

Time allowed : 3 hours ] [ Maximum marks : 80

Note : Attempt five questions, selecting one question from each unit. Unit-V is compulsory. All questions carry equal marks.

Unit-I

1. (a) Define Weierstrass Primary factors and show

$$|\log E(z, p)| \leq 2|z|^{p+1} \text{ for } |z| \leq \frac{1}{2} \quad 8$$

- (b) Construct Euler's Gamma function and hence define Euler's constant. 8

2. (a) Define Riemann's zeta function. Also find its domain of convergence. 4

- (b) State and prove stirling formula. 12

Unit-II

3. (a) State and prove the uniqueness of direct analytic continuation. 8

- (b) State and prove Monodromy theorem. 8

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[P.T.O.]

4. (a) What is Poisson Kernel? Show that the Poisson Kernel  $P_r(\theta)$  satisfies the properties:

$$(i) \quad \frac{1}{2\pi} \int_{-\infty}^{\infty} P_r(\theta) d\theta = 1$$

- (ii)  $P_r(\theta) > 0$  for all  $\theta$ ,  $P_r(-\theta) = P_r(\theta)$  and  $P_r(\theta)$  is periodic in  $\theta$  with period  $2\pi$ . 8

- (b) Outline the Dirichlet problem for a unit disc. 8

### Unit-III

5. (a) State and prove Harnack's theorem for harmonic functions. 8  
 (b) Prove that if  $f(z)$  is an entire function of order  $\rho$  then for every  $\epsilon > 0$ , the inequality  $N(r) \leq r^{\rho+\epsilon}$  holds for all sufficiently large  $r$ . 8
6. (a) What do you mean by order of an entire function? Find the order of  $\sin z$  and  $\cos \sqrt{z}$ . 8  
 (b) Prove that the order of a canonical product is equal to the exponent of convergence of its zeros. 8

### Unit-IV

7. (a) Let  $f$  be analytic function on a region containing the closure of  $B(0; R)$  then  $f(B(0; R))$  contains a disk of radius  $\frac{1}{72} R |f'(0)|$ . 8  
 (b) State and prove Schottky's theorem. 8

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8. (a) State and prove Montel Caratheodory theorem. 8  
 (b) Write note on univalent functions and hence state Bieberbach's conjecture. 8

### Unit-V

9. (a) Given that  $\sin^2 z + \cos^2 z = 1$  holds for all real values of  $z$ , prove that it also holds for all complex values of  $z$ .  $8 \times 2 = 16$   
 (b) Define Natural boundary for an analytic function.  
 (c) State Weierstrass factorization theorem.  
 (d) Define Green function.  
 (e) Write Riemann's functional equation.  
 (f) State Montel Caratheodory theorem.  
 (g) Define Landau's constant and specify its range.  
 (h) Show that a function which is meromorphic in the entire  $z$ -plane is the quotient of two integral functions.

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