Roll No. ....

74452

M. Sc. Mathematics 2nd Semester

Examination – May, 2016

REAL ANALYSIS - II

Paper : MM-422

Time : Three Hours] *[Maximum Marks : 80*

Before answering the question, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt any ***five*** questions in all, selecting ***one*** question from each Section. Question No. 9 is ***compulsory*** from Section - V.

SECTION – I

1. (a) State and prove Riemann's theorem of rearrangement of terms of a series. 8

(b) Show that the sequence $\{fn\}$ where $f_n(x) = \frac{nx}{1+n^2x^2}$ is not uniformly convergent on any interval containing zero. 8

2. (a) If a series $\sum_{n=1}^{\infty} fn$ converges uniformly to a function f in $[a,b]$ and x_o is a point in $[a,b]$ such that $\lim_{x \to x_0} f_x(x) = a_n, (n = 1,2, ...)$. Then prove that : 8

(i) $\sum_{n=1}^{\infty} a_n$ converges

(ii) $\lim_{x \to x_0} f(x) = \sum_{n=1}^{\infty} a_n$

(b) Let $< f_n >$ be a sequence of differentiable functions on [a, b] such that it converges at least at one point $x_0 \in [a,b]$. If the sequence of differentiales $< f_n' >$ converges uniformly to G on $[a,b]$, then prove that the given sequence $< f_n >$ converges uniformly on $[a,b]$ to f and $f'(x) = G(x)$. 8

SECTION - II

3. (a) Let $\Sigma a_n x^n$ be a power series such that $\lim |a_n|^{1/n} = \frac{1}{R}$. Then show that power series is convergent with radius of convergence R. Further if $\Sigma a_n x^n$, power series converges for $x = x_0$, then prove that it is absolutely convergent for $x = x_1$ when $|x_1| < |x_0|$. 8

(b) Give an example to show that the partial derivative need not always be differential coefficient. 8

4. (a) Consider the function $f : R^2 \to R$ defined by : 8

$$f(x,y)=\begin{cases}\dfrac{x^3+y^3}{x-y} & \text{if } x\neq y\\ 0 & \text{if } x=y\end{cases}$$

Show that this function has second order partial derivative at a point without being continuous at that point.

(b) State and prove Schwartz's Theorem. 8

SECTION – III

5. (a) Let W be a function of two variables x and y, then transform the expression $\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}$ by the formula of polar transformation $x=u\cos v, y=u\sin v$. 8

(b) Examine the function x^3+y^3-3axy for maxima and minima. 8

6. (a) If the variables x, y, z satisfy the equation $\phi(x)\phi(y)\phi(z)=k^3$ and $\phi(a)=k>0$. Show that the function $f(x)+f(y)+f(z)$ has a maximum value when $x=y=z=a$ provided that

$$f'(a)\left[\frac{\phi''(a)}{\phi'(a)}-\frac{\phi'(a)}{\phi(a)}\right]>f''(a).$$ 8

(b) Prove that the functions : 8

$$f_1=x+y+z+t,\ f_2=x^2+y^2+z^2+t^2,$$

$$f_3=x^3+y^3+z^3+t^3$$

$f_4=xyz+xyt+xzt+yzt$ are dependent. Also establish the relation between the four functions.

SECTION – IV

7. (a) Let f be a bounded and measurable function on $[a,b]$ and $F(x)=\int_a^x f(t)\,dt+F(a)$. Then prove that $F'(x)=f(x)$ a. e in $[a,b]$. 8

(b) Prove that a function of bounded variation is bounded but converse need not be true. 8

8. (a) Let ϕ be a convex function on (a, b) and $a < s < t < u < b$ then show that: 8

$$\frac{\phi(t)-\phi(s)}{t-s} \le \frac{\phi(u)-\phi(s)}{u-s} \le \frac{\phi(u)-\phi(t)}{u-t}$$

further if ϕ is strictly convex, equality will not occur.

(b) If $1<p<\infty$, $1<q<\infty$ such that $\frac{1}{p}+\frac{1}{q}=1$ and a, b, be two non negative real numbers then, prove that :

$a^{1/p}b^{1/q} \le \frac{a}{p}+\frac{b}{q}$. 8

SECTION – V

9. (a) State M_n – Test for uniform convergence.

(b) State Dinis Theorem of Differentiation for uniform convergence.

(c) Find radius of convergence of the series :

$$1 + x + \lfloor 2\, x^2 + \lfloor 3\, x^3 + \ldots.$$

(d) State Taylor's Theorem of Power Series.

(e) Define Continuously Differentiable Mapping.

(f) State Young's Theorem.

(g) Give an example of a function which is continuous and need not be of bounded variation.

(h) Define convex function. $8 \times 2 = 16$