Roll No. ....

74454

M. Sc. Mathematics 2nd Semester
Examination – May, 2016

ORDINARY DIFFERENTIAL EQUATIONS

Paper : MM-424

Time : Three Hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***one*** question from each Unit. Question No. 9 is ***compulsory.***

UNIT – I

1. State and prove Cauchy-Euler theorem for constructing an $\in$-approximate solution of an initial value problem of first order.

2. (a) Use Picard method to obtain solution of $\frac{dy}{dt} = y,\ y(1) = 1$.

UNIT – V

9. (a) Equicontinuous family.
   (b) $\in$-approximate solution.
   (c) Adjoint systems.
   (d) Fundamental matrix.
   (e) Critical points.
   (f) Stability of critical points.
   (g) Periodic solutions.
   (h) Index of a critical point.

(b) Define Lipschitz condition. Let $f(t, y)$ be a function such that $\frac{\partial f}{\partial y}$ exists and is bounded in a convex domain D. Prove that $f(t, y)$ satisfies a Lipschitz condition w. r. t. y in D.

UNIT – II

3. (a) Use the matrix method to solve the system :

$$\frac{dx}{dt} = 3x - y, \quad \frac{dy}{dt} = 4x - y$$

(b) Solve the system :

$$\frac{dx_1}{dt} = 3x_1 + x_2 - x_3$$
$$\frac{dx_2}{dt} = x_1 + 3x_2 - x_3$$
$$\frac{dx_3}{dt} = 3x_1 + 3x_2 - x_3$$

by using matrix method.

4. (a) State and prove Liouville's theorem.

(b) State and prove Sturms separation theorem.

UNIT – III

5. Discuss the classification of critical points of a plane autonomous system.

6. Consider the system :

$$\frac{dx}{dt} = -3x + 2y, \quad \frac{dy}{dt} = x - 2y$$

(a) Solve this system by matrix method.

(b) Find the critical points of this system and also determine their nature and stability.

UNIT – IV

7. (a) Determine the type and stability of the critical point O(0, 0) of the non-linear system :

$$\frac{dx}{dt} = -x + y + x^3 y, \quad \frac{dy}{dt} = -2x - 3y - x^2 y^2$$

(b) Examine the critical points of the system

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = x^2 - 4x + \lambda,$$

λ being a parameter. Also find the critical values of λ.

8. (a) Define Liapunov function and construct a Liapunov function for the system :

$$\frac{dx}{dt} = -x + y^2, \quad \frac{dy}{dt} = -y + x^2$$

(b) State and prove Bendixson non-existence theorem for closed paths.