(c) Give example of noetherian ring which is not Artinian.

(d) State Hilbert basis theorem.

(e) Define primary modules.

(f) State Wedderburn-Artin theorem.

(g) Define invariant subspace of a L. T.

(h) Define cycle subspace of a nilpotent L. T.

Roll No. ....

74451

M. Sc. Mathematics 2nd Semester Examination – May, 2016

ADVANCED ABSTRACT ALGEBRA-II

Paper : MM-421

Time : Three Hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting at least *one* question from each Unit.

UNIT – I

1. Prove that any finite abelian group is a direct product of cyclic groups. 16

2. (a) Let M be a finitely generated free module over a commutative ring R, then prove that all bases of M are finite. 8

(b) State and prove fundamental theorem of R-module homomorphism. 8

UNIT – II

3. (a) In a left noetherian ring, every nil ideal is nilpotent. 8

(b) A Boolean noetherian ring is finite and is a finite direct product of fields with two elements. 8

4. (a) Prove that an R-module M is artinian iff every non-zero family of submodules has a minimal element. 12

(b) Let R be a P. I. D. and $I \neq \{0\}$ be an ideal of R, then R/I is both noetherian and artinian. 4

UNIT – III

5. (a) Let R be a left Artinian ring with unity having no non-zero nilpotent ideals. Then, every left ideal of R is generated by an idempotent. 8

(b) Let M be a noetherian module or any module over a noetherian ring. Then, each non-zero submodule of M contains a uniform module. 8

6. (a) State and prove Maschke theorem. 8

(b) Let M be a finitely generated module over a commutative noetherian ring R. Then, there exists a finite family $N_1, N_2, \ldots, N_t$ of submodules of M such that : 8

$$\bigcap_{i=1}^{t} N_i = \{0\} \text{ and } \bigcap_{\substack{i=1 \\ i \neq i_0}}^{t} N_i \neq \{0\} \text{ for all } 1 \leq i_0 \leq t$$

UNIT – IV

7. (a) Let V be a vector space and $T \in A(V)$ be such that the matrix of T in two different bases be A and B. Prove that A and B are similar matrices. 8

(b) Let $T \in A(V)$ and suppose $\lambda_1, \lambda_2, \ldots, \lambda_k \in f$ be the distinct roots of f(x) the minimal polynomial of T. Then obtain the Jordan form of T. 8

8. State and prove primary decomposition theorem. 16

UNIT – V

9. (a) Define semi-simple module. 16

(b) Prove that intersection of two submodules is a submodule.