Roll No. ....

78451

M. Sc. (Mathematics) 4th Semester

Examination – December, 2014

FUNCTIONAL ANALYSIS - II

Paper : MM-521

Time : Three hours] *[Maximum Marks : 80*

Before answering the question, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question No. 9 from Section V is *compulsory*.

SECTION – I

1. (a) State and prove Hahn Decomposition Theorem. 10

   (b) Give an example to show that the Hahn decomposition need not be unique and is unique except for nullsets. 6

2. (a) Prove that product of two measures is again a measure. 4

(b) State and prove Fubini's Theorem. 12

SECTION – II

3. (a) State and prove Riesz Markov Theorem. 12

(b) Define regularity of Baire Measure. 4

4. (a) Prove that $c[a,b]$ is not a Hilbert space. 12

(b) Prove that every one dimensional normed space is an miner product space. 4

SECTION – III

5. (a) Give an example of a convex set. Show that a closed convex subsect C of a Hilbert space H contains a unique vector of smallest norm. 8

(b) Show that every nonzero Hilbert space contains a complete orthognormal set. 8

6. (a) State and prove Riesz Representation Theorem for Hilbert spaces. 12

(b) Define conjugate of a Hilbert space. 2

(c) State Bessel's inequality. 2

SECTION – IV

7. (a) Define the adjoint operation of linear operator on a Hilbert space. Prove that the set of all self adjoint operators on a Hilbert space H form a closed linear subspace of the space of linear operators on H which is a real Banach space and contains identity transformation. 12

(b) Define normal operator on a Hilbert space. 2

(c) State Spectral Theorem. 2

8. (a) Show that an operator on H is unitary if and only if it is an isometric isomorphism of H onto itself. 8

(b) Give an example of an operator on a certain Hilbert space which is an isometry but is not a unitary operator. 4

(c) Show that if N is a normal opearator on the Hilbert space H, then : 4

$$\|N^2\| = \|N\|^2$$

SECTION – V

9. (i) State Lebesgue Decomposition Theorem.

(ii) State Projection Theorem.

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(iii) Define Borel sets.

(iv) Define Baire Measure.

(v) Define Lebesgue Stieltjes Integral.

(vi) Is every Hilbert space reflexive ? Discuss.

(vii) Define orthonormal sets.

(viii) In sum of two projections always a projection ? Discuss. 16