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M. Sc. 4th Semester Examination,

May–2016

MATHEMATICS

Paper–MM–524 (A2)

Advanced Diserete Mathematics

Time allowed : 3 hours] *[Maximum marks : 80*

Note : *Attempt five questions in all.* ***Question No. 9 from Section–V*** ***is compulsory.***

Section–I

1. (a) Construct a truth table for each of the following compound propositions.

(i) $p \to (\sim q \vee r)$

(ii) $\sim p \to (q \to r)$

(iii) $(p \to q)(\sim p \to r)$

(iv) $(p \to q) \wedge (\sim p \to r)$ 8

(b) Show that $(p \to q) \to r$ and $(p \to (q \to r))$ are not equivalent. 4

(c) Determine whether

(i) $(\sim p \wedge (p \leftrightarrow q)) \to \sim q$

(ii) $(\sim q \wedge (p \to q)) \to \sim p$

are tautology. 4

2. (a) Express the statements as a logical expression.

(i) Some student in this class has visited Mexico.

(ii) Every student in this class has visited either Canada or Mexico.

(iii) Everyone has exactly one best friend.

(iv) If somebody is female and is a parent, then this person is some-one's mother.

(v) There is a woman who has taken a flight on every airline in the world.

(vi) There is a student in this class who can speak Hindi.

(vii) Every student in this class knows how to drive a car.

(viii) Some student in this class has visited Alaska but has not visited Hawaii. 8

(b) Write Negation of following statements :

(i) $\exists y \exists x P(x, y)$

(ii) $\forall x \exists y P(x, y)$

(iii) $\exists y (Q(y) \wedge \forall y R(x, y))$

(iv) $\exists y (\exists x R(x, y) \vee \forall x S(x, y))$

(v) $\exists y (\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z))$

(vi) $\forall x \forall y P(x, y)$

(vii) $\forall y \exists x P(x, y)$

(viii) $(\exists x \exists y \sim P(x, y) \wedge \forall x \forall y Q(x, y))$ 8

Section–II

3. (a) Let $S = N \times N$. Let $*$ be the operation on S defined by $(a, b) * (a', b') = (aa', bb')$

(i) Show that $*$ is associative

(ii) Define $f : (S, *) \rightarrow (Q, x)$ by $f(a, b) = a/b$. Show that f is a homomorphism.

(iii) Find the congruence relation $\sim$ in S determined by the homomorphism f, that is, where $x \sim y$ if $f(x) = f(y)$

(iv) Describe $S/\sim$. Does $S/\sim$ have an identity element ? Does it have inverses ? 8

(b) Prove that the intersection of two subsemigroups of a Semigroup $(S, *)$ is subsemigroup of $(S, *)$ but need not be true in case of union. Explain with example. 8

4. (a) Let $(Z, +)$ and $(Z, \cdot)$ be the Semigroups of Integers. Consider the relation R defined on Z by aRb if and

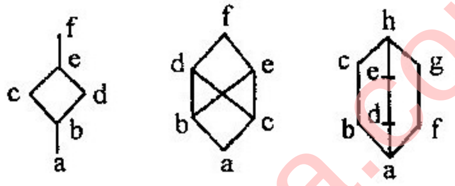
only if $a \equiv b \pmod{m}$. Then show that Relation R is congruence in both semigroups. 8

(b) State and Prove fundamental theorem of semigroup homomorphism. 8

Section–III

5. (a) Determine whether the posets represented by Hasse diagrams are lattices.

(i)



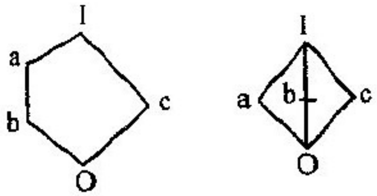
(ii) Is the poset $(Z, /)$ a lattice.

(iii) Determine whether each of the posets $(\{1, 2, 3, 4, 5,\}, 1)$ and $(\{1, 2, 4, 8, 16\}, 1)$ is a lattice.

(iv) Determine whether $(P(S), \leq)$ is a lattice where S is a set. 8

(b) If $(L_1, \leq)$ and $(L_2, \leq)$ are lattices. Then prove that $(L, \leq)$ is a lattice, where $L = L_1 \times L_2$ and partial order $\leq$ of L is the product partial order. 8

6. (a) Prove that the following lattices are non-distributive.



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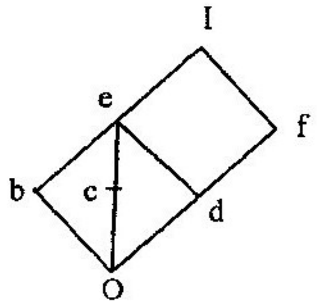
(b) Consider the bounded Lattices L in Fig.



(i) Find the complements, if they exist, of e and f

(ii) Express I in an irredundant joinirreducible decomposition in as many ways as possible.

(iii) Is L distributive ?

(iv) Describe the Isomorphisms of L with itself. 8

Section–III

7. (a) Consider the Boolean algebra D_{110}

(i) List its elements and draw its diagram.

(ii) Find all its subalgebras.

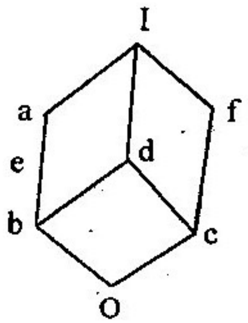
(iii) Find the number of sublattices with four elements.

(iv) Find the set A of atoms of D_{110}

(v) Give the isomorphic mapping

$f : D_{110} \rightarrow P(A)$ 10

(b) Show that the lattice whose diagram is



is not a Boolean algebra. 6

8. (a) Find Prime Implicants using Consensus Method of following Boolean expression.

(i) $E = xyz + \bar{x}\bar{z} + xy\bar{z} + \bar{x}\bar{y}z + \bar{x}y\bar{z}$

(ii) $E = x\bar{y} + \bar{x}y + \bar{y}z$ 8

(b) Find Prime implicants and a minimal sum of products form for each of the following complete sum of products Boolean expressions :

(i) $E = xyz + xy\bar{z} + \bar{x}y\bar{z} + \bar{x}\bar{y}z$

(ii) $E = xyz + xy\bar{z} + x\bar{y}z + \bar{x}yz + \bar{x}\bar{y}z$

(iii) $E = xyz + xy\bar{z} + \bar{x}yz + \bar{x}\bar{y}z$ 8

9. (a) Draw truth table for conjunction operation.

(b) Show that $p \wedge \sim p$ is a contradiction.

(c) Define semigroup Isomorphism.

(d) Define concatenation operation.

(e) Write Absorption property.

(f) Define Joinirreducible element.

(g) Define Boolean Homomorphism.

(h) Draw table for NOR gate. 16