M.Sc. 4th Semester Examination,
May–2016
MATHEMATICS
Paper–MM–522
Classical Mechanics

Time allowed : 3 hours] *[Maximum marks : 80*

Note : *Attempt **five** questions in all, selecting **one** question from each unit. **Question No. 9 of Unit–V is compulsory**. All questions carry equal marks.*

Unit–I

1. Define free and constrained systems. How they are classified ?

2. (a) Derive Lagrange's equations of first kind.

   (b) State principle of Virtual work and prove D'Alembert's Principle.

Unit–II

3. (a) A particle of mass m moves in a plane. Determine its equations of motion in plane polar coordinates.

   (b) Prove that for a scleronomic system, such that the potential energy is not explicitely dependent on time, the derivative of the total energy with respect to time is equal to the power of the non-potential forces.

4. (a) Derive Lagrange's equation of motion for a particle moving under the inverse square law of attraction towards a fixed point.

(b) Define Lagrangian and Hamiltonian variables and show that Hamiltonian variables can be expressed in terms of the Lagrange variables.

Unit–III

5. (a) Using spherical polar coordinates, derive the Hamilton's equations of motion for a particle of mass m, moving in a force field of potential $V(r, \theta, \phi)$.

(b) State and prove Poisson's identity for Poisson's Brackets.

6. (a) Derive Whittaker's equations.

(b) Derive Jacobi's equations.

Unit–IV

7. (a) Discuss the necessary and sufficient condition for a transformation

$$\tilde{q}_i = \tilde{q}_i\,(t, q_k, p_k)\,; \quad \tilde{p}_i = \tilde{p}_i(t, q_k, p_k)\,;$$

$$\frac{\partial(\tilde{q}_1, \tilde{p}_1, \tilde{q}_2, \tilde{p}_2, \ldots, \tilde{q}_n, \tilde{p}_n)}{\partial(q_1, p_1, q_2, p_2, \ldots, q_n, p_n)} \neq 0 \text{ to be Canonical.}$$

(b) Discuss Free Canonical transformations.

8. (a) Prove that Poisson brackets are invariant under univalent Canonical transformations.

(b) Show that the transformation

$$Q = \log\left(\frac{1}{q}\sin p\right)\,; \quad P = q\cot p \quad \text{is Canonical.}$$

Unit–V

9. (a) Define possible displacement.

(b) Define degrees of freedom.

(c) Define ideal constraints.

(d) Define Hamiltonian function and write down Hamilton Canonical equations.

(e) Define Poisson Bracket and show that for two arbitrary functions $\phi(t, q_i, p_i)$ and $\psi(t, q_i, p_i)$;

$$\frac{\partial}{\partial t}(\phi\psi) = \left(\frac{\partial\phi}{\partial t}\psi\right) + \left(\phi\frac{\partial\psi}{\partial t}\right)$$

(f) State Hamilton's principle.

(g) Show that the transformation

$\tilde{q}_i = \alpha q_i$, $\tilde{p}_i = \beta p_i$ $(i = 1, 2, \ldots n\,; \alpha \neq 0, \beta \neq 0)$ is Canonical.

(h) Define Lagrange Bracket. How they are different from Poisson Bracket?