Roll No. ....

78454

M. Sc. Mathematics 4th Sem.

Examination – May, 2014

ADVANCED DISCRETE MATHEMATICS

Paper : MM-524 (A2)

Time : Three hours] *[Maximum Marks : 80*

Before answering the question, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* questions from each Section. Section V are *compulsory*.

SECTION – I

1. (a) Verify that the proposition $pv[(p\wedge\sim q)\rightarrow r]$ is a tautology. 8

   (b) Show that $p\rightarrow(qvr)$ and $(p\rightarrow q)v(p\rightarrow r)$ statements are logically equivalent. 8

2. (a) Without using truth tables show that : 8

$$(p\,v\,q)\wedge\sim p\equiv\sim p\wedge q$$

(b) Negate each of the statements :

(i) $\sim(\exists x \in A)(x+3=10)$

(ii) $\sim(\forall x \in A)(x+3<10)$

(iii) $\sim(\exists x \in A)(x+3<5)$

(iv) $\sim(\forall x \in A)(x+3 \le 7)$

SECTION – II

3. (a) Consider the set N of positive integers and let * be the operation of lcm on N. 8

(i) Find 4 * 6, 3 * 5, 9 * 18 and 1 * 6

(ii) Is (N, *) a semigroup ? Is it commutative ?

(iii) Find the identity elements of *.

(b) Prove that Homomorphic image of an abelian semigroup is abelian. Is converse true ? Explain, 8

4. (a) Let (Z, +) be a semigroup of integers and + defines an ordinary addition relation let $f(x)=x^2-5x+6$. Define a relation R on Z such that aRb iff f(a) = f(b) Is R a congruence relation. 8

(b) How many distinct integers must be chosen to assure that there are atleast 10 having the same congruence class modulo 7 ? 8

SECTION – III

5. (a) If $(L_1,\le)$ and $(L_2,\le)$ are lattices. Then show that $(L,\le)$ is a lattice where $(L, L_1 \times L_2)$ and $\le$ partial order of L is product partial order. 8

(b) Let $(L_1 \;\le)$ be a lattice. Then show that for a, b, c d $\in$ L if $a \le b$ and $c \le d$ then (i) $a \vee c \le b \vee d$ and (ii) $a \wedge c \le b \wedge d$. 8

6. (a) Let L be a bounded distributive lattice prove that if a complement of any element exists, H is unique. 8

(b) Let L be a complemented lattice with unique complements then prove that the irreducible elements of L other than o are its atoms. 8

SECTION – IV

7. (a) If $n = p_1 p_2 \dots p_k$ where p_i's are distinct primes. Prove that D_n is a Boolean algebra. 8

(b) Prove De-Margan's Law in Boolean algebra. 8

8. (a) Let B be a finite Booalean algebra and A be the set of atoms in B. If P(A) is the Booleanalgebra of all subsets of the set of atoms, then the mapping f : B $\to$ P (A) is an isomorphism. Verify this result with an example. 8