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END TERM EXAMINATION

SECOND SEMESTER [BCA], MAY – 2011

Paper Code : BCA 102	Subject : Mathematics-II
Paper Id : 20102	

Time : 3 Hours Maximum Marks : 75

Note : Q.No.1 is compulsory. Attempt One question from each section.

Q. 1. (a) Write the power set of the set $A = \{1, 2, 3, 4\}$ and determine its cardinality. (3)

(b) Let $\mathbb{R}$ denote the set of all real numbers. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined as $f(x) = x^2$ for all $x \in \mathbb{R}$.
Prove that f is neither one-to-one nor onto. (3)

(c) Let $A = \{2, 3, 7, 8\}$, $B = \{1, 3, 5\}$ and $C = \{3, 5, 9, 11\}$. Verify $A \times (B \cap C) = (A \times B) \cap (A \times C)$. (3)

(d) Let R denote the relation on $N = \{1, 2, 3, ...\}$ where $R = \{(x,y) : x + 3y = 12\}$. Write R as a set of ordered pairs. Compute RoR. (3)

(e) Show that : $\int_0^\infty dx \int_x^\infty \frac{e^{-y}}{y} dy = \int_0^\infty \frac{e^{-y}}{y} dy \int_0^y dx = 1$. (3)

(f) The locus of points common to a sphere and a plane is a .... (1)

(g) Find the equation of the line through the point (1,2,3) parallel to (3)

SECTION - A

Q. 2. (a) Consider the set $S = \{1, 2, 3\}$. Write all partitions of S. **(3)**

(b) Let $A = \{1, 2, 3, 4\}$ and B denote the class of all those subsets of A which contain 2 and two other elements of A. Determine B. **(3.5)**

(c) Let $N=\{1, 2, 3, ...\}$ denote the set of all positive integers. For each positive integer $n \in N$, let

$D_n=\{n, 2n, 3n, ...\}$

denote the set of all positive multiples of n. If $D_6 \cap D_8 = D_x$ and $D_3 \cup D_{12} = D_y$, find x and y. **(6)**

Q. 3. (a) Given the relation R on $A = \{1, 2, 3\}$ where $R=\{(1,2), (2,3), (3,3)\}$. Compute the transitive closure of R. **(6.5)**

(b) Show that the relation $R = \{(1,1), (1,2), (2,1), (3,3), (2,2)\}$ is an equivalence relation on the set $A = \{1, 2, 3\}$. Determine the R-equivalence classes. **(6)**

SECTION - B

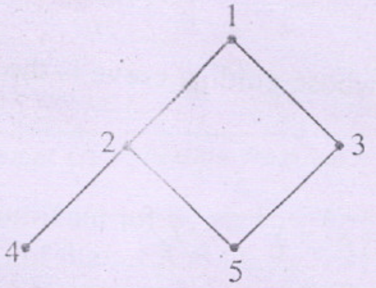
Q. 4. (a) Let N denote the set of all positive integers and $a \in N$, $b \in N$. Let a/b mean "a divides b" in the sense that there exists a positive integer c such that $b = ac$. Prove that " | " is a partial order on N. Let A={1,2,3,4,6,8,9,12,1824}. Draw the Hasse diagram with respect to the partial order " | ". Find any two chains in A. **(6.5)**

(b) Let $L = (L, \wedge, \vee)$ be a lattice under the binary operations meet $(\wedge)$ and join $(\vee)$. Prove that

(i) $a \wedge b = a$ if and only if $a \vee b = b$

(ii) the relation $\leq$ defined on L as $a \leq b$ if $a \wedge b = a$ is a partial order on L. **(6)**

Q. 5. (a) Consider the ordered set A with the Hasse diagram (edges slanting upwards)



Find all minimal and maximal elements of A. Does A have a first element as a last element.

Let $\lfloor(A)$ denote the collection of all linearly ordered subsets of A with two or more elements. Order $\lfloor(A)$ by set inclusion. Draw the Hasse diagram of $\lfloor(A)$. **(6.5)**

(b) Let P be a partially ordered set in which inf(a,b) and sup(a,b) exist for all a, b in P. Let $a \wedge b = \inf(a, b)$ and $a \vee b = (\sup(a,b)$. Prove that $(P, \wedge, \vee)$ is a lattice. **(6)**

SECTION - C

Q. 6. (a) If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, show that

(i) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$

(ii) $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = (1 - 4\sin^2 u)\sin 2u$ **(6.5)**

(b) Find the equations of the spheres which pass through the circle $x^2 + y^2 + z^2 - 2x + 2y + 4z - 3 = 0, 2x + y + z = 4$ and touch the plane $3x + 4y = 14$. **(6)**

Q. 7. (a) Find the equation of the cylinder whose generators are parallel to the line

$\frac{x}{1}=\frac{y}{-2}=\frac{z}{3}$ and whose guiding curve is the ellipse $x^2+2y^2=1$, $z=3$. **(6.5)**

(b) Examine $f(x,y)=x^3+y^3-3xy$ for maximum and minimum values. **(6)**

SECTION - D

Q. 8. (a) Evaluate : $\iint_S \sqrt{xy-y^2}\, dxdy$ where S denotes the triangle with vertices (0, 0), (10, 1) and (1,1). **(6.5)**

(b) Evaluate : $\iiint_A (x+y+z)\, dxdydz$ where

$$A=\{(x,y,z):0\le x\le 1, 1\le y\le 2, 2\le z\le 3\}.$$ **(6)**

Q. 9. (a) Evaluate : $\int_{\theta=0}^{\theta=\frac{\pi}{2}} \left\{ \int_{r=a(1-\cos\theta)}^{r=a} r^2 dr \right\} d\theta$ **(6.5)**

(b) Evaluate : $\int_0^{2\pi} d\phi \int_0^{\pi/4} \sin\theta\, d\theta \int_0^a r^2 dr.$ **(6)**

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