UNIT – 4

7. (a) Solve the variational problem :

$$J[y] = \int_1^2 \left[\frac{1}{x}\sqrt{1+(y')^2}\right] dx, \quad y(1)=0, \quad y(2)=1.$$

(b) Find the extremal of the functional

$$J[y,z] = \int_0^{\pi/2} \left[(y')^2 + (z')^2 + 2yz\right] dx$$

$$y(0)=0, \; y(\pi/2)=1, \; z(0)=0, \; z(\pi/2)=-1$$

8. (a) Find the geodesics of a sphere of radius a.

(b) Among all curves of length 'l' in the upper half-plane passing through the points $A(-q, 0)$ and $B(a, 0)$, find that curve which together with the interval $[-a, a]$ encloses the largest area.

UNIT – 5

9. (a) Isoperimetric problems.

(b) Brachistochrone problem.

(c) Self-adjoint equation.

(d) Sturm – Liouville boundary value problem.

(e) Laplace transform.

(f) Laplace convolution.

(g) Separable kernel.

(h) Approximation of a kernel.

Roll No. ....

72454

M. Sc. (Mathematics) 1st Semester Examination – December, 2015

INTEGRAL EQUATIONS AND CALCULUS OF VARIATIONS

Paper : MM-414

Time : Three Hours] ***[Maximum Marks : 80***

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : The candidates are required to attempt ***one*** question from each Unit I-IV. Question No. 9 is ***compulsory***.

UNIT – 1

1. (a) Transform the IVP :

$$y''(x) + xy(x) = 1, \; y(0)=0, \; y'(0)=0$$

to an equivalent integral equation.

(b) Solve the integral equation :

$$y(x) = 1 + \int_0^x (x+\xi)\, y(\xi)\, d\xi$$

by using successive approximations.

P. T. O.

2. (a) Use Laplace transform to solve :

$$y(x) = e^{-x} + \int_0^x \sin(x-\xi)u(\xi)d\xi.$$

(b) Find the Neumann resolvent kernel to solve the integral equation :

$$u(x) = x + \lambda \int_0^x x\xi\, u(\xi)d\xi.$$

UNIT – 2

3. (a) Transform the integral equation :

$$u(x) = \lambda \int_0^1 K(x,t)u(t)dt$$

with

$$K(x,t) = \begin{cases} x(1-t); & 0 \le x \le t \\ t(1-x); & t \le x \le 1 \end{cases}$$

to an equivalent B.V.P.

(b) Find the resolvent kernel for integral equation :

$$u(x) = f(x) + \lambda \int_0^1 e^{a(x^2-\xi^2)} u(\xi)\, d\xi$$

by using Neumann series.

4. (a) Solve the integral equation :

$$u(x) = 1 + \lambda \int_0^1 (1-3x\xi)\, u(\xi)\, d\xi$$

by the method of iteration.

(b) Use the method for a degenerate kernel to solve the integral equation.

$$u(x) = \lambda \int_0^{2\pi} \sin(x+\xi)u(\xi)\, d\xi.$$

UNIT – 3

5. What do you mean by the Green's function for the BVP ?

$$\frac{d}{dx}\left[r(x)\frac{du}{dx}\right] + [q(x) + \lambda\rho(x)]u(x) = 0 \text{ in } a < x < b$$

$$\alpha_1 u(a) + \alpha_2 u^1(a) = 0,$$

$$\beta_1 u(b) + \beta_2 u^1(b) = 0.$$

Use the method of variation of parameters to construct the Green's function for the above BVP.

6. (a) Construct the Green's function for the B.V.P. :

$$\frac{d^2u}{dx^2} = f(x) \text{ in } 0 < x < l$$

$$u(0) = 0,\ u(l) = 0$$

(b) Obtain the series expansion of the kernel $K(x, t)$ of the integral equation $u(x) = \lambda \int_0^1 K(x,t)\, u(t)\, dt$

with $K(x,t) = \begin{cases} x(1-t) & x \le t \\ t(1-x) & t \le x \end{cases}$.