

7. (a) Form a p.d.e. by eliminating the function  $f$  from

$$z = x^p f\left(\frac{y}{x}\right).$$

- (b) Solve  $x^2 p + y^2 q = (x + y)^2$ .

#### Section-IV

8. (a) Solve  $(D^2 - D D^1 - 2 D^1^2) z = (y - 1) e^x$ .  
 (b) Solve  $(D - D^1 - 1)(D - D^1 - 2) z = \sin(2x + 3y)$
9. (a) Solve two dimensional heat equation by method of separation of variables.  
 (b) Solve two dimensional Laplace's equation by Method of Separation of variables.

Time allowed : 3 hours ] [ Maximum marks : 40

Note : Attempt five questions in all, selecting one from each section. Question No. 1 is compulsory.

#### Compulsory Question

1. (a) Prove that the function  $\sin |x|$  is continuous.  
 (b) State Rolle's theorem.  
 (c) Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be defined as

$$f(x, y) = \begin{cases} \frac{xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

Show that  $\lim_{(x, y) \rightarrow (0, 0)} f(x, y)$  does not exist.

- (d) If  $u = xy f\left(\frac{y}{x}\right)$ , show that  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2u$ .

(e) Solve  $3 \frac{dy}{dx} = \frac{4x}{y^2}$ .

(f) Solve  $\frac{d^2y}{dx^2} - 6 \frac{dy}{dx} + 9y = 0$ .

(g) Solve  $p^2 - q^2 = k^2$ .

(h) Define one dimensional heat equation.

### Section-I

2. (a) Every function defined and continuous on a closed interval attains its bounds in that interval.

(b) Show that the  $f(x) = \frac{1}{x^2}$  is not uniformly continuous on  $(0, 1]$

3. (a) Verify Rolle's theorem for the function  $f(x) = \sin x - \sin 2x$  in  $[0, \pi]$

(b) Show that

$$x - \frac{x^2}{2} < \log(1+x) < x - \frac{x^2}{2(1+x)}, x > 0.$$

### Section-II

4. (a) Show that the function

$$f(x, y) = \begin{cases} \frac{xy}{\sqrt{x^2+y^2}}; & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$$

is continuous at  $(0, 0)$ .

(b) If  $u = \tan^{-1} \frac{x^2+y^2}{x+y}$  show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{1}{2} \sin 2u.$$

5. (a) Find  $\frac{d^2y}{dx^2}$  where relation is  $x^5 + y^5 - 5a^3xy = 0$ .

(b) Expand  $e^x \sin y$  in powers of  $x$  and  $y$  as far as terms of third degree.

### Section-III

6. (a) Solve  $\frac{dy}{dx} + y = xy^3$

(b) Solve  $\frac{d^2y}{dx^2} - 4 \frac{dy}{dx} + 3y = \sin 3x \cos 2x$ .