60248

B.Sc. 3rd Semester (Hons) Old Scheme Examination, December–2015

MATHEMATICS

Paper–BHM–235, Opt. (i)

Probability Distribution

Time allowed : 3 hours] *[Maximum marks : 60*

Note : *Attempt five questions in all, selecting one question from each section. **Question number 9 is compulsory.***

Section–I

1. (a) State and prove Chebychev's inequality. 6

(b) Find the moment generating function of the random variable X having probability density function

$$f(x) = \begin{cases} x & \text{for } 0 \le x < 1 \\ 2 - x & \text{for } 1 \le x < 2 \\ 0 & \text{elsewhere} \end{cases}$$

Also determine μ'_1, μ_2, μ_3 and μ_4. 6

2. (a) Let $X_1, X_2, ..., X_n$ be jointly normal with $E(X_i) = 0$ and $E(X_i^2) = 1$ for all i

and $Cov(X_i, X_j) = \rho$ if $|j - i| = 1$ and $= 0$, otherwise.

Examine if WLLN holds for the sequence $\{X_n\}$. 6

(b) Let the r.v. X assume the value 'r' with the probability law :

$$P(X = r) = q^{r-1} p \; ; \; r = 1, 2, 3, ...$$

Find the m.g.f. of X and hence its mean and varience. 6

Section–II

3. (a) If X is a Poisson variate such that

$$P(X = 2) = 9 P(X = 4) + 90 P(X = 6).$$

Find λ, the mean of X and β, the co-efficient of skewness. 6

(b) Derive Poisson distribution as a limiting form of a binomial distribution. Hence find β_1 and β_2 of the distribution. 6

4. (a) What is hypergeometric distribution ? Find the mean and variance of this distribution. 6

(b) Show that mode of the distribution

$$p(x) = \left(\frac{1}{2}\right)^x ; \; x = 1, 2, 3, ...$$

is 1. 6

Section–III

5. (a) If $X_1, X_2, X_3, ..., X_n$ are independent random variables, X_i having an exponential distribution with parameter θ_i ; $i = 1, 2, 3, ..., n$, then prove that $Z = \min(X_1, X_2, ..., X_n)$ has an exponential distribution with parameter $\sum_{i=1}^{n} \theta_i$. 6

(b) If X and Y are independent Gamma variates with parameters μ and ν respectively, show that the variables

$$U = X + Y \text{ and } V = \frac{X}{Y} \text{ are independently}$$

distributed, U as a $\gamma(\mu + \nu)$ variate and V as a $\beta_2(\mu, \nu)$ variate. 6

6. (a) Define the Beta variate of the first kind. Obtain its mean and variance. 6

(b) Show that if $X \sim N(\mu, \sigma^2)$, then $\frac{1}{2}\left(\frac{X-\mu}{\sigma}\right)^2$ is a $\gamma\left(\frac{1}{2}\right)$ variate. 6

Section–IV

7. (a) Show that any linear combination of n independent normal variates is also a normal variate. 6

(b) If X is a normal variate with mean 30 and S.D. 5. Find the probabilities that

$$26 \le X \le 40\,; \quad X \ge 45 \text{ and } |X - 30| > 5.$$ 6

8. (a) Prove that for the normal distribution, the quartile deviation, the mean deviation and standard deviation are approximately 10 : 12 : 15. 6

(b) If X, Y are independent normal variates with means 6, 7 and variances 9, 16 respectively, determine λ such that $P(2X + Y \le \lambda) = P(4X - 3Y \ge 4\lambda)$. 6

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Section–V

9. (a) The mean and variance of binomial distribution are 4 and 4/3 respectively. Find $P(X \ge 1)$

(b) State Central limit theorem.

(c) If $\log_{10} X$ is normally distributed with mean 4 and variance 4, find the probability of

$$1.202 < X < 83180000.$$

(given $\log_{10} 1202 = 3.08$, $\log_{10} 8318 = 3.92$)

(d) If X is a Poisson variate with mean m, what would be the expectation of $e^{-kX}\, kX$, k being a constant.

(e) Define convergence in probability.

(f) The probability of a man hitting a target is $\frac{1}{4}$ and he fires 7 times. What is the Probability of his hitting the target atleast twice ? [6×2=12]