Roll No. ....

78452

M. Sc. (Mathematics) 4th Semester Examination – December, 2014

CLASSICAL MECHANICS

Paper : MM-522

Time : Three Hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Unit. Question No. 9 (Unit - V) is *compulsory*. All questions carry equal marks.

UNIT – I

1. (a) Discuss free and constrained systems. Explain with the help of examples.

(b) Discuss general equation of dynamics and derive Lagrange's equations of first kind.

2. Two ponderable particles M_1 and M_2 of identical masses m = 1, are joined by a rod of invariable length l and negligible small mass. The system is constrained to move in the vertical plane and only in such a manner that the velocity of the mid point of rod is directed along it. Determine the motion of particles M_1 and M_2.

UNIT – II

3. (a) A particle of mass m moves in a plane. Find its equations of motion in plane polar coordinates.

   (b) Prove that the kinetic energy of a scleronomic system can be expressed as a homogeneous function of second degree in the generalized velocities.

4. (a) Define Lagrangian and Hamiltonian variables. Show that Hamiltonian variables may be expressed in terms of Lagrange variables.

   (b) State and prove Donkin's theorem.

UNIT – III

5. (a) Using cylindrical, polar coordinates, derive Hamilton's equations of motion for a particle of mass m moving in a force field of potential $V(r, \theta, z)$.

   (b) Derive Whittaker's equations.

6. (a) Derive Jacobi equations.

   (b) Define Lagrange action and prove the principle of Least action. Also establish the relation between Lagrange action and Hamilton action.

UNIT – IV

7. (a) State and prove the necessary and sufficient condition for the transformation :

$$\tilde{q}_i = \tilde{q}_i(t, q_k, p_k)$$

$$\tilde{p}_i = \tilde{p}_i(t, q_k, p_k) \text{ ; with the}$$

condition $\dfrac{\partial(\tilde{q}_1, \tilde{p}_1, \tilde{q}_2, \tilde{p}_2, \ldots\ldots\ldots \tilde{q}_n, \tilde{p}_n)}{\partial(q_1, p_1, q_2, p_2, \ldots\ldots\ldots q_n, p_n)} \neq 0$

to be canonical.

   (b) Show that the transformation $Q = \log\left(\frac{1}{q}\sin p\right)$, $P = q \cot p$ is canonical.

8. (a) Derive Hamilton - Jacobi equations.

   (b) Prove that Lagrange's Bracket is invariant under a free univalent canonical transformation.

UNIT – V

9. (a) Define virtual displacement.

(b) State D'Alembert's principle.

(c) Define ideal constraints.

(d) Define Hamiltonian function.

(e) Define Poisson bracket and Lagrange bracket.

(f) State Hamilton's Principle.

(g) Define complete integral.

(h) Prove that the necessary and sufficient condition for the function $f(t, q_i, p_i)$ to be the integral of the equations $\frac{dq_i}{dt} = \frac{\partial H}{\partial p_i}, \frac{dp_i}{dt} = -\frac{\partial H}{\partial q_i}$ $(i = 1, 2, —, n)$ is that $\frac{\partial f}{\partial t} + (f, H) = 0$.