Roll No. ....

78453

M. Sc. (Mathematics) 4th Semester Examination – December, 2014

COMPLEX ANALYSIS - II

Paper : MM-523

Time : Three Hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Unit. Questions No. 9 is *compulsory*. All questions carry equal marks.

UNIT – I

1. (a) State and prove Weierstrass factorization theorem.

(b) Construct Euler's Gamma function and hence define Euler's constant.

2. (a) Prove that :

$$\sqrt{\pi}\,\Gamma(2z) = 2^{2z-1}\,\Gamma(z)\,\Gamma\left(z+\frac{1}{2}\right)$$

(b) State and prove Mittag - Leffler's theorem.

UNIT – II

3. (a) If $f(z) = \sum_{n=0}^{\infty} z^{2n}$, show that $f(z) = z + f(z^2)$ and $|z| = 1$ is a natural boundary of this function.

(b) State and prove Schwarz's Reflection Principle.

4. (a) What is Poisson Kernel ? Describe its various properties.

(b) If $u : G \to R$ is continuous function which has the mean value property then prove that u is harmonic.

UNIT – III

5. (a) State and prove Poisson - Jensen formula.

(b) Find the order of the functions $\cos\sqrt{z}$ and $\sin z$.

6. (a) Using Hadmard's factorization theorem, show that :

$$\sin \pi z = \pi z \prod_{n=1}^{\infty}\left(1-\frac{z^2}{n^2}\right)$$

(b) Show that the order of a canonical product is equal to the convergence exponent of its zeros.

UNIT – IV

7. (a) If f is analytic on a region containing the closure of the disk $D = \{z : |z| < 1\}$ and $f(0) = 0$, $f'(0) = 1$ then prove that $f(D)$ contains a disk of radius L, where L is Landau's constant.

(b) If f is an entire function that omits two values then show that f is a constant.

8. (a) State & prove Schottky' theorem.

(b) Define univalent functions and show that if $f(z)$ is univalent in D then $f'(z) \neq 0$ in D.

UNIT – V

9. (i) Find the order of the functions :

$$f(z) = a_0 + a_1 z + a_2 z^2 + \ldots\ldots\ldots + a_n z^n,\ a_n \neq 0$$

(ii) Define subharmonic and superharmonic functions.

(iii) State Montel coratheodory theorem.

(iv) Prove that $\Gamma(1-z)\,\Gamma(z) = \dfrac{\pi}{\sin \pi z}$

(v) Define exponent of convergence of zeros of an entire function $f(z)$.

(vi) State Monodromy theorem.

(vii) Define Natural boundary for an analytic function.

(viii) Define FEP homotopic.
