Error analysis

Steady state error

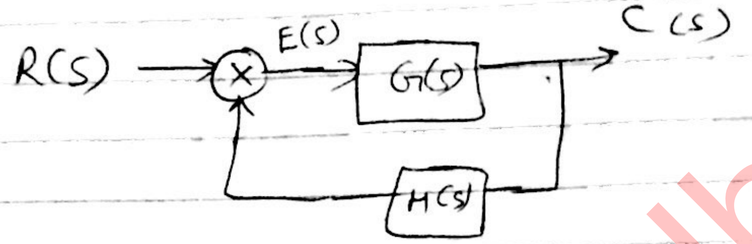
It is a measure of system accuracy

It is the diff. b/w i/p & o/p of the sys

during steady state

for accuracy steady state error

should be min.



$$\frac{C(S)}{R(S)} = \frac{G(S)}{1 + G(S)H(S)}$$

$$C(S) = E(S) \cdot G(S)$$

$$\frac{E(S)}{R(S)} = \frac{1}{1 + G(S)H(S)}$$

$$E(S) = \text{steady state error} = \frac{R(S)}{1 + G(S)H(S)}$$

final value theorem

can be used to find steady state error "ess".

Eqⁿ for unity feedback

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s) = \lim_{s \to 0} \frac{sR}{1+}$$

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Static error coefficients

Static Position error constant (K_p)
for unit step $= \frac{1}{s} = R(s)$

$$e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1 + G(s)H(s)}$$

$$R(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \to 0} \frac{s \cdot 1/s}{1 + G(s)H(s)} = \lim_{s \to 0} \frac{1}{1 + G(s)H(s)}$$

$$e_{ss} = \frac{1}{1 + \underbrace{\lim_{s \to 0} G(s)H(s)}_{\rightarrow K_p}}$$

$$K_p = \lim_{s \to 0} G(s)H(s)$$

$$e_{ss} = \frac{1}{1 + K_p}$$

Static velocity error constant (K_v)
$R(s) = \frac{1}{s^2}$

$$e_{ss} = \lim_{s \to 0} \frac{s R(s)}{1 + G(s)H(s)} \qquad R(s) = \frac{1}{s^2}$$

$$= \lim_{s \to 0} \frac{1}{s + s G(s)H(s)}$$

$$e_{ss} = \frac{1}{0 + \underbrace{\lim_{s \to 0} s G(s)H(s)}_{\rightarrow K_v}}$$

$$e_{ss} = \frac{1}{K_v} \qquad K_v = \lim_{s \to 0} s G(s)H(s)$$

3) Static accelaration error constant

$$R(S) = \frac{1}{s^3}$$

$$e_{ss} = \lim_{s \to 0} \frac{s R(S)}{1 + G(S) H(S)}$$

$$e_{ss} = \lim_{s \to 0} \frac{1}{s^2 + s^2 G(S) H(S)}$$

$$= \frac{1}{0 + \lim_{s \to 0} s^2 G(S) H(S)}$$

$$\boxed{e_{ss} = \frac{1}{K_a}}$$

$$\boxed{K_a = \lim_{s \to 0} s^2 G(S) H(S)}$$

* open loop transfer fnt.

$$G(S) H(S) = \frac{K(1+ST_1)(1+ST_2) \ldots}{S^m (1+ST_a)(1+ST_b) \ldots}$$

m determines type of system

m = 0 Type '0' Control system
1 Type (1) "
2 Type (2) "

Error analysis in diff. types of systems.

	Type 0	Type 1	Type 2
p	$\frac{1}{1+K}$	0	0
p	∞	$1/K$	0
[illegible]	∞	∞	$1/K$

System type is defined as the no. of pure integrator in a system.

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☆ Type 0 unit step
$R(S) \to \frac{1}{s}$

$$G(S)\,H(S) = \frac{K(1+ST_1)(1+ST_2)\ldots}{(1+sT_a)(1+ST_b)\ldots}$$

forr unit step

$$e_{ss} = \frac{1}{1+K_p}$$

$$K_p = \lim_{s \to 0} G(s)\,H(s)$$

$$K_p = \lim_{s\to 0} \frac{K(1+ST_1)(1+ST_2)}{(1+ST_a)(1+ST_b)}$$

$$\underline{K_p = K}$$

$$\boxed{e_{ss} = \frac{1}{1+K}}$$

☆ Type 0 ramp

$$R(s) = \frac{1}{s^2}$$

$$K_v = \lim_{s\to 0} s\,G(s)\,H(s)$$

$$K_v = 0$$

$$e_{ss} = \frac{1}{K_v} = \frac{1}{0} = \infty$$

☆ Type 0 parabolic

$$R(s) = \frac{1}{s^3}$$

$$K_a = \lim_{s\to 0} s^2\,G(s)\,H(s) \qquad e_{ss} = \frac{1}{K_a}$$

$$\boxed{K_a = 0} \qquad \boxed{e_{ss} = \infty}$$

Type 1 unit step

$R(s) = \frac{1}{s}$

Type $\Rightarrow G(s)H(s) = \frac{K(1+sT_1)(1+sT_2)\ldots}{s(1+sT_a)(1+sT_b)\ldots}$

$K_p = \lim_{s \to 0} G(s)H(s)$

$K_p = \lim_{s \to 0} \frac{K(1+sT_1)(1+sT_2)\ldots}{s(1+sT_a)(1+sT_b)\ldots}$

$K_p = \infty$

$e_{ss} = \frac{1}{1+K_p} = \frac{1}{\infty} = 0$

Type 1 ramp

$R(s) = \frac{1}{s^2}$

$K_v = \lim_{s \to 0} s\,G(s)H(s)$

$K_v = \lim_{s \to 0} \frac{\cancel{s}\,K(1+sT_1)(1+sT_2)\ldots}{\cancel{s}(1+sT_a)(1+sT_b)\ldots}$

$K_v = K$

$e_{ss} = \frac{1}{K_v} = \frac{1}{K}$

Type 1 parabolic

$R(s) = \frac{1}{s^3}$

$K_a = \lim_{s \to 0} s^2 G(s)H(s) = 0$

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☆ Type 2 step

$R(s) = \frac{1}{s}$

for type '2' ⇒ $G(s)H(s) = \frac{K(1+sT_1)(1+sT_2)}{s^2(1+sT_a)(1+sT_b)}$.

$K_p = \lim_{s \to 0} G(s)H(s) = \infty$

$$\boxed{e_{ss} = \frac{1}{1+K_p} = 0}$$

☆ Type 2 ramp

$R(s) = \frac{1}{s^2}$

$K_v = \lim_{s \to 0} s\,G(s)H(s) = \infty$

$$\boxed{e_{ss} = \frac{1}{K_v} = 0}$$

☆ Type 2 parabolic

$R(s) = \frac{1}{s^3}$

$K_a = \lim_{s \to 0} s^2 G(s)H(s) = K$

$$\boxed{e_{ss} = \frac{1}{K_a} = \frac{1}{K}}$$

$$\boxed{\omega_r = \omega_n\sqrt{1-2\xi^2}}$$ resonance freq.

$M_r = \frac{1}{2\xi\sqrt{1-\xi^2}}$

Q→ $G(s) = \frac{50}{(1+0.1s)(s+10)}$ $H(s) = 1$

$$K_p = \lim_{s\to 0} G(s)\,H(s)$$

$$= \lim_{s\to 0} \frac{50}{(1+0.1s)(s+10)}$$

$$\boxed{K_p = 5}$$

$$K_v = \lim_{s\to 0} \frac{s\,50}{(1+0.1s)(s+10)} = 0$$

$$K_a = \lim_{s\to 0} s^2 \frac{50}{(1+0.1s)(s+10)}$$

$$K_a = 0$$

☆ Transient response spec for Second order ...

→ Delay time → It is the time reqd. for the respon... to reach 50% of the final val... in first time

→ Rise time → It is the time reqd. of resp... to rise from 10% to 90% of its final val... for overdamped & 0 to 100% for ...

→ Peak time → It is the time reqd. for the res... to reach the first peak of the time res... or first peak overshoot.

→ Max. overshoot → It is the normalized ... b/w the peak of the time response & st... o/p.

Max. % overshoot = $\frac{C(t_p) - C(\infty)}{C(\infty)}$

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) **settling time** → The settling time is the time reqd. for the response to reach and stay within specified range (2% to 5%) of its final value

Polar Plots

→ Polar plot of sinusoidal T.F $G(j\omega)$ is a plot of magn. of $G(j\omega)$ v/s the phase angle of $G(j\omega)$ on polar coordinates as 'ω' varied from '0' to '∞'.

→ Magn. of $G(j\omega)$ is plotted as the distance from the origin

→ Phase angle is measured from +ve real axis
- +ve angle measured counter-clockwise
- -ve angle measured Clockwise

→ Procedure to sketch the Polar Plot

1.) Determine the T.F $G(s)$ of the system.

2.) Put $s = j\omega$ in transfer fnt to obtain $G(j\omega)$

3.) At $\omega = 0$ & $\omega = \infty$. Calculate $|G(j\omega)|$ by $\lim_{\omega \to 0} |G(j\omega)|$ & $\lim_{\omega \to \infty} |G(j\omega)|$

4.) Calculate the phase angle of $G(j\omega)$ at $\omega = 0$ & $\omega = \infty$ by $\lim_{\omega \to \infty} \angle G(j\omega)$ & $\lim_{\omega \to 0} \angle G(j\omega)$.

5.) Rationalize $G(j\omega)$ and separate real and imaginary parts.

6.) ($Im\,|G(j\omega)| = 0$), determine freq. at which polar plot intersects the real axis and calculate value $G(j\omega)$ at this pt. by substituting determined 'ω' in $G(j\omega)$.

7.) $Re\,|G(j\omega)| = 0$, determine freq. at which polar plot intersects the imaginary axis and calculate the value of $G(j\omega)$ at this pt by subst. determined 'ω' in $G(j\omega)$

Polar plot for Type '0' system

$$G(S) = \frac{K}{(1+sT_1)(1+ST_2)}$$

$s = j\omega$

$$G(j\omega) = \frac{K}{(1+j\omega T_1)(1+j\omega T_2)}$$

write eq^n. in polar form

$$= \frac{K}{\sqrt{1+(\omega T_1)^2}\sqrt{1+(\omega T_2)^2}} \angle \tan^{-1} -\omega T_1 - \tan^{-1}\omega T$$

phase Angle

$\omega = 0 \quad \lim_{\omega \to 0} |G(j\omega)| = \frac{K}{\sqrt{1+0}\sqrt{1+0}} = K$

$\omega = \infty \quad \lim_{\omega \to \infty} |G(j\omega)| = \frac{K}{\infty} = 0$

Phase

$\omega = 0 \quad \lim_{\omega \to 0} -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 = 0$

$\omega = \infty \quad \lim_{\omega \to \infty} -\tan^{-1}\omega T_1 - \tan^{-1}\omega T_2 = -180°$

270 am

Equate real part to = 0

$$K(1-\omega^2 T_1 T_2) = 0$$

$$\omega = \frac{1}{\sqrt{T_1 T_2}}$$ (freq. at which polar plot intersect imag. axis)

$G(j\omega)$ at $\omega = \frac{1}{\sqrt{T_1 T_2}}$

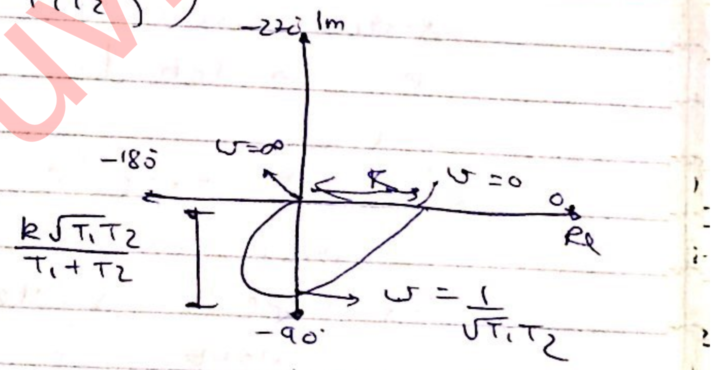
$$\frac{K\left(1 - \frac{T_1 T_2}{T_1 T_2}\right) - jK\left(\frac{T_1 + T_2}{\sqrt{T_1 T_2}}\right)}{1 + \frac{T_1^2}{T_1 T_2} + \frac{T_1^2}{T_1 T_2} + \frac{T_1^2 T_2^2}{T_1^2 T_2^2}} = \frac{-jK\sqrt{T_1 T_2}}{T_1 + T_2}$$

$$|G(j\omega)| = \frac{K\sqrt{T_1 T_2}}{T_1 + T_2}$$

ange at $\omega = 1/\sqrt{T_1 T_2}$

$$\phi = \tan^{-1}\left(\frac{-K\omega(T_1 + T_2)}{K(1-\omega^2 T_1 T_2)}\right)$$

$$\phi = -90^\circ$$



Polar plot for standard fns

i) $G(S) = 1/s$ ii) $G = 1/s^2$ iii) $G(s) = \frac{1}{s+1}$