Roll No. ....

91026

B. Sc. (Hons.) Physics 1st Semester Examination – December, 2015

MATHEMATICAL PHYSICS - I

Paper : Phy - 101

Time : Three Hours] *[Maximum Marks : 40*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting at least *two* questions from each Section. All questions carry equal marks.

SECTION – I

1. (a) Express $\vec{a}, \vec{b}, \vec{c}$ in terms of $\vec{b}\times\vec{c}, \vec{c}\times\vec{a}, \vec{a}\times\vec{b}$

   (b) Prove that $\left[\vec{a}\times\vec{b}\ \ \vec{b}\times\vec{c}\ \ \vec{c}\times\vec{a}\right]=\left[\vec{a}\ \vec{b}\ \vec{c}\right]^2$

2. (a) The necessary and sufficient condition for the vector function $\vec{f}$ of a scalar variable t to have constant magnitude is $\vec{f}\cdot\frac{d\vec{f}}{dt}=0$

(b) Find a unit tangent vector to any point on the curve $x = a\cos t, y = a\sin t, z = bt$

3. (a) If $\phi = 3x^2y$ and $\psi = xz^2 - 2y$; find :

(i) grad (grad ϕ. grad Ψ)

(ii) curl (grad $\phi \times$ grad Ψ)

(b) Find the directional derivative of $\phi(x,y,z) = x^2yz + 4xz^2$ at the point (1,–2,1) in the direction $2\hat{i} - \hat{j} - 2\hat{k}$

4. (a) Evaluate $\int_c \vec{f}.d\vec{r}$, where $\vec{f} = (x^2+y^2)\hat{i} - 2xy\hat{j}$, the curve c is the rectangle on xy plane bounded by $y = 0, x = 0, y = b\ x = b$.

(b) Verify Gauss's divergence theorem for the function $\vec{f} = 4xz\hat{i} - y^2\hat{j} + yz\hat{k}$ over the surface S of the cube bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$

SECTION – II

5. (a) Transform the function $\vec{f} = P\hat{e}_p + P\hat{e}_\phi$ from cylindrical to Cartesian coordinates.

(b) Prove that on cylinderical coordinates :

$$\nabla.\vec{F} = \frac{1}{P}\frac{\partial}{\partial P}(Pf_1) + \frac{1}{P}\frac{\partial f_2}{\partial \phi} + \frac{\partial f_3}{\partial z}$$

6. (a) If (r, θ, φ) are spherical co-ordinates, show that :

$$\nabla\phi = \nabla \times (r \operatorname{cosec}\theta\, \nabla\theta)$$

(b) Find the extremal of the functional $\int_0^1 (y')^2 + (z')^2 dx$ that satisfy the boundary conditions $y(0) = 0$, $z(0) = 0$, $y(1) = 1, z(1) = 2$

7. (a) Find the extremal of the function $\int_0^\pi (y')^2 dx$, $y(0) = 0$, $y(\pi) = 0$ subject to the condition $\int_0^\pi y^2 dx = 1$

(b) Find the values of x, y, z for which $\frac{5xy^2}{x+2y+4z}$ is maximum given that xyz = 8

8. (a) Show that the volume of the tetrahedron bounded by $x = 0, y = 0, z = 0$ and $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$ is $\frac{abc}{6}$

(b) Evaluate $\int_0^\infty \int_0^x xe^{-x^2/y} dy\, dx$
