(ii) What is ment by Bieberbach's conjecture ?

(iii) List the properties satisfied by Poisson Kernel.

(iv) Define FEP homotopic relation.

(v) Find the order of the function.

$$f(z) = a_0 + a_1 z + a_z z^2 + \dots + anz^n, an \neq 0$$

(vi) Define Green's function.

(vii) State $\frac{1}{4}$ theorem.

(viii) What is natural boundry ?

Roll No. ....

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M. Sc. Mathematics 4th Sem.

Examination – May, 2014

COMPLEX ANALYSIS - II

Paper : MM-523

Time : Three hours] *[Maximum Marks : 80*

Before answering the question, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* questions from each Unit. Question No. 9 is ***compulsory***. All questions carry equal marks.

UNIT – I

1. (a) If $z_1, z_2, \ldots, z_n, \ldots$ be any sequence of numbers whose only limit point is the point at infinity, then it is possible to construct an integral function which vanishes at each of the point z_n and nowhere else.

   (b) Derive stirling formula.

2. (a) Prove that :

$$\varsigma(z) = 2(2\pi)^{z-1}\Gamma(1-z)\varsigma(1-z)\sin\left(\frac{1}{2}\pi z\right)$$

(b) Derive Riemann's functional equation.

UNIT – II

3. (a) Show that the circle of convergence of the power series $f(z) = 1 + z + z^2 + z^4 + z^8 + \ldots$ is a natural boundry of its sum function.

(b) State and prove uniquencess of Analytic continuation along a curve.

4. (a) State and prove monodromy theorem.

(b) Outline the Dirichlet problem for a unit disc.

UNIT – III

5. (a) State and prove Harnack's theorem.

(b) Let ρ be the order of an integral function f(z) then show that :

$$\rho = \lim_{r\to\infty} \sup \frac{\log \log (\mu/r)}{\log r}$$

Where $\mu(r) = \max / f(z) /$ on $|z| = r$.

6. (a) Prove that if $f(z)$ is an entire function of order ρ and convergence exponent σ then $\sigma \le \rho$.

(b) Let $p(z)$ be a canonical product of finite order ρ and $q > 0, \epsilon > 0$. Then show that for all sufficiently large $|z|$.

$$|z - z_i| > |z_i|^{-q} \text{ implies } \log|p(z)| > -|z|^{\rho+\epsilon}$$

UNIT – IV

7. (a) Let f be analytic function on a region containing the closure of B(0; R); then f(B(0; R)) contains a disk of radius $\frac{1}{72}R|f'(0)|$.

(b) State and prove Little Picard Theorem.

8. (a) Let f be an analytic function in a region containing the closure of the dise $D = \{z : |z| < 1\}$ and $f(0) = 0$, $f'(0) = 1$. Show that f(D) contains a dise of radius L, where L is Landan's constant.

(b) State and prove great picard theorem.

UNIT – V

9. (i) State Mittag leffler's theorem.