(3)

DC - PREDICTION FILTER

Linear Predictive Coding

LPC is a tool used mostly in audio signal proce & speech processing for representing the spectral envelopment of a digital signal of speech in compressed form, using the info of a linear prediction model.

LP is a mathematical opⁿ where future values of a discrete time signal are estimated as a linear function of previous sample.

In digital signal processing linear prediction is often called LPC & can thus be viewed as a subset of filter theory. Filter design a process of designing a signal processing filter that satisfies a set of requirements, some of which are contradictory. The process is to fi a realization of the filter that meets each of the requirements to a sufficient degree to make it usefull.

$$\hat{x}(n) = \sum_{i=1}^{p} a_i \, x(n-i)$$

$\hat{x}(n)$ = predicted signal value.

$x(n-i) \rightarrow$ Previous observed val.

a_i = Predictor coefficients.

error generic

$e(n) = x(n) - \hat{x}(n)$

$x(n) \Rightarrow$ true signal value.

he filter design process can be defined as an optimizer problem where each requirement contributes to of error func. which should be minimized. Certain parts of designed process can be automated, but normally an experienced electrical is needed to get a good result.

In system analysis LP can be viewed as a part of mathematic modeling or optimization. It is the selection of a best element from some set of available alternatives. In the simplest case, an optimization problem consists of max. & min. a real func. ~~while~~ by systematically choosing i/p values from within a allowed set & computing the value of the func.

LPC starts with the assumption that a speech signal is produced by a buzzer at the end of a tube, with occasional added hissing & popping sounds. Although apparently ~~viewed~~ crude this model is actually a close approximation of the reality of speech production.

LPC analysis the speech signal by estimating the formants, removing their effects from the speech signal & estimating the intensity & frequ of remaining buzz. The process of removing the formants is called inverse filtering & the rema signal after the subtraction of the filtered mode signal is called the residue.

The no. which describe the intensity & freq of the buzz, the formants & the residue signal can be stored or transmitted somewhere else. LPC sympasizes speech signal by reversing the proc case the buz parameter & the residue. to cre a ~~signal~~ source signal use the formants to crea filter & run. The source signal through the fi resulting in speech.

Linear prediction of speech.

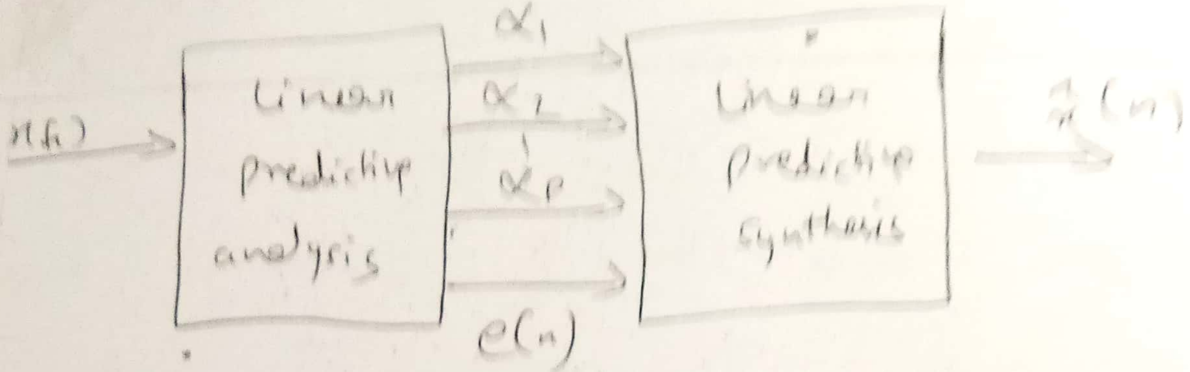
$$e(n) = x(n) - \sum_{k=1}^{p} \alpha_k x(n-k)$$

$x(n-1)$

$x(n-2)$

$\vdots$

$x(n-p)$



$$e(n) = x(n) - \sum_{k=1}^{p} \alpha_p x(n-k)$$

Taking Z transform

$$E(z) = X(z) - \sum_{k=1}^{p} \alpha_k z^{-k} X(z)$$

$$= \left(1 - \sum_{k=1}^{p} \alpha_k z^{-k}\right) X(z)$$

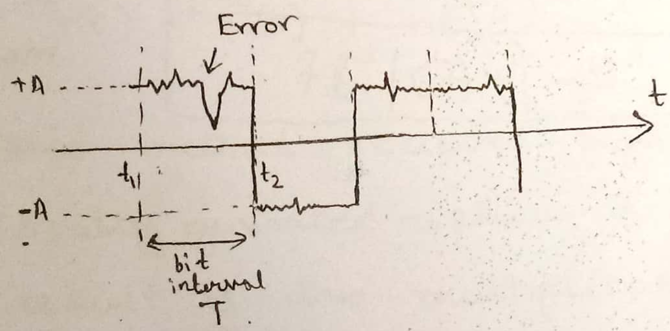
$$\boxed{X(z) = \frac{E(z)}{1 - \sum_{k=1}^{p} \alpha_k z^{-k}}}$$

DCS ASSIGNMENT

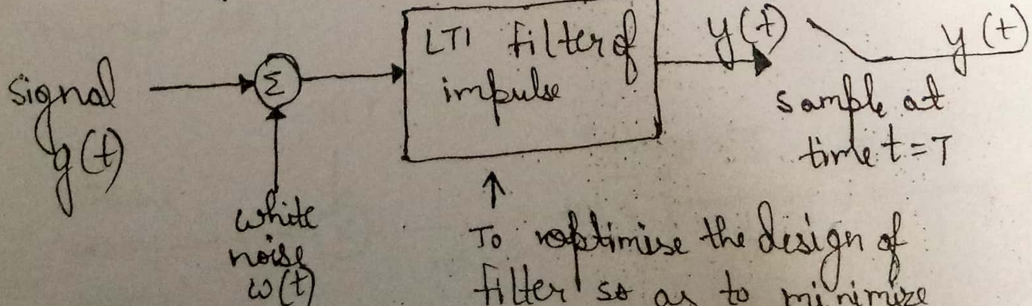
MATCHED FILTER

NEED OF RECEIVERS & FILTERS

The data transmission system uses sequence of binary digits in encoded form by different patterns NRZ unipolar, bipolar RZ and NRZ, AMI and Manchester.
The receiver signal is corrupted by noise. Hence, the receiver can make error in deciding whether a 1 or a 0 was transmitted. Noise gets superimposed on the signal. Thus we need different filters to maximise signal amplitude & increase SNR.



- The objective of a matched filter is to maximise the signal to noise ratio & minimise probability of undetected errors received from a signal.
- White gaussian is a noise that has uniform power across the frequency band for the information system.



To optimise the design of filter so as to minimize the effect of noise at filter O/P and improve the detection of pulse signal

$$SNR = \frac{|g_o(T)|^2}{\sigma_n^2} = \frac{|g_o(T)|^2}{E[n^2(t)]}$$ → instantaneous power of filtered signal $g(t)$ at point $t=T$

We sampled $t=T$ because that gives you the max power of the filtered signal.

filter

$$S_x(f) \longrightarrow \boxed{H(f)} \longrightarrow S_x(f)|H(f)|^2 = S_n(f)$$

PSD of white gaussian noise, $S_x(t) = \frac{N_0}{2}$

$$S_n(f) = \frac{N_0}{2}|H(f)|^2$$

$$\boxed{SNR = \frac{\left|\int H(f)G(f)e^{j2\pi ft}\,df\right|}{\frac{N_0}{2}\int|H(f)|^2\,df}}$$

→ we have to maximise this equation

→ Correlator reciever

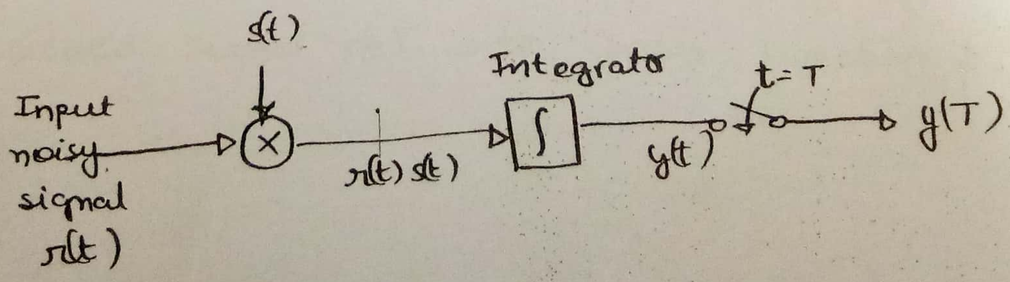
We use a correlator reciever to design optimum reciever.

The transmitted signal $s(t)$ is corrupted by the AWGN channel, hence an optimum reciever is is designed to minimize probability of making an error in detection process.

Hence the recieved signal $r(t)$ is

$$r(t) = s(t) + n(t)$$

In this method recieved signal $r(t)$ is multiplie to locally generated replica of input signal $s(t)$. The result of this multiplication is is integrated. The output of integrator is sampled at $t = T$. Then based on this sampled value, decision is made. This is how correlator works. It is known as correlator because it correlates recieved signal $r(t)$ with stored replica of known signal $s(t)$.



In block diagram product $r(t)\,s(t)$ is integrated over one symbol period i.e T

Thus output $y(t)$ will be,

$$y(t) = \int_0^T r(t).\, s(t)\, dt$$

At $t = T$ output of correlator

$$y(T) = \int_0^T r(t)\, s(t)\, dt$$

Let us represent recieved signal as N-dimensi vector

$$r(t) = \sum_{i=1}^{N} r_i\, \phi_i(t)$$

where $\phi_i(t)$ are the orthonormal function or correlators which produce the ~~the~~ locally generated function $s(t)$ by operating on $r(t)$,

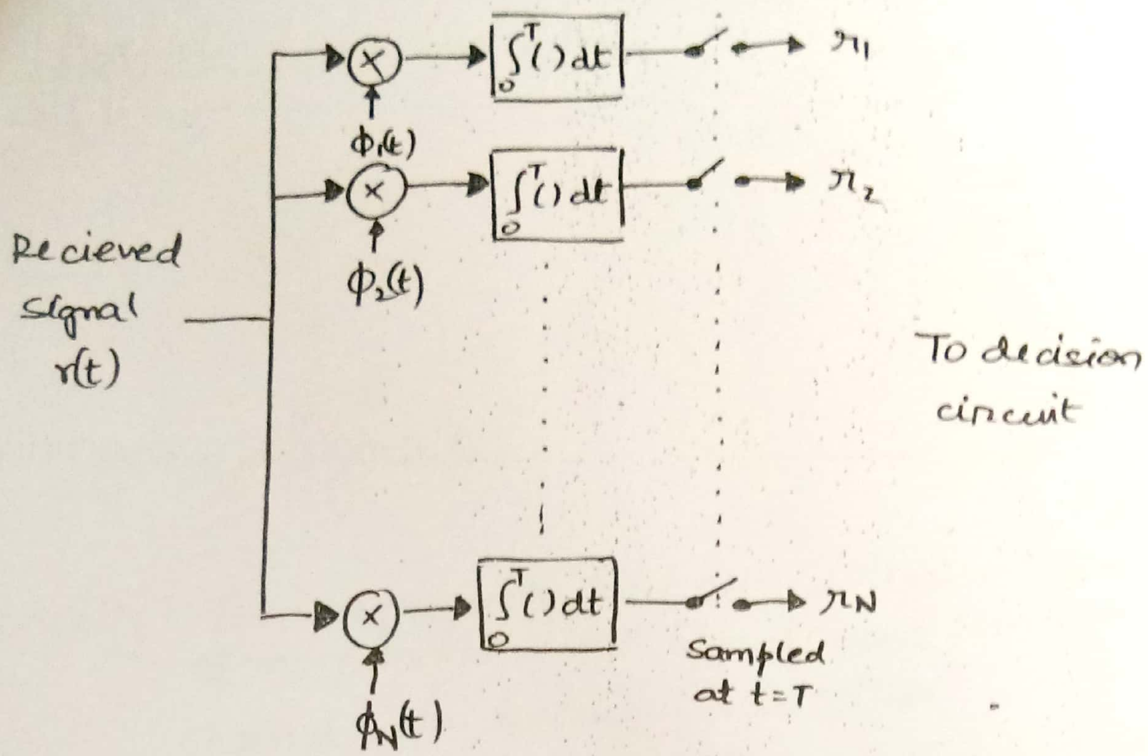
and recieved signal vector

$$r = [\, r_1 \,,\, r_2 \quad \dots \; r_N \,]$$

where

$$r_i = \int_0^T r(t)\, \phi_i(t)\, dt \qquad t \in [0, T]$$

This suggests that i^{th} element of recieved vector $r(t)$ is obtained by correlating the recieved signal $r(t)$ with basis function $\phi_i(t)$.



correlation demodulator

The figure shows a demodulator that implements $r_i = \int_0^T r(t)\,\phi_i(t)\,dt$ with a set of N correlators. In each branch r(t) is multiplied with $\phi_i(t)$ and integrated over symbol period & result is sampled at end of period T.

The output of correlator & matched filter are identical. Matched filter provides the same output in a single step as compared to the correlator which gives the output by multiplying the signal $r_i(t)$ with basis function $\phi_i(t)$ & integrating it over sampling period T.

Digital Communication

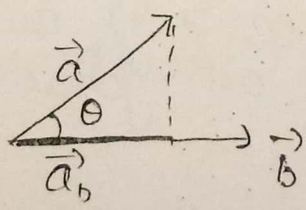
Orthogonal Signals :-

* Orthogonality :- It is the property that allows transmission of more than one signal over a common channel with successful detection.

* Orthogonal Signals :- Those signals are said to be orthogonal which are mutually in dependent.

* Orthogonal vectors :- Let there be two vectors $\vec{a}$ and $\vec{b}$ and let the angle between them be θ

(I)



$\vec{a}_b$: projection of $\vec{a}$ over $\vec{b}$

$|\vec{a}_b|$: magnitude of projection of $\vec{a}$ over $\vec{b}$

$|\vec{a}_b| = \vec{a}\cos\theta$

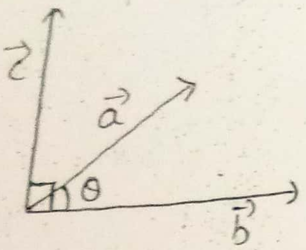
multiply eq both the sides by $|\vec{b}|$ we get

$|\vec{a}||\vec{b}|\cos\theta \rightarrow$ RHS

$\vec{a}\cdot\vec{b} \neq 0$

hence they are not mutually in dependent, hence they are not orthogonal.

II) Let there be another vector $\vec{c}$



Projection of $\vec{c}$ on $\vec{b}$

$\theta = 90°$

$\vec{c} \cdot \vec{b} = |\vec{c}||\vec{b}| \cos 90 = 0$

Therefore $\vec{c}$ and $\vec{b}$ are mutually independent

In case of signal space, the inner product is definite integral.

Case 1: If the signals are aperiodic.

if $\int_{-\infty}^{\infty} x_1(t) \cdot x_2(t)\, dt = 0$, then $x_1(t)$ and $x_2(t)$ are orthogonal

Case 2: If the signals are periodic.

If $\int_{0}^{T} x_1(t) \cdot x_2(t)\, dt = 0$, then $x_1(t)$ and $x_2(t)$ are orthogonal

* Properties of Orthogonal Signals

Property-1 Two Harmonics of different frequencies are always orthogonal.

$$x_1(t) = \sin(n\omega_0 t + \phi_1)$$

$$x_2(t) = \sin(m\omega_0 t + \phi_2)$$

$$\int_0^T \sin(n\omega_0 t + \phi_1)\,\sin(m\omega_0 t + \phi_2)\,dt = 0$$

Hence orthogonal.

Property-2 Sine and Cosine functions with same phase and frequencies are orthogonal.

$$\int_0^T \sin(n\omega_0 t + \phi)\cos(n\omega_0 t + \phi)\,dt = 0$$

Property-3 dc value and sine function are also orthogonal

let dc value be a

$$\int_0^T a \sin(n\omega_0 t + \phi)\,dt = 0$$

Property-4 Effect of orthogonality on energy and power of the signal.

Let two signals be $x_1(t)$ and $x_2(t)$

$$\int_{-\infty}^{\infty} x_1(t)\cdot x_2(t)\,dt \text{ or } \int_0^T x_1(t)\cdot x_2(t)\,dt = 0 \text{, for orthogonality}$$

(if aperiodic) (if periodic)

Let there be a signal $y(t)$, such that

$y(t) = x_1(t) + x_2(t)$

then $P_{y(t)} = P_{x_1(t)} + P_{x_2(t)}$

$\therefore$ { if $x_1(t)$ and $x_2(t)$ are energy signals, then

$E_{y(t)} = E_{x_1(t)} + E_{x_2(t)}$

if $x_1(t)$ and $x_2(t)$ are power signals, then

$E_{y(t)} = \infty$ }

As for power signals, Power is finite and energy is infinite.

Il periodic signals are power signals, because they do not merge to a finite value so their energy is infinite and their power is finite.

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Numerical

Ques: find the average power and Total energy of the following signals.

(1) $5\cdot\sin(2\pi 100)t$ and $1\cdot\cos(2\pi 100)t$

(2) $2\sin(4\pi 100)t$ and $3\sin(2\pi 100)t$

(3) $2 + \sin(2\pi 100)t$

Ans: (1) $x_1(t) = \sin(2\pi 100)t$

$x_2(t) = \cos(2\pi 100)t$

By property (2), $x_1(t)$ and $x_2(t)$ are orthogonal signals,

Proof:
$$\int_0^T x_1(t)\, x_2(t)\, dt$$

$$\omega = 2\pi/T$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{2\pi 100} = 1/100$$

$$= \int_0^{1/100} \sin(2\pi 100)t \cos(2\pi 100)t\, dt$$

$$= \frac{1}{2}\int_0^{1/100} \sin 2(2\pi 100)t\, dt$$

$$= \frac{1}{2}\int_0^{1/100} \sin(4\pi 100)t\, dt$$

$$\therefore -\frac{1}{2}\left[\frac{\cos(4\pi 100)t}{4\pi 100}\right]_0^{1/100} = \frac{-1}{8\pi 100}\left[\cos(4\pi) - \cos(0)\right]$$

$$= \frac{-1}{8\pi 100}[1-1] = 0$$

hence $x_1(t)$ and $x_2(t)$ are orthogonal.

As $x_1(t)$ and $x_2(t)$ are periodic, hence they are power signals.

$$P_{x_1(t)} = \frac{(5)^2}{2} = \frac{25}{2} = 12.5$$

$$P_{x_2(t)} = \frac{(1)^2}{2} = \frac{1}{2} = 0.5$$

$$P_{y(t)} = P_{x_1(t)} + P_{x_2(t)} = 13 \Big\} \text{ average power}$$

$E_{y(t)} = \infty$; as power signal

i) By property-1 & two harmonics of different frequencies are orthogonal.

let $x_1(t) = 2\sin(4\pi 100)t$ and $x_2(t) = 3\sin(2\pi 100)t$

$$\int_0^T x_1(t) \cdot x_2(t)\, dt$$

$$T_1 = \frac{2\pi}{4\pi 100} = \frac{1}{200} \text{ and } T_2 = \frac{2\pi}{2\pi 100} = \frac{1}{100}$$

$\frac{T_1}{T_2} = \frac{1/200}{1/100} = \frac{1}{2}$, therefore period is

$$LCM(1/100, 1/200) = 1/200$$

$$6\int_0^{1/200} \sin(4\pi 100t)\sin(2\pi 100t)\,dt$$

$$\frac{6}{2}\int_0^{1/200} 2\sin(4\pi 100t)\sin(2\pi 100)t\,dt$$

$$\frac{6}{2}\int_0^{1/200} \cos(2\pi 100)t\,dt - \frac{6}{2}\int_0^{1/200} \cos(6\pi 100)t\,dt$$

$$3\int_0^{1/200} \cos(2\pi 100)t\,dt - 3\int_0^{1/200} \cos(6\pi 100)t\,dt$$

$$\frac{3}{2\pi 100}\left[\sin(2\pi 100)t\right]_0^{1/200} - \frac{3}{6\pi 100}\left[\sin(6\pi 100)t\right]_0^{1/200}$$

$$\frac{3}{2\pi 100}\left[\sin\pi - \sin 0\right] - \frac{3}{6\pi 100}\left[\sin 3\pi - \sin 0\right]$$

$$= 0$$

Hence orthogonal

Average Power $= P_{x_1(t)} + P_{x_2(t)}$

$$= \frac{(2)^2}{2} + \frac{(3)^2}{2} = \frac{4}{2} + \frac{9}{2}$$

$$= \frac{13}{2} = 6.5$$

Total Energy $= \infty$; as power signals.

③ $2 + \sin(2\pi 100)t$

$\downarrow$ dc value $\quad \downarrow$ sine funcⁿ

By Property-3, it is orthogonal

$$\int_0^{1/100} 2 \cdot \sin(2\pi 100)t$$

$$-2\left[\frac{\cos(2\pi 100)t}{2\pi 100}\right]_0^{1/100} = \frac{-2}{2\pi 100}\left[\cos(2\pi) - \cos 0\right]$$

$$= \frac{-2}{2\pi 100}[1-1] = 0$$

$$\text{Average Power} = \frac{(2)^2}{2} + \frac{(1)^2}{2} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2} = 2.5$$

Energy = ∞; Power Signal

Applications of Orthogonal Signals :

∴ Orthogonal signals are used extensively in communication dustry. They range from a simple sine/cosine quadrature signals to multiple signals whose inner product is zero. orthogonal signals can be used for several different oplications :

1) Quadrature signals can be used to send and receive eparate information channels on each orthogonal signal with minimum interference between them.

or example in [illegible] QPSK

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In QPSK modulator is used in a system, two different data streams, one for channel I (0, 180 degrees and one for the Q channel (90, 270 degrees), can be sent simultaneously and received on the other end as separate data streams.

2) Orthogonality can also be used in an antenna system. Two signals can be transmitted on different polarizatio or two parallel channels can be used with the same frequency for increased data rates.

3) Orthogonal signals are also used to separate desired signals from the jammers using a Gram Schmidt orthogonalizer (GSO). The GSO forces the desired signals to be orthogonal to the jamming signal so that it can be used to eliminate the jamming signal.

4) Another application for orthogonal signals is to preven adjacent channel interference. This application allows for the system to be able to overlap multiple frequen channel signals with minimum interference between them and still guarantee reception and detection of phase-shift keyed signals. Thus, more data can b

* Gram-Schmidt Orthogonalization Procedure

To understand the Gram-Schmidt Procedure, we first need to know the orthogonality property of functions as well as normalization and the completeness of an orthogonal set: the Fourier Series.

(1) Orthogonal Representation of a Signal

- useful technique to represent any arbitrary signal in terms of orthogonal basis function.

Let us consider a set of functions $g_1(x)$, $g_2(x)$ ... g defined over interval $x_1 \leq x \leq x_2$ and are related in such a way that any two different ones satisfies the condition,

$$\int_{x_1}^{x_2} g_i(x) g_j(x) dx = 0$$

Hence, a set of functions which has this property is described as being orthogonal over the interval from x_1 to x_2 (just like vectors)

Now, consider some arbitrary fn. $f(x)$ in the range x_1 to x_2 i.e., in which $g(x)$ is orthogonal then,

$$f(x) = C_1 g_1(x) + C_2 g_2(x) + \cdots C_n g_n(x) + \cdots$$

Now, to evaluate C_n we multiply $g_n(x)$ on both sides & integrate over range x_1 to x_2,

$$\Rightarrow \int_{x_1}^{x_2} f(x)\, g_n(x)\, dx = c_1 \int_{x_1}^{x_2} \cancel{g_1(x)\, g_n(x)\, dx}^{\,0} + c_2 \int_{x_1}^{x_2} \cancel{g_2(x)\, g_n(x)\, dx}^{\,0} \ldots$$

$$c_n \int_{x_1}^{x_2} g_n^2(x)\, dx$$

$$\Rightarrow \int_{x_1}^{x_2} f(x)\, g_n(x)\, dx = c_n \int_{x_1}^{x_2} g_n^2(x)\, dx.$$

$\Rightarrow$ This mechanism is called "orthogonality sieve".

Now, $g_n(x)$ is considered normalized.

$$\therefore \int_{x_1}^{x_2} g_n^2(x)\, dx = 1.$$

$$\therefore \boxed{c_n = \int_{x_1}^{x_2} f(x)\, g_n(x)\, dx}$$

A set of f^{ns} which is both orthogonal and normalized is called an orthonormal set.

1.) Completeness of an Orthogonal Set: The Fourier Series

let, $f(x) = c_1 S_1(x) + c_2 S_2(x) + c_3 S_3(x) + \ldots$

& $f(x) = c_1 S_1(x) + c_3 S_3(x) + \ldots$

So, this ~~is~~ seems absurd by the above procedure so, to represent an arbitrary function, an orthogonal set has to be complete i.e., it should contain all the functions necessary to allow an error-free expansion.

One of such expansion is fourier series of sine & cos functions. For variable t & interval T,

$$v(t) = \sum_{n=0}^{\infty} A_n \cos \frac{2\pi nt}{T} + \sum_{n=0}^{\infty} B_n \sin \frac{2\pi nt}{T}$$

Since $\cos 0 = 1$ & $\sin 0 = 0$

$$v(t) = \frac{A_0}{\sqrt{T}} + \sum_{n=1}^{\infty} A_n \sqrt{2/T} \cos \frac{2\pi nt}{T} + \sum_{n=1}^{\infty} B_n \sqrt{2/T} \sin \frac{2\pi nt}{T}$$

∴ orthonormal f^n are,

$$\frac{1}{\sqrt{T}}, \sqrt{\frac{2}{T}} \cos \frac{2\pi nt}{T} \text{ and } \sqrt{\frac{2}{T}} \sin \frac{2\pi nt}{T}.$$

$$A_0 = \frac{1}{\sqrt{T}} \int_T v(t)\, dt$$

$$A_n = \sqrt{\frac{2}{T}} \int_T v(t) \cos \frac{2\pi nt}{T}\, dt \quad n \neq 0$$

$$B_n = \sqrt{\frac{2}{T}} \int_T v(t) \cos \frac{2\pi nt}{T}\, dt.$$

(3) Gram-Schmidt Procedure

For finite no. of functions in the orthogonal set, this procedure allows us to construct this set.

time functions ; $s_1(t), s_2(t), \ldots s_N(t)$ and

orthonormal functions ; $u_1(t), u_2(t) \ldots u_N(t)$.

$$\Rightarrow s_1(t) = s_{11} u_1(t) + s_{12} u_2(t) + \ldots s_{1N} u_N(t) \quad — (1)$$

$$s_2(t) = s_{21} u_1(t) + s_{22} u_2(t) + \ldots s_{2N} u_N(t) \quad — (2)$$

$$\vdots$$

$$s_N(t) = s_{N1} u_1(t) + s_{N2} u_2(t) + \ldots s_{NN} u_N(t) \quad — (3)$$

or $$s_i(t) = \sum_{j=1}^{N} s_{ij} u_j(t) \text{ and } i = 1, 2 \ldots N. \quad — (4)$$

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The orthogonality of the f^{ns} $u(t)$ over interv...

$$\int_T u_j(t)\, u_k(t)\, dt = \begin{cases} 1 & , j=k \\ 0 & , j\neq k \end{cases} \quad — (5)$$

Process :-

Step 1: ~~Put~~ Make all coeff. in (1) zero except S_{11}.

$$s_1(t) = s_{11} u_1(t)$$

we know, $u_1(t)$ is normalized f^n.

$$\therefore \int_T u_1^2(t)\, dt = 1.$$

$$\therefore \quad S_{11} = \left[\int_T s_1^2(t)\, dt\right]^{1/2} \quad — (6)$$

$$\Rightarrow u_1(t) = \frac{s_1(t)}{s_{11}}$$

Step 2; Make all coeff. in (2) zero except S_{21} & S_{22}.

$$\Rightarrow S_2(t) = S_{21} u_1(t) + S_{22} u_2(t) \quad — (7)$$

Multiply both sides by $u_1(t)$ & integrate over T.

$$\therefore \quad S_{21} = \int_T s_2(t)\, u_1(t)\, dt \quad — 7(a)$$

$$\Rightarrow S_{22} u_2(t) = S_2(t) - S_{21} u_1(t) \quad — 7(b)$$

Squaring and integrating both sides of 7(b).

$$S_{22}^2 \int_T \cancel{u_2^2(t)}^{\,1}\, dt = \int_T \left[S_2(t) - S_{21} u_1(t)\right]^2 dt$$

$$S_{22} = \left[\int_T \left[S_2(t) - S_{21} u_1(t)\right]^2 dt\right]^{1/2}$$

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$$\Rightarrow u_2(t) = \frac{1}{S_{22}}\left[s_2(t) - S_{21} u_1(t) \right]$$

$$= \frac{1}{S_{22}}\left[s_2(t) - S_{21} \frac{s_1(t)}{S_{11}} \right] \quad [\because \text{Using } \ldots$$

Step 3: Similiarly,

$$s_3(t) = S_{31} u_1(t) + S_{32} u_2(t) + S_{33} u_3(t) \quad - (8)$$

$$S_{31} = \int_T s_3(t) u_1(t)\, dt$$

$$S_{32} = \int_T s_3(t) u_2(t)\, dt$$

$$S_{33} = \left\{ \int_T \left[s_3(t) - S_{31} u_1(t) - S_{32} u_2(t) \right]^2 dt \right\}^{\frac{1}{2}}$$

finally, $u_3 = \dfrac{s_3(t) - S_{31} u_1(t) - S_{32} u_2(t)}{S_{33}}$.

Step 4: Further continuing we can find $u_1(t), u_2(t) \ldots u_N(t)$ & we also know, $s_1(t), s_2(t) \ldots s_N(t)$'s coefficien

Assumption: N fns. $s(t)$ are linearly independent.

Suppose, $s_3(t) = c_1 s_1(t) + c_2 s_2(t)$ where c_1 & c_2 are constt.

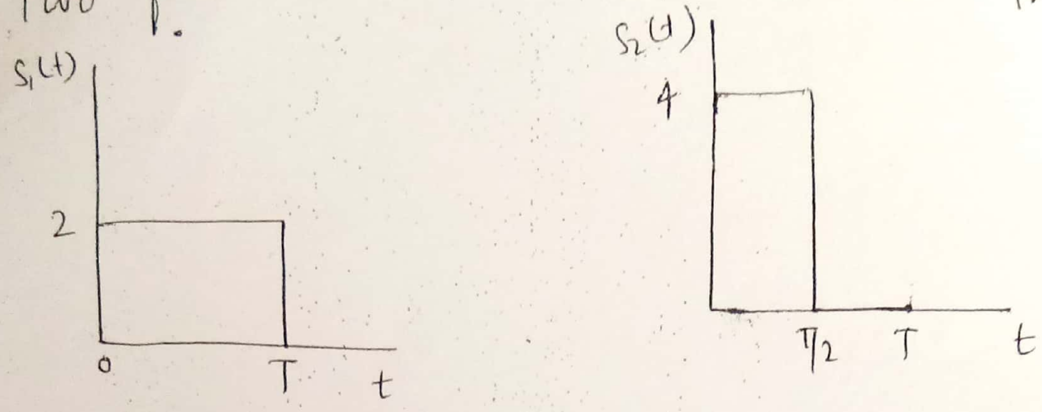
$\Rightarrow$ s_1 & s_2 can be expressed in terms of u_1 & u_2 & hence so can s_3 $\Rightarrow$ Eqn (8) has $S_{33} = 0$

$\Rightarrow$ $u_3(t)$ will not be generated.

$\Rightarrow$ If there are N fns. $s_i(t)$ but only M of them are linearly independent, then procedure will generate $M (\leq N)$ orthonormal functions in terms of which $s(t)$ will be expressed upto N.

Example :

Two functions $s_1(t)$ and $s_2(t)$ are given in figure in interval 0 to T.



Use Gram-Schmidt procedure to express these functions in terms of orthonormal components.

Solution).
$$s_{11} = \left[\int_0^T s_1^2(t)\,dt\right]^{1/2} = \left[\int_0^T 4\,dt\right]^{1/2} = 2\sqrt{T}$$

$$\Rightarrow u_1(t) = \frac{s_1(t)}{s_{11}} = \frac{s_1(t)}{2\sqrt{T}} \quad — \quad (1)$$

Also,
$$s_{21} = \int_0^T s_2(t)\,u_1(t)\,dt = \int_0^{T/2} 4\left(\frac{2}{\sqrt{4T}}\right)dt = 2\sqrt{T}$$

$$\Rightarrow s_{22} = \left\{\int_0^T \left[s_2(t) - s_{21}u_1(t)\right]^2 dt\right\}^{1/2}$$

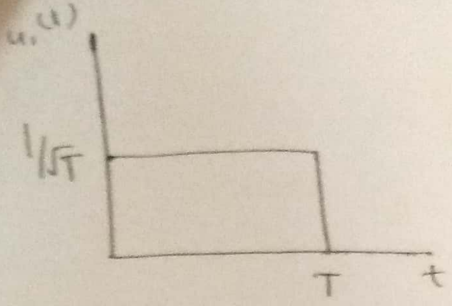
$$= \left(\int_0^T 4\,dt\right)^{1/2} = 2\sqrt{T}.$$

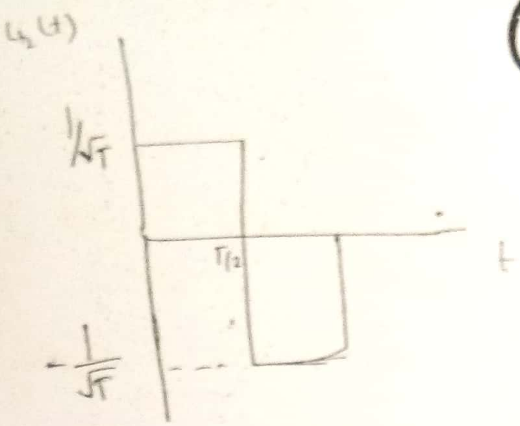
$$\Rightarrow u_2(t) = \frac{1}{2\sqrt{T}}\left\{s_2(t) - \frac{2\sqrt{T}\,s_1(t)}{2\sqrt{T}}\right\}$$

$$= \frac{1}{2\sqrt{T}}\left[s_2(t) - s_1(t)\right]$$

$$\boxed{s_2(t) = 2\sqrt{T}\,u_1(t) + 2\sqrt{T}\,u_2(t)}$$ ~~and $s_1(t) = 8$~~

Scanned by CamScanner





(4) Signal energy

On correspondence with vectors, as, the magnitude of vector A can be calculated by its components.

$$\rightarrow \quad |A| = \left[A_x^2 + A_y^2 + A_z^2\right]^{1/2}$$

"Magnitude" of $s(t)$ can be anologously defined by

$$|s(t)| = \left[\int_T s^2(t)\,dt\right]^{1/2}.$$

$$|s(t)|^2 = \left[c_1^2 + c_2^2 + \cdots c_N^2\right].$$

It is called signal energy.

Maximum Likelihood Reciever

Whatever receiver we do study is to minimise one facto ie. probability of error (P_e). Detector is the recieving end circuit to minimise error & maximum likelihooc is that symbol which is having maximum probabili will be transmitted

$$\left.\begin{array}{ll} 0 & P(D_0/T_0) \\ 1 & P(D_1/T_1) \end{array}\right\} \text{Signals} \qquad \left.\begin{array}{l} P(D_1/T_0) \\ P(D_0/T_1) \end{array}\right\} \text{Fo} \ \text{er}$$

D → data recieved   T → Data transmitte

There is one factor which decides which symbol to transmitted is threshhold or decision threshold denoted by λ.

If we consider V & $-V$ for 1 & 0 then λ u be 0 assuming gausian PDF and probability of oc both the symbol is equal.

Optimum Decision Rule:

The estimate $\hat{m_i} = m_i$ if

$$P(M_i \text{ sent}/n) \geq P(m_k \text{ sent}/n) \text{ for } k \neq i$$

This is called maximum aposteriori probability

ML detector computes the metric of each transmitted message, compares them & make decision.

$$P_e = P(D_1/T_0)\,P(T_0) + P(D_0/T_1)\,P(T_1)$$

As $P(D_0/T_0) + P(D_1/T_0) = 1$

$$P(D_1/T_0) = 1 - P(D_0/T_0)$$

$$P_e = P(T_0)\left(1 - P(D_0/T_0)\right) + P(T_1)\,P\left(D_0/T_1\right)$$

$$P_e = P(T_0)\left(1 - \int_{D>\lambda} P(D/T_0)\right) + P(T_1)\int_{D>\lambda} P(D/T_1)$$

$$P_e = P(T_0) + \int_{D>\lambda} P(T_1)\,P(D/T_1) - \int_{D>\lambda} P(T_0)\,P(D/T_0)$$

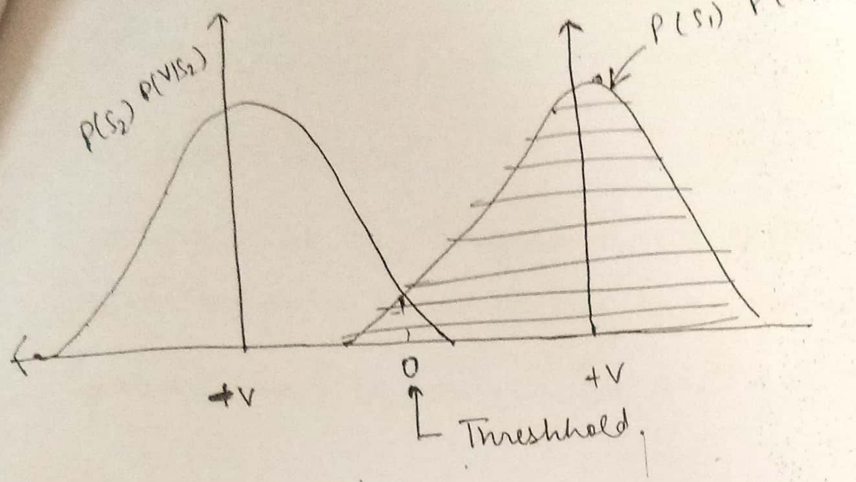
P_e is to be minimum for every $V > \lambda$

$$P(T_1)\,P(D/T_1) < P(T_0)\,P(D/T_0)$$

$$\frac{P(D/T_0)}{P(D/T_1)} > \frac{P(T_1)}{P(T_0)}$$

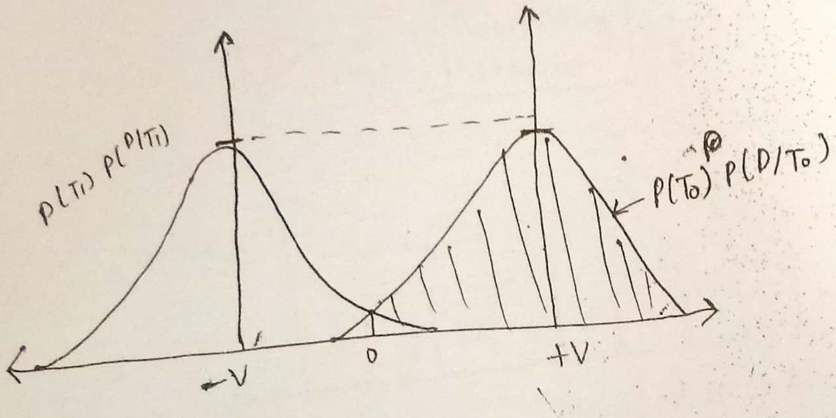
At $D = \lambda$,

$$\frac{P(\lambda/T_0)}{P(\lambda/T_1)} = \frac{P(T_1)}{P(T_0)}$$



If $P(T_0) = P(T_1)$

$\therefore$



If unequal

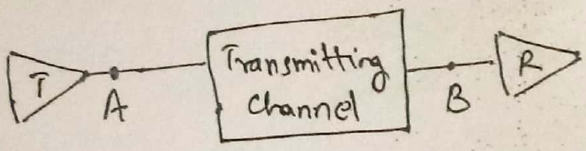
$(P(T_0) > P($

Eye Pattern

⇒ What is Eye Pattern?

Eye Pattern or Eye Diagram is an oscilloscope display of a data sig passed through a transmission line, clocked synchronously to the dat symbols, used to analyze the quality of transmission.

Eye pattern can be further explained by the figure below:-



$f = 1/t_p$

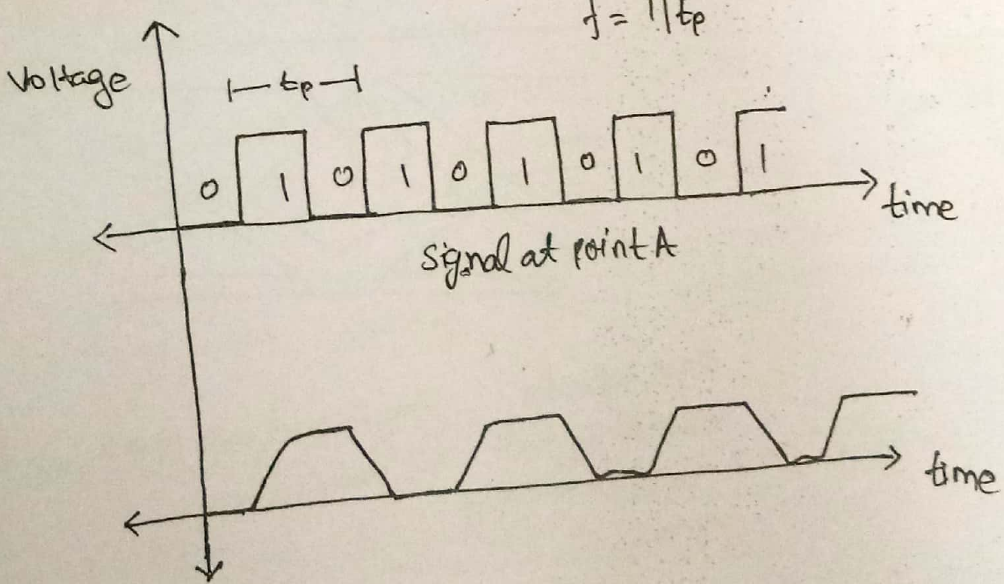


fig 1

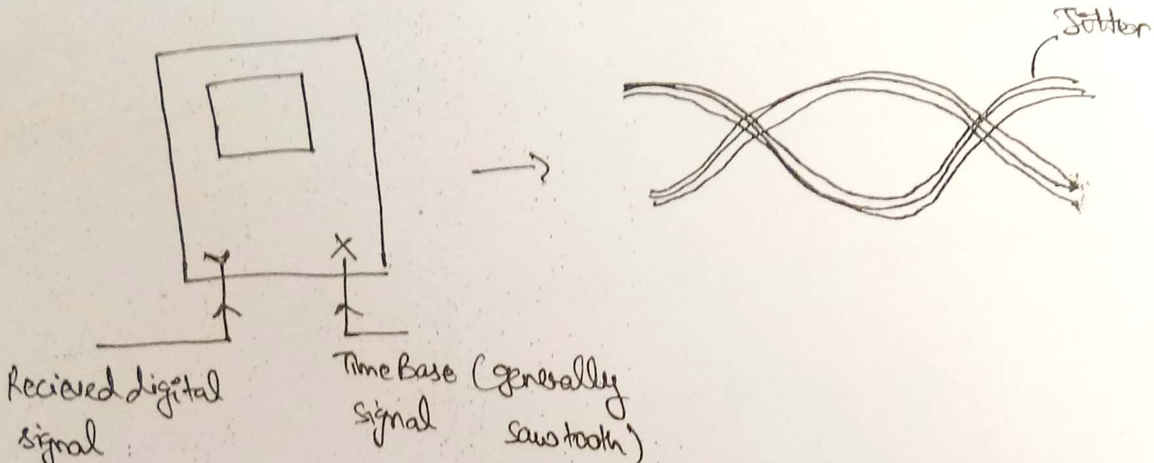
Signal at point B (distorted due to noise in the channel)

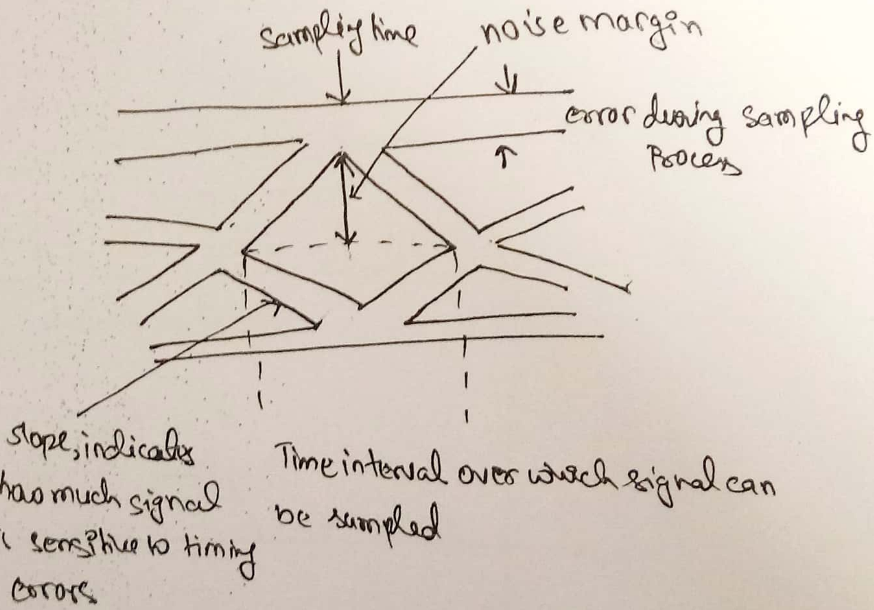
As the bandwidth decreases and the intersymbol interferance i the signal becomes distorted and the more the signal is distored of the eye pattern is observed on CRO.

* The ideal eye pattern have no band width constraint and also ha no inte...

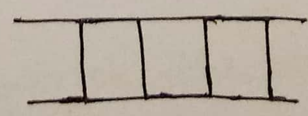
... is used to correct errors in ...

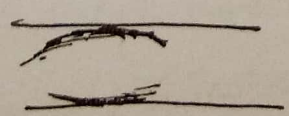
Now, if the bit rate of transmission is increased in fig 1, then signal point B becomes more close and the relative bits starts interfering with each other, and that is called intersymbol interferance.





for fig 1 signal,

f there is no interferance →  at CRO

ight interferance or noise → 

re noise →