

78453

M. Sc. (Mathematics) 4th Semester Examination - December, 2014

COMPLEX ANALYSIS - II

Paper: MM-523

Time: Three Hours 1

I Maximum Marks: 80

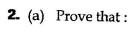
Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note: Attempt five questions in all, selecting one question from each Unit. Questions No. 9 is compulsory. All questions carry equal marks.

UNIT - I

- 1. (a) State and prove Weierstrass factorization theorem.
 - (b) Construct Euler's Gamma function and hence define Euler's constant.

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$$\sqrt{\pi} \Gamma(2z) = 2^{2z-1} \Gamma(z) \Gamma\left(z + \frac{1}{2}\right)$$

(b) State and prove Mittag - Leffler's theorem.

UNIT - II

- 3. (a) If $f(z) = \sum_{n=0}^{\infty} z^{2n}$, show that $f(z) = z + f(z^2)$ and |z| = 1 is a natural boundary of this function.
 - (b) State and prove Schwarz's Reflection Principle.
- **4.** (a) What is Poisson Kernel? Describe its various properties.
 - (b) If $u: G \to R$ is continuous function which has the mean value property then prove that u is harmonic.

UNIT - III

- 5. (a) State and prove Poisson Jensen formula.
 - (b) Find the order of the functions cos \sqrt{z} and sin z.
- **6.** (a) Using Hadmard's factorization theorem, show that:

$$\sin \pi z = \pi z \prod_{n=1}^{\infty} \left(1 - \frac{z^2}{n^2} \right)$$

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(b) Show that the order of a canonical product is equal to the convergence exponent of its zeros.

UNIT - IV

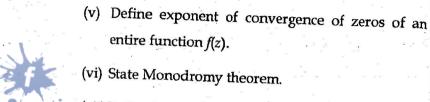
- 7. (a) If f is analytic on a region containing the closure of the disk $D = \{z : |z| < 1\}$ and f(0) = 0, f'(0) = 1 then prove that f(D) contains a disk of radius L, where L is Landau's constant.
 - (b) If *f* is an entire function that omits two values then show that *f* is a constant.
- 8. (a) State & prove Schottky' theorem.
 - (b) Define univalent functions and show that if f(z) is univalent in D then $f'(z) \neq 0$ in D.

UNIT - V

9. (i) Find the order of the functions:

$$f(z) = a_0 + a_1 z + a_2 z^2 + \dots + a_n z^n, a_n \neq 0$$

- (ii) Define subharmonic and superharmonic functions.
- (iii) State Montel coratheodory theorem.
- (iv) Prove that $\Gamma(1-z) \Gamma(z) = \frac{\pi}{\sin \pi z}$



(vii) Define Natural boundary for an analytic function. (viii) Define FEP homotopic.

