \left(\frac{3}{2}\right)^1 - \left(\frac{3}{2}\right)^2 - \left(\frac{3}{2}\right)^3 - ....$$

and $\sum_{n=0}^{\infty} b_n = 1 + \left(2 + \frac{1}{2^2}\right) + \frac{3}{2}\left(2^2 + \frac{1}{2^3}\right) + \left(\frac{3}{2}\right)^2\left(2^3 + \frac{1}{2^4}\right) + ...$

is convergent. 6