B.Sc. 3rd Semester (Hons) (Old Scheme)

Examination, December–2015

MATHEMATICS

Paper–BHM–234

Differential Geometry

Time allowed : 3 hours] *[Maximum marks : 60*

Note : *Attempt five questions in all, selecting **one** question from each section. **Question No. 9 is compulsory**. All questions carry equal marks.*

Section–I

1. (a) Show that the envelope of the family of paraboloids $x^2+y^2=4a(z-a)$ is the circular cone $x^2+y^2=z^2$, where 'a' is a paraboloid. 6

(b) Find the edge of regression of the envelope of the family of planes

$$x \sin\theta - y\cos\theta + z = a\theta,$$ where θ is a parameter. 6

2. (a) Define a developable surface. What is the necessary and sufficient condition that the surface $z = f(x, y)$ should represent a developable surface. 6

(b) If w is the angle between the parametric curves u = costt. and v = constt., then find sin w and cos w.

(c) Write the relation between $\vec{t}, \vec{n}, \vec{b}$ using "Serret-Fernet Formulae".

(d) Define L, M, N in terms of $\vec{r}_1$ and $\vec{r}_2$, where $\vec{r}$ (u, v) is the equation of the surface.

(e) Prove that surface $xy=(z-c)^2$ is developable.

(f) Write the Weingarten Equations in terms of $\vec{r}_1$ and $\vec{r}_2$. [2×6=12]

(b) Determine that the surface $xyz = a^3$ is developable or not. 6

Section–II

3. (a) Find the equation of the envelope of system of surfaces whose equations involve two parameter. 6

(b) Find the envelope of a plane that forms with the rectangular co-ordinates planes a tetrahedron of constant volume $c^3/6$. 6

4. (a) Calculate the fundamental magnitudes for the surface :-

$x = a(u + v),\ \ y = b(u - v),\ \ z = uv$ 6

(b) Find the angle θ between the parameter curves $u = \text{constant}$ and $v = \text{constant}$. 6

Section–III

5. (a) Derive the formula for normal curvature in terms of Fundamental magnitudes. 6

(b) Find the equations giving the principal curvatures K_a and K_b. 6

6. (a) Find the equation of Dupin's indicatrix and show it is conic section. 6

(b) Calculate the first and second curvature of the helicoid $x = u\cos v,\ y = u\sin v,\ z = f(u) + cv$. 6

Section–IV

7. (a) Show that the necessary and sufficient condition for a curve u = costant to be geodesic on the general surface is

$$GG_1 + FG_2 - 2\,GF_2 = 0$$ 6

(b) Prove that the curve $u + v = \text{constant}$; are geodesics on a surface with metric

$$(1 + u^2)\,du^2 - 2uv\,du\,dv + (1 + v^2)\,dv^2$$ 6

8. (a) Discuss the nature on a sphere. 6

(b) Prove that straight lines on a surface are the only asymptotic lines, which are geodesics. 6

Section–V

9. (a) Define Curvature and Torsion.