Roll No. ....

78454

M. Sc. (Mathematics) 4th Semester

Examination – December, 2014

ADVANCED DISCRETE MATHEMATICS

Paper : MM-524 (A2)

Time : Three Hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in all, selecting ***one*** question from each Unit – I, II, III, IV. Unit – V (Q. No. 9) is ***compulsory***. All questions carry equal marks.

UNIT – I

1. (a) Show that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology. 8

   (b) Check for validity, the argument $p \to q ; \sim q, \therefore \sim p$. Also verify it with an example. 8

2. (a) Use a diagram to show the invalidity of the following : 8

   (i) All man are strong,

   (ii) Ritu is strong,

   (iii) $\therefore$ Ritu is a man.

(b) Define universal conditional statement and give an example of it. 8

UNIT – II

3. (a) Define semi-group and show that the inverse, if exists, of every element in a semi-group with identity 'e' is unique. 8

(b) Let Z be the set of integers and T be the set of all even integers. Then, show that (Z, +) is a semi-group and (T, +) is its sub-semi group. 8

4. (a) Let (Z, +) be the semi-group of integers and let $f(x) = x^2 - x - 2$. Let R be the relation defined by aRb iff $f(a) = f(b)$. 8

Show that it is an equivalence relation but not congruence.

(b) If (S, *) and (T, *') are monoids. Then, show that (S × T, *") is also a monoid w.r.t. *" operation defined as $(a_1, b_1) *'' (a_2, b_2) = (a_1 * a_2, b_1 *' b_2)$ 8

UNIT – III

5. (a) Let L be a lattice. Then for every a, b in L, prove the following : 8

(i) $a \vee b = b$ iff $a \leq b$

(ii) $a \wedge b = a$ iff $a \vee b = b$

(b) Show that a lattice with three or fewer elements is a chair. 8

6. (a) Show that D_{12} is a distributive lattice whereas (A, ≤) where A = {1, 2, 3, 4, 12} and '≤' is partial order relation of divisibility is not distributive. 8

(b) Let L be a complemented lattice with unique complements. Then, the join irreducible elements of L, other than O, are atoms. 8

UNIT – IV

7. (a) Show that the poset D_{30} = {1, 2, 3, 5, 6, 10, 15, 30} with $a \vee b$ = lcm {a, b} ; $a \wedge b$ = gcd {a, b} and $a' = \frac{30}{9}$ is a Boolean algebra. Find its zero and unit element. 8

(b) Let a be any element of a Boolean algebra B. Then prove : 8

(i) Complement of 'a' is unique,

(ii) $(a')' = a$

(iii) $0' = 1$ and $1' = 0$

8. (a) Show that :

(i) $(x_1' x_2' x_3' x_4') + (x_1' x_2' x_3' x_4) + (x_1' x_2' x_3 + x_4) + (x_1' x_2' x_3 x_4') = x_1' x_2'$

(ii) Express $P(x, y, z) = x(y'z)'$ in its complete sum-of-product form in three variables x, y, z. 8

P. T. O.

(b) Show that the following Boolean expressions are equivalent to one-another. Obtain their sum-of-product canonical form. 8

(i) $(x+y)(x'+z)(y+z)$

(ii) $xz+x'y$

(iii) $(x+y)(x'+z)$

UNIT – V

9. (a) Define a tautology. 16

(b) Define a quantifier.

(c) Define semigroup.

(d) Differentiate between equivalence relation and congruence relation.

(e) State Pigeonhole principle.

(f) Give example of a complete lattice.

(g) Differentiate Atoms and Minterms.

(h) Give example for AND Gate to explain it.