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72456

M. Sc. (Mathematics) 1st Semester Examination – December, 2015

MATHEMATICAL STATISTICS

Paper : MM-415-B

Time : Three Hours] *[Maximum Marks : 80*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt *five* questions in all, selecting *one* question from each of the *four* Units(**I**, **II**, **III** and **IV**) and question No. **9** is *compulsory*.

UNIT – I

1. (a) Give axiomatic definition of probability for any two events A and B, prove that :

$$P(\overline{A} \cap B) = P(B) - P(A \cap B).$$

(b) Show that for two events A and B,

$$P(A \cap B) \le P(A) \le P(A \cup B) \le P(A) + P(B).$$

(c) State and prove multiplication theorem of probability. 5, 5, 6

2. (a) For 'n' events $A_1, A_2, \ldots, A_n$ prove that :

$$P\left[\bigcup_{i=1}^{n} A_i\right] \leq \sum_{i=1}^{n} P(A_i).$$

(b) A and B alternatively cut a pack of cards and the pack is shuffled after each cut. If A starts and the game is continued until one cuts a diamond, what are the chances of A first cutting a diamond ?

(c) Consider a lot of electric bulbs that contains 30% of brand A, 45% of brand B and 25% of brand C. It is known that 2% of brand A, 3% of brand B and 1% of brand C are defective. What is the probability that a randomly selected bulb from the lot is defective ? If the drawn bulb is defective, what is the probability that this was of the brand A ? 4, 6, 6

UNIT – II

3. (a) Define the following terms :

(i) Random variable,

(ii) probability density function,

(iii) marginal probability mass function, and

(iv) conditional density function.

(b) A random variable 'X' has the following probability distribution :

Value of X, x :	0	1	2	3	4	5	6	7	8
p(x) :	k	3k	5k	7k	9k	11k	13k	15k	17k

(i) Determine the value K,

(ii) Find $P(X \geq 3)$, and

(iii) What is the smallest value of 'x' for which $P(X \leq x) > 0.5$?

(c) What do you mean by mathematical expectation of a random variable ? A coin is tossed until a head appears. What is the expectation of the number of tosses required ? 4, 6, 6

4. (a) For two independent random variables X and Y, prove that :

$$V(X \pm Y) = V(X) + V(Y).$$

(b) Define moment generating function of a random variable. Discuss the effect of change of origin and scale on it.

(c) What do you mean by 'covariance' between two variables ? Show that :

$$Cov\left(\frac{X+a}{c}, \frac{Y+b}{d}\right) = \frac{1}{cd} Cov(X, Y)$$

Where a, b, c and d are constants. 6, 5, 5

UNIT – III

5. (a) Define binomial distribution. Obtain its moment generating function. Hence find its mean and variance.

(b) If X is a Poisson variate such that :

$$P(X = 1) = 2P(X = 2)$$

Find :

(i) the variance of X,

(ii) $P(X = 0)$, and

(iii) $P(X = 2)$.

(c) Write a short note on exponential distribution.

7, 5, 4

6. (a) What are the chief characteristics of normal curve and normal distribution ?

(b) Obtain mean and variance of the uniform distribution :

$$f(x) = \frac{1}{2c}, -c \le x \le c$$

(c) Explain Bernoull distribution and find its r^{th} raw moment. 8, 5, 3

UNIT – IV

7. (a) Define the following and give one example of each :

(i) Parameter,

(ii) Sampling distribution,

(iii) Point estimator, and

(iv) Composite hypothesis.

(b) Show that $\frac{\sum x_i(\sum x_i - 1)}{n(n-1)}$ is an unbiased estimate of θ^2, for the sample $x_1, x_2, \ldots, x_n$ drawn on X which takes the values 1 or 0 with respective probabilities θ and $(1 - \theta)$.

(c) What are two types of errors in testing of hypothesis ? Explain. Also define 'size' and 'power' of the test. 6, 6, 4

8. (a) Explain testing of significance of 'single proportion' and 'difference of two proportions' in case of large samples.

(b) In a random sample of 500, the mean is found to be 20. In another independent sample of 400, the mean is 15. Test whether the samples have been drawn from the same population with standard deviation 4. [Given $z_{.05} = 1.96$].

(c) Write a short note on 'critical region'. 8, 6, 2)

UNIT – V

9. (a) Define 'equally likely' and 'mutually exclusive' events.

(b) What do you mean by 'conditional probability' ? Give suitable example.

(c) Give important properties of distribution function of a random variable.

(d) State Chebychev's inequality. Give its importance.

(e) Mention important properties of a geometric distribution.

(f) Under what conditions, normal distribution is limiting forms of 'Binomial' and 'Poisson' distribution ?

(g) Explain 'efficiency' of an estimator.

(h) Differentiate between 'one-tailed' and 'two-tailed' tests. 2 × 8
