

Roll No.

76456

**M.Sc. Mathematics 3rd Semester
Examination-December, 2015**

ANALYTICAL NUMBER THEORY-I

Paper : MM-515(CI)

Time : 3 hours Max. Marks : 80

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard will be entertained after the examination.

Note : Attempt **five** questions in all selecting **one** from each Unit.

UNIT - I

1. (a) Prove that every natural number $N > 1$ has a prime divisor and, hence, primes are infinite. 8
- (b) If $\frac{h}{k}, \frac{h'}{k'}$ are two consecutive members of F_n , then prove that $k \neq k'$ if $n > 1$. 8

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- (b) Let a be an odd integer, then prove that 5

- (i) $a \in Q_2$
- (ii) $a \in Q_4$ iff $a \equiv 1 \pmod{4}$
- (iii) If $c \geq 3$, then $a \in Q_{2^c}$ iff $a \equiv 1 \pmod{8}$
- (c) Prove that Q_n is a subgroup of U_n . 5

UNIT - V

9. (a) Define Mersenne numbers. 2
- (b) State Hurwitz Theorem 2
- (c) Define $g(k)$ and $G(k)$. 2
- (d) Define Disphantine equations and give two examples. 2
- (e) Define Legendre's symbol 2
- (f) Prove that Z_n is a group 2
- (g) Find generators of U_{25} 2
- (h) Find order of Q_{256} 2

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UNIT - III

2. (a) Let α be any rational number then prove that $\exists$ only a finite number of pairs of integers (x, y) such that

$$x > 0, \gcd(x, y) = 1 \text{ and } \left| \alpha - \frac{y}{x} \right| < \frac{1}{x^2}$$

- (b) Prove that F_n is composite and $F_n \equiv 7 \pmod{10}$ for $n \geq 2$, where F_n represents n th Fermat number.

UNIT - II

3. (a) State and prove Euler's theorem and, hence, deduce Fermat's theorem.

- (b) If all primes of type $4k + 3$ occur to an even degree in the prime factorization of a natural number n , then n can be written as a sum of two squares.

4. (a) Prove that $G(2) = 4$.

- (b) Let p be a prime such that $p > 2$, then

$$G(p^0(p-1) \geq p^{0+1})$$

5. (a) Let p be an odd prime and let $\gcd(a, p) = 1$, then

$$\left[\frac{p-1}{2} \right]! \equiv - \left(\frac{a}{p} \right) a^{\frac{p-1}{2}} \pmod{p}$$

- (b) State and prove Gauss lemma.

6. State and prove quadratic law of reciprocity and, hence, evaluate

$$\left(\frac{202}{257} \right)$$

UNIT - IV

7. (a) If p is an odd prime, then prove that U_{p^c} is cyclic for all $c \geq 1$.

- (b) If $e \geq 3$, then $U_{2^e} = \{ \pm 5^i : 0 \leq i \leq 2^{e-2} \}$

8. (a) Let $n = n_1 n_2 \dots n_k$, where the integers n_i are mutually co-prime. Then $a \in Q_n$ iff $a \in Q_{n_i}$ for each i .