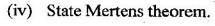
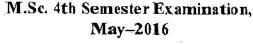
(iii) Give example of field in which the fundamental theorem is false. Explain.



- (v) What are magnitude and average order of  $\sigma(n)$ ?
- (vi) Define Mobius function.
- (vii) Prove that  $\sum_{i=1}^{n} i \approx \frac{n \phi(n)}{2}$

(viii) State Bertrand Postulate.



## **MATHEMATICS**

Paper-MM-525 (C-2)

Analytical Number Theory-II

Time allowed: 3 hours ]

[ Maximum marks: 80

Note: Attempt one question from each of the Unit-I, II, III, IV. Unit-V is compulsory.

### Unit-I

- 1. (a) Using Riemann zeta function, prove that there are infinitely many primes.
  - (b) Prove that if b (s) is a Riemann zeta function, then evaluate b (2k), where  $k \ge 1$ .
- 2. (a) Show that  $\sum_{n=1}^{\infty} \frac{\tau(n)}{n^s} (\xi(s))^2 \text{ for } s > 1.$

(b) Suppose that 
$$F(s) = \sum_{n=1}^{\infty} \frac{f(n)}{n^s}$$
,  $G(s) = \sum_{n=1}^{\infty} \frac{g(n)}{n^s}$ 

and H (s) =  $\sum_{n=1}^{\infty} \frac{h(n)}{n^s}$  where f, g, hare arithmetic

functions such that h = f \* g, then

 $H(s) = F(s)G(s) \forall s \text{ such that } F(s) \text{ and } G(s) \text{ both converge absolutely.}$ 

Also, Using above theorem, show that

$$\sum_{n=1}^{\infty} \frac{\mu(n)}{n^s} = \frac{1}{\xi(s)}$$

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### Unit-2

- 3. (a) State and prove Division Algorithm in Q(i).
  - (b) Prove that if k is a quadratic field,  $\exists$  a non-zero square free integer m such that  $k = Q(\sqrt{m})$ .
- 4. Prove that the prime of  $Q(\sqrt{2})$  can be divided into 3 classes
  - (i) The prime  $\sqrt{2}$  and its associates.
  - (ii) The rational primes of the form  $8n \pm 3$  and their associates.
  - (iii) The prime factor  $a + b\sqrt{2}$  of the rational primes of the form  $8n \pm 1$  and their associates.

# Unit-3

- 5. (a) Prove that d(n) and  $\sigma(n)$  are multiplicative functions. Also, find formulae for d(n) and  $\sigma(n)$ .
  - (b) Prove that if f(n) is a multiplicative function and  $f(n) \neq 0$ . Then f(n) = 1.
- 6. (a) What is a General principle? By using this principle prove that

$$\phi(n) = n \left(1 - \frac{1}{p_1}\right) \left(1 - \frac{1}{p_2}\right) \dots \left(1 - \frac{1}{p_k}\right)$$

where n > 1 and let  $n = p_1^{\alpha_1} p_2^{\alpha_2} \dots p_k^{\alpha_k}$  where  $p_1, p_2, \dots, p_k$  are distinct prime.

(b) Find the sum of the cubes of the integers ≤ n and relatively coprime to n where n > 1. Also state clearly results used by you.

(3)

#### Unit-IV

7. (a) Prove that if x > 0, then

$$0 \le \frac{\psi(x)}{x} - \frac{\theta(x)}{x} \le \frac{(\log x)^2}{2\sqrt{x} \log 2}$$

Also define Chebyshev's functions  $\psi$  (x) and  $\theta$  (x).

- (b) State and prove Abel's identity. Also give its applications.
- 8. (a) Prove that there is a constant A such that

$$\sum_{p \le x} \frac{1}{p} = \log \log x + A + O\left(\frac{1}{\log x}\right)$$

for all  $x \ge 2$ .

(b) State and prove Selberg's asymptotic formula.

## Unit-V

- 9. (i) Show that  $\lambda = 1 w$  is a prime and 3 is associated with  $\lambda^2$ .
  - (ii) What do you understand by Dirichlet's Series?

P.T.O.