(b) Define simple closed curve. 2

(c) State monotonicity property of outer measure. 2

(d) Show that [0, 1] is uncountable. 2

(e) Define step function with an example. 2

(f) State Approximation Theorem. 2

(g) Define Restriction of a function. 2

(h) State Lebesgue Bounded Convergence Theorem. 2

Roll No. ....

72452

M. Sc. 1st Sem. (Mathematics)

Examination – December, 2015

REAL ANALYSIS-I

Paper : MM-412

Time : Three Hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each Section. Question from **Section – V** is *compulsory.*

SECTION – I

1. (a) If f is continuous on $[a, b]$. Then prove that for every $\in > 0$, there exists a $\delta > 0$ such that $\left|\sum_{i=1}^{n} f(t_i)\Delta\alpha_i - \int_a^b f d\alpha\right| < \in$ for every partition P of $[a, b]$ with max $\Delta\alpha_i < \delta$ and for every choice of points $t_i \in [x_{i-1}, x_i]$. 8

(b) If $f \in R(\alpha_1)$ and $R(\alpha_2)$ on $[a, b]$, then prove that $f \in R(\alpha_1 + \alpha_2)$ on $[a, b]$ and : 8

$$\int_a^b fd(\alpha_1 + \alpha_2) = \int_a^b fd\alpha_1 + \int_a^b fd\alpha_2$$

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2. (a) If $\lim_{\|p\|\to 0} S(P, f, \alpha)$ exists then prove that $f \in R(\alpha)$ and $\lim_{\|p\|\to 0} S(P, f, \alpha) = \int_a^b f d\alpha$. 8

(b) State and prove second mean value theorem for Riemann Stieltjes Integral. 8

SECTION – II

3. (a) If E_1 and E_2 are measurable sets. Then prove that $E_1 \cup E_2$ is also measurable. 8

(b) Prove that Every Borel set in IR is measurable. 8

4. (a) Let $\{E_i\}$ be an infinite increasing sequence of measurable sets. Then show that : 8

$$m\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m(E_n)$$

(b) Let $E \subset [0, 1]$ be a measurable set and $y \in [0, 1]$ be given. Then prove that set $E \overset{\circ}{+} y$ is measurable and $m\left(E \overset{\circ}{+} y\right) = m(E)$. 8

SECTION – III

5. (a) Prove that a continuous function defined on a measurable set is measurable. 8

(b) Prove that a function f is measurable if and only if f^+ and f^- are measurable. 8

6. (a) If a function f defined on a measurable set E is continuous a.e. Then show that it is measurable on E. 8

(b) Let $\{f_n\}$ be a sequence of measurable functions that converges in measure to f then prove that there is a subsequence $\{f_{n_k}\}$ of $\{f_n\}$ which converges to f a.e. 8

SECTION – IV

7. (a) Let f be bounded function defined on a measurable set E with measure finite. Then prove that f is measurable if and only if : 8

$$\inf_{\Psi \geq f} \int_E \Psi(x) dx = \sup_{\phi \leq f} \int_E \phi(x)\, dx$$

for all simple functions ϕ and Ψ.

(b) If $\int_E f = 0$ and $f \geq 0$ on E then show that $f = 0$ a.e. 8

8. (a) State and prove Fatou's Lemma. 8

(b) Let f be a non-negative function which is integrable over a set E. Then prove given $\epsilon > 0$, there exist a $\delta > 0$ such that for every $A \subset E$ with $m(A) < \delta$, we have : 8

$$\int_A f < \epsilon$$

SECTION – V

9. (a) State first mean value theorem for Riemann Stieltjes integral. 2