UNIT – V

8. (i) Describe Fourier series representation of a function and list necessary conditions. Represent the function $f(x) = \pi + x$ when $-\pi \le x < 0$; and $f(x) = \pi - x$ when $0 < x \le \pi$. 12

(ii) Obtain inverse Laplace transform of the function 4

$$g(s) = (s^2)/(s^2 + a^2)(s^2 + b^2)$$

9. (i) Describe the mechanism of Fourier transform of Derivatives and obtain the transformation of a nth order derivative. 10

(ii) Differentiate between Fourier Sine and Fourier cosine transforms. Find out the Fourier sine transform of e^{-3t} 6

Roll No. ....

72601

M. Sc. (Physics) 1st Sem.

Examination – December, 2015

MATHEMATICAL PHYSICS

Paper : I

Time : Three Hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Unit. Question No. 1 is ***compulsory***. All questions carry equal marks.

UNIT – I

1. (i) If B is a skew-symmetric matrix and $B^2 + I = Z$, then show that B is an orthogonal matrix. 4

(ii) Show that between any *two* consecutive zeroes of $J_n(x)$ there is one and only one zero of $J_{n+1}(x)$. 4

(iii) Show that $P_n(\cos\theta)$ can be expressed as a series consisting of cosines of even or odd integer multiples of θ. 4

(iv) Find the Fourier series for the periodic function $f(x)$ defined by $f(x) = -\pi$ if $-\pi < x < 0$ and $f(x) = \pi$ if $0 < x < \pi$. 4

UNIT – II

2. (i) Show that the eigenvectors corresponding to distinct eigenvalues of a matrix are linearly independent. 4

(ii) Using the process of similarity transformation obtain the normalized eigenvectors of the matrix

$$C = \begin{matrix} 1 & 3 & -2 \\ 1 & 2 & 4 \\ -2 & 5 & 2 \end{matrix}$$

Also check if the eigenvectors are orthogonal. 12

3. (i) For α, β, γ being a linearly independent set of vectors, find out if α – β, β – γ, γ – α are linearly independent. 4

(ii) Define a linear vector space. Describe dimension and basis of a linear vector space and give at least two examples. Obtain a set of four orthonormal vectors from the following four linearly independent vectors : P = (1, 0, 0, 1) Q = (1, 1, 0, 2) R= (1, 1, 2, –3) S = (1, 1, 1, 1). 12

UNIT – III

4. Using the method of Frobenius, obtain linearly independent solutions of the equation $x^2 d^2y/dx^2 + x(dy/dx) + (x^2 - m^2) = 0$: m = constant Discuss various possibilities arising due to choice of m. Whether the two solutions are linearly independent, in case, m is an integer ? Justify your answer. 16

5. (i) Define Wronskian and discuss its application. Show by means of Wronskian that a linear, second order, homogeneous differential equation

$$y'' + P(x)y' + Q(x)y = 0$$

can not have three independent solutions. 10

(ii) Differentiate between removable and non-removable singularity and give suitable examples. Determine the nature of a point at infinity in case of 6

$$d^2y/dx^2 + 4x\, dy/dx + 4y = 0$$

UNIT – IV

6. (i) State and prove orthogonality property of Legendre polynomials and derive Rodrigue's relation. 10

(ii) Show that for Laguerre polynomials : 3

(a) $L_n'(x) = nL'_{n-1}(x) - nL_{n-1}(x)$

(b) $(2m-1)\, P_{m-1}(x) + P'_{m-2}(x) = P'_m(x)$

(iii) Prove that for Bassel functions : 3

$(\pi x/2)\, J_{3/2}(x) = (\sin x/x) - \cos x$

7. (i) State and prove orthogonality relation for Bessel functions. 12

(ii) Prove that $J_0^2(x) + 2\sum_{m=1}^{\infty}(-1)^m J_m^2(x) = J_0(2x)$ 4