(b) Let X and Y be normed space and $T : X \to Y$ be a compact operator. Suppose : $x_n \xrightarrow{w} x$, Then $<T(x_n)>$ is strongly convergent in Y and $y = T(x) = \lim_{h\to\infty} T(x_n)$ (6)

UNIT - V

9. (a) Give example of a space which is metric but not normed linear space. (2)

(b) Define incomplete normed linear space. (2)

(c) Differentiate between bounded linear transformation and continuous linear functional. (2)

(d) Define conjugate space of a normed linear space. (2)

(e) State uniform boundedness principle. (2)

(f) State open mapping theorem. (2)

(g) Define equivalent norms and compact operators. (2)

(h) Define weak and strong convergence. (2)

Roll No. ....

76451

M.Sc. Mathematics 3rd Semester Examination–December, 2015

FUNCTIONAL ANALYSIS-I

Paper : MM-511

Time : 3 hours ***Max. Marks : 80***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after the examination.

Note : Attempt five questions in all, selecting one question from each Unit I,II,III and IV. Unit-V is compulsory.

UNIT - I

1. (a) State and prove Riesz-Fisher theorem. (8)

(b) Let M be a closed linear subspace of a normed linear space N. If the norm of $x + M \in N/m$ is defined by $||x + M|| = \inf \{||x + m|| : m \in M\}$. Then N/m is a normed linear space. Further prove that if N is a Banach space, then so is N/m. (8)

2. (a) Prove that the space l_p^n is a Banach space. (12)

(b) Prove that in normed linear space, every convergent sequence is Cauchy sequence. (4)

UNIT - II

3. (a) Prove that every finite dimensional subspace Y of X of a normal space X is complete. (10)

(b) Let N and N′ be normed linear spaces and let T be a linear transformation of N into N′. Then, the inverse T^{-1} exists and is continuous iff ∃ a constant m > 0 such that $m\|x\| \le \|T(x)\| \ \forall x \in N$. (6)

4. (a) State and prove Hahn Banach theorem. (12)

(b) A non-empty subset X of a normed linear space N is bounded iff f(x) is a bounded set of numbers for each f in N*. (4)

UNIT - III

5. (a) Prove that $\left(l_\infty^n\right)^* = l_1^n$ (8)

(b) State and prove closed graph theorem. (8)

6. (a) If P is a projection on a Banach space B, and if M and N are range and null spaces, then M and N are closed linear subspaces of B such that B = M ⊕ N. (6)

(b) State and prove Riez-Representation theorem for bounded linear functionals on C[a,b]. (10)

UNIT - IV

7. (a) Prove that on a finite dimensional space, all norms are equivalent. (12)

(b) In a finite dimensional space prove that the notion of weak and strong convergence are equivalent. (4)

8. (a) State and prove closed range theorem. (10)